

Machine Learning Exercise Sheet 1

Math Refresher

Group_369

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Problem 7

Let $\phi(x_1, x_2) = x_1 x_2$, we can observe that for all crosses $\phi(x_1, x_2) \leq 0$ and for all circles $\phi(x_1, x_2) \geq 0$, which means it is linearly separable. We can separate the crosses and circles with a single hyperplane $\phi(x_1, x_2) = 0$.

Problem 8

On the boundry Γ , the \mathbf{x} must realize

$$p(y = 1 | \mathbf{x}) = p(y = 0 | \mathbf{x})$$

It is equivalent to

$$\log \frac{p(y = 1 | \mathbf{x})}{p(y = 0 | \mathbf{x})} = 0$$

We expand

$$\begin{aligned} \log \frac{p(y = 1 | \mathbf{x})}{p(y = 0 | \mathbf{x})} &= \frac{\frac{1}{(2\pi)^{\frac{D}{2}} |\Sigma_1|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_1)^T \Sigma_1^{-1} (\mathbf{x} - \boldsymbol{\mu}_1)} \cdot \pi_1}{\frac{1}{(2\pi)^{\frac{D}{2}} |\Sigma_0|^{\frac{1}{2}}} e^{-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu}_0)^T \Sigma_0^{-1} (\mathbf{x} - \boldsymbol{\mu}_0)} \cdot \pi_0} \\ &= \frac{1}{2} \mathbf{x}^T (\Sigma_0^{-1} - \Sigma_1^{-1}) \mathbf{x} + \mathbf{x}^T (\Sigma_1^{-1} \boldsymbol{\mu}_1 - \Sigma_0^{-1} \boldsymbol{\mu}_0) \\ &\quad - \frac{1}{2} \boldsymbol{\mu}_1^T \Sigma_1^{-1} \boldsymbol{\mu}_1 + \frac{1}{2} \boldsymbol{\mu}_0^T \Sigma_0^{-1} \boldsymbol{\mu}_0 + \log \frac{\pi_1}{\pi_0} + \frac{1}{2} \log \frac{|\Sigma_0|}{|\Sigma_1|} \\ &= \mathbf{x}^T \mathbf{A} \mathbf{x} + \mathbf{b}^T \mathbf{x} + c \end{aligned}$$

where we define

$$\begin{aligned}\mathbf{A} &= \frac{1}{2} (\boldsymbol{\Sigma}_0^{-1} - \boldsymbol{\Sigma}_1^{-1}) \\ \mathbf{b} &= \boldsymbol{\Sigma}_1^{-1} \boldsymbol{\mu}_1 - \boldsymbol{\Sigma}_0^{-1} \boldsymbol{\mu}_0 \\ c &= -\frac{1}{2} \boldsymbol{\mu}_1^T \boldsymbol{\Sigma}_1^{-1} \boldsymbol{\mu}_1 + \frac{1}{2} \boldsymbol{\mu}_0^T \boldsymbol{\Sigma}_0^{-1} \boldsymbol{\mu}_0 + \log \frac{\pi_1}{\pi_0} + \frac{1}{2} \log \frac{|\boldsymbol{\Sigma}_0|}{|\boldsymbol{\Sigma}_1|}\end{aligned}$$