PS #1

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1 Classify a model from a journal

1.1 Part a & b

The model this report focuses on is Heterogeneous Agent New Keynesian(HANK) model from **Monetary Policy According to HANK** by Kaplan et al.(2018) published on *American Economic Review* VOL. 108, NO. 3, March 2018. The innovation of HANK model, contrasted to previous RANK(Representative Agent New Keynesian) and TANK(Two Agent New Keynesian) models, is that households face uninsurable idiosyncratic income risk which they can self-insure through two savings instrument with different degree of liquidity and return rates.

The mathematical foundation of this model is as following:

1.2 Part c: Model Setting

1.2.1 Household

$$\max_{\{c_t,l_t,k_t\}} E_0 \int_0^\infty e^{-(\rho+\zeta)t} u\left(c_t, \ell_t\right) dt$$

s.t.

$$\begin{aligned}
\dot{b}_t &= \underbrace{\left(1-\tau_t\right) w_t z_t \ell_t}_{\text{Income}} + \underbrace{r_t^b\left(b_t\right) b_t}_{\text{Capital}} + \underbrace{T_t}_{\text{Transfer}} - \underbrace{d_t}_{\text{Deposit}} - \underbrace{\chi\left(d_t, a_t\right)}_{\text{Transaction cost}} - \underbrace{c_t}_{\text{Consumption}} \\
\dot{a}_t &= r_t^a a_t + d_t \\
b_t &\geq -\underline{b}, \quad a_t \geq 0
\end{aligned}$$

where ζ is rate of an exodogenous death Poisson process,and upon death households give birth to an offspring with wealth a_0 and productivity $Z' \sim \mathcal{N}\left(0, \sigma^2\right)$. b refers to household's liquid asset with interest rate r^b, and a refers to household's illiquid asset with interest rate r^a. Labor earnings are taxed at rate τ.

Households maximize life-long utility by choosing decision path $\{c_t, l_t, k_t\}_{t\geq 0}$, taking as given the time path of prices and policies $\{\Gamma_t\}_{t\geq 0} := \left\{r_t^b, r_t^a, w_t, \tau_t, T_t\right\}_{t\geq 0}$.

Referring to methods in HACT[1], it is easy for us to construct the corresponding HJB and KF function.

HJB :

$$\begin{aligned}
(\rho+\zeta) V(a, b, y) = & \max_{c,\ell,d} u(c, \ell) + V_b(a, b, y)\left[(1-\tau) w e^y \ell + r^b(b) b + T - d - \chi(d, a) - c\right] \\
& + V_a(a, b, y)\left(r^a a + d\right) \\
& + V_y(a, b, y)(-\beta y) + \lambda \int_{-\infty}^{\infty}(V(a, b, x) - V(a, b, y)) \phi(x) dx
\end{aligned}$$

KF:

$$\begin{aligned}
0 = & -\partial_a\left(s^a(a, b, y) g(a, b, y)\right) - \partial_b\left(s^b(a, b, y) g(a, b, y)\right) \\
& - \partial_y(-\beta y g(a, b, y)) - \lambda g(a, b, y) + \lambda \phi(y) \int_{-\infty}^{\infty} g(a, b, x) dx \\
& - \zeta g(a, b, y) + \zeta \delta\left(a-a_0\right) \delta\left(b-b_0\right) g^*(y)
\end{aligned}$$

where δ is the Dirac delta function, (a_0, b_0) are starting assets and $g^*(y)$ is the stationary distribution of y. The last term implies that so as to have the equilibrium to be achieved, after the origin household dies, the new-born households, with initial wealth, should have the stationary distribution of income.

1.2.2 Producers

Final-Good Producers uses a continuum of intermediate inputs indexed by $j \in [0,1]$ and produce with constant elasticity of substitution ε (CES). *Intermediate Good Producers* produce intermediate good j in monopolistically competitive market. Households' illiquid assets can be invested into two assets: capital k_t and equity share of intermediate firms s_t, which implies $a_t = k_t + q_t s_t$, where q_t denotes the share price. Since households can switch costlessly between capital and equity, the return of both assets should be equal, which implies

$$\frac{\Pi_t + \dot{q}_t}{q_t} = r_t^k - \delta = r_t^a$$

And **this connects r_t^k with r_t^a**.

1.2.3 Monetary Authority

According to the Taylor Rule and the Fisher Equation:

$$r_t^b = i_t - \pi_t = \bar{r}^b + \phi\pi_t + \epsilon_t - \pi_t$$

And **this connects r_t^b with ϵ_t**.

1.2.4 Government

The governmental budget balance condition gives us:

$$\dot{B}_t^g + G_t + T_t = \tau_t \int w_t z \ell_t(a,b,z) d\mu_t + r_t^b B_t^g$$

where μ_t is the measure of households. (of distribution of income?)

1.2.5 Equilibrium

- Decisions: $\{a_t, b_t, c_t, d_t, \ell_t, n_t, k_t\}_{t\geq 0}$
- States: $\{\Gamma_t\}_{t\geq 0} := \left\{\omega_t, r_t^k, r_t^a, r_t^b, q_t, \pi_t, \tau_t, \boldsymbol{T}_t, \boldsymbol{G}_t, \boldsymbol{B}_t, \mu_t\right\}_{t\geq 0}$
  - Liquid asset clears: $B_t^h + B_t^g = 0$
  - Illiquid asset clears: $K_t + q_t = A_t$, with the total number of shares normalized to 1
  - good market clears: $Y_t = C_t + I_t + G_t + \Theta_t + \chi_t + \kappa \int \max\{-b, 0\} d\mu_t$, the last term reflects borrowing costs.

1.3 Monetary Transmission in HANK

Consider an expansionary monetary shock ($\epsilon_0 < 0$). We define aggregate consumption as

$$C_t\left(\{\Gamma_t\}_{t\geq 0}\right) = \int c_t\left(a, b, z; \{\Gamma_t\}_{t\geq 0}\right) d\mu_t$$

which can be total differentiated into:

$$dC_0 = \underbrace{\int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt}_{\text{direct effect } (\approx 19\%)} + \underbrace{\int_0^\infty \left(\frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial \tau_t} d\tau_t + \frac{\partial C_0}{\partial T_t} dT_t\right) dt}_{\text{indirect effects} (\approx 81\%)}$$

1.4 Part d - f

In this model, exogenous variables includes: monetary shock (η_0) and initial value of State variables
($\{\Gamma_t\}_{t\geq 0} := \left\{\omega_t, r_t^k, r_t^a, r_t^b, q_t, \pi_t, \tau_t, \boldsymbol{T}_t, \boldsymbol{G}_t, \boldsymbol{B}_t, \mu_t\right\}_{t\geq 0}$). This model is dynamic, nonlinear and stochastic model, since time and stochastic income jumps, monetary shock are involved.

In HANK model, productivity z varies between households, while the aggregation remains unchanged in time. Thus **stochastic aggregated Y_t(income) and Z_t(productivity)** can be taken for further consideration.

2 Make my own model

2.1 Part a - c

The model I made is a simple logistic model:

$$Y_i = g(\beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{3i} + T)$$

where

$$g(x) = \frac{1}{\exp(-x)}$$

and variables are defined as :

$$\begin{aligned} Y &:= \text{marital status of individual (1: married, 0: unmarried} \\ X_1 &:= \text{age of individual} \\ X_2 &:= \text{which country that individual from} \\ X_3 &:= \text{job of individual} \\ T &:= \text{time fixed effect} \end{aligned}$$

The data for variables above can be obtained by survey or demographic reports. X_1(age) is a variable with multiple discreet values. X_2 and X_3 are categorical variables, which can be included in the model with one-hot dummy variables. The model is a complete data generating process.

2.2 Part d

The key factors here are people's origin country and their jobs. People from different part of the world are under different marital convention and economic situation, their willingness to marry can be therefore heterogeneously influences. [2] The job factor is also important since people under diverse workload may have different amount of time to date or meet new people, and the sex ratio of workplace may also influence the odd of getting married.

2.3 Part e

The other factors, such as salary and appearance are also possible factors. The reason that they are not included in the model is that the data of wealth can be inauthentic, since people may tend to boost their financial situation; the appearance, on the other hand, is hard to measure.

2.4 Part f

Firstly, we need to collect a dataset containing over 100 individuals' data. Each individual's data includes marital status, age, original country and job indicator. Then, run the logistic regression of the marriage variable on the other three independent variables with time fixed effect. We can see from the p-value whether the three variables are significant or not. We can also use scatterplots matrix to see the relationship.

References

[1] Yves Achdou, Jiequn Han, Jean-Michel Lasry, Pierre-Louis Lions, and Benjamin Moll. Income and wealth distribution in macroeconomics: A continuous-time approach. Working Paper 23732, National Bureau of Economic Research, August 2017.

[2] Susheela Singh and Renee Samara. Early marriage among women in developing countries. *International Family Planning Perspectives*, 22(4):148–175, 1996.