Machine Learning Exercise Sheet 1

Math Refresher

Group_369

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November 10, 2021

Problem 9

The MAP estimation of the parameter λ is:

$$\begin{aligned}
\lambda_{MAP} &= \arg\max_{\lambda} \mathrm{p}(\lambda \mid x, a, b) \\
&= \arg\max_{\lambda} \log \mathrm{p}(\lambda \mid x, a, b) \\
&= \arg\max_{\lambda} \log(\mathrm{p}(x \mid \lambda)\, \mathrm{p}(\lambda \mid a, b)) \\
&= \arg\max_{\lambda} \log(\frac{b^a \lambda^{a-1} \exp(-b\lambda)}{\Gamma(a)} \frac{\lambda^x \exp(-\lambda)}{x!}) \\
&= \arg\max_{\lambda} (a - 1 + x) \log \lambda - (b + 1)\lambda + const
\end{aligned}$$

In order to maximize the function, compute the derivative:

$$\frac{\partial}{\partial \lambda}((a - 1 + x) \log \lambda - (b + 1)\lambda + const) = \frac{a - 1 + x}{\lambda} - b - 1 \stackrel{!}{=} 0$$

Then we get

$$\lambda = \frac{x + a - 1}{b + 1}$$

Hence $\lambda_{MAP} = \frac{x+a-1}{b+1}$.