Supplementary Material for "Congestion-Aware Rebalancing and Vehicle-to-Grid Coordination of Shared Electric Vehicles: An Aggregative Game Approach"

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In the supplementary material, we give the detailed information of the numerical examples and proof for Proposition 4.1 in the paper "Congestion-Aware Rebalancing and Vehicle-to-Grid Coordination of Shared Electric Vehicles: An Aggregative Game Approach".

I. Simulation Setting

A. Coupled Power and Transportation Networks

Figure S1 shows the network of Nguyen–Dupuis (N-D) transportation network that contains 13 vertices and 19 bidirectional road segments. The N-D transportation network is coupled with a 33-node power distribution network, as shown in Fig. S1 [S1].

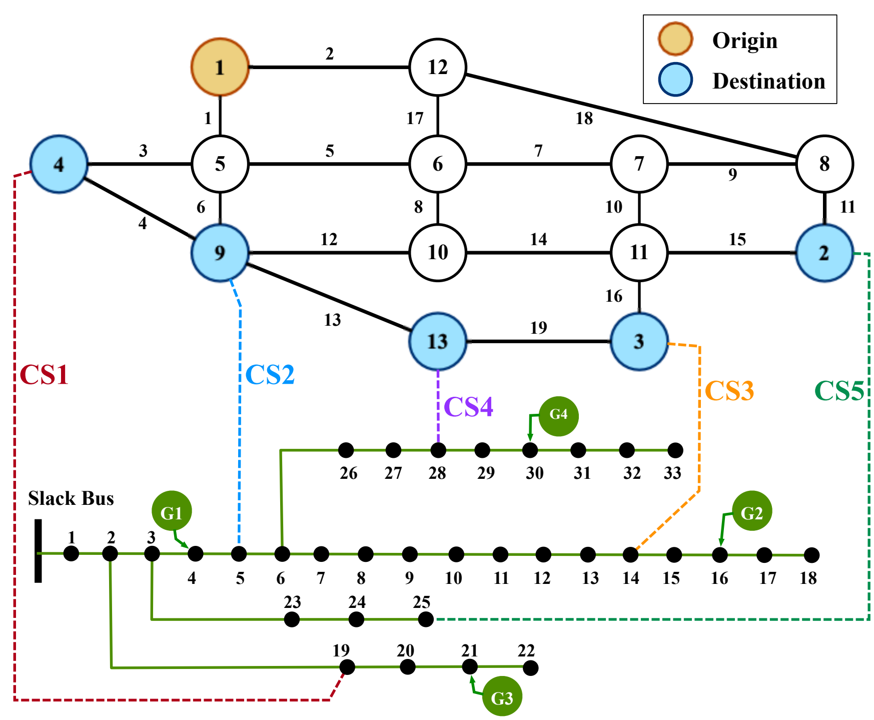

Fig. S1. The Nguyen-Dupuis transportation network coupled with the 33-bus power distribution network. The coupling relationship in the networks is denoted in dashes.

There are 5 charging stations (CSs) located in the coupled systems and their connection with the transportation and power networks is denoted by the dashes in Fig. S1 and presented in Table S1.

TABLE S1. Coupling relationship between the power and transportation networks (CS: Charging Station).

No.	Node in Transportation Network	Bus of Power Network
CS 1	4	19
CS 2	9	5
CS 3	3	14
CS 4	13	28
CS 5	2	25

The EVs depart from Node 1 (denoted as the orange one in Fig. S1), and reach the destination at Nodes 4, 9, 13, 3, 2 (denoted as the blue ones in Fig. S1). The origin-destination pair is listed in Table S2.

TABLE S2. Origin-destination pairs.

No.	Origin	Destination
1	1	4
2	1	9
3	1	3
4	1	13
5	1	2

The travel demand originating from Node 1 over a twenty-four-hour period is comparable to an average day in Beijing [S2], as illustrated in Fig. S2 and Table S3.

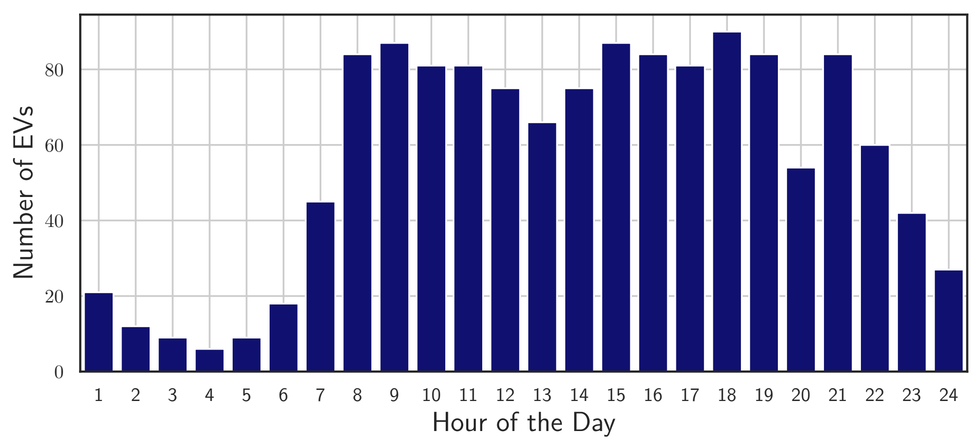

Fig. S2. The number of EVs departing from Node 1 for twenty-four hours of a day.

TABLE S3. The number of EVs departing from Node 1 for twenty-four hours of a day.

Time periods	1	2	3	4	5	6	7	8	9	10	11	12
Number of vehicles	21	12	9	6	9	18	45	84	87	81	81	75
Time periods	13	14	15	16	17	18	19	20	21	22	23	24
Number of vehicles	66	75	87	84	81	90	84	54	84	60	42	27

B. Other Parameter Setting

In the base case, equal preference, denoted as $\tilde{\mathbf{w}}$, is assigned to each destination. For instance, in our simulation with 5 charging stations, the probability of preference for selecting each charging station is 0.2. Based on this probability of destination choice, equal values are assigned to the preference for road segments, denoted as $\tilde{\mathbf{v}}$. In other words, in the base case scenario, drivers have the same level of preference for all destinations and road segments.

To account for the heterogeneity among EV drivers, the parameters for travel time cost, rebalancing utility, and discharging utility are randomly generated. Specifically, the parameters $\omega_i^{\text{time}}, \omega_i^{\text{reb}}, \beta_{1,i}, \beta_{2,i}$ are assigned random values.

II. Proof for Proposition 4.1

For ease of illustration, **Proposition 4.1** in the manuscript is presented as follows:

Furthermore, the game Ξ has the following properties:

(a) For each EV driver i, the individual constraint set Ω_i is non-empty, closed and convex.

(b) The aggregate cost function $f_i(\mathbf{x}_i, \text{avg}(\mathbf{x}))$ is continuously differentiable in $\mathbf{x}$ and convex in $\mathbf{x}_i$ for any fixed $\text{avg}(\mathbf{x})$.

(c) The global set $\mathcal{X}$ in (1) is nonempty and satisfies Slater's constraint qualification [S3].

$$\mathcal{X} := \prod_{i=1}^{N} \Omega_i \bigcap \mathcal{C} \tag{1}$$

- First, for **Proposition 4.1(a):** The set Ω_i denotes the EV driver's local decision set:

$$\Omega_i = \{\mathbf{x}_i | \mathbf{x}_i \text{ satisfies (3)-(4)}\} \tag{2}$$

where (3) denotes the traffic flow continuity constraint and (4) denotes the non-negative constraint, shown as follows:

$$\sum_{e:(j,k)\in\mathcal{E}} v_i^e - \sum_{e:(k,m)\in\mathcal{E}} v_i^e = \begin{cases} -1 & \text{if } k = o_i, \\ w_i^d & \text{if } k = d, \\ 0 & \text{otherwise} \end{cases} \qquad \forall k \in \mathcal{V},\ \forall d \in \mathcal{D} \tag{3}$$

$$\mathbf{v}_i \in [0,1]^{|\mathcal{V}|},\ \mathbf{w}_i \in [0,1]^{|\mathcal{D}|}, \qquad \forall i \in \mathcal{N} \tag{4}$$

By (4) and definition $\mathbf{x}_i := \text{col}\{(\mathbf{v}_i, \mathbf{w}_i)\}$, it is easy to deduce that Ω_i is non-empty and closed. Observe that (3)-(4) are both linear constraints. Therefore, Ω_i is a convex set. As such, for each EV driver i, the individual constraint set Ω_i is non-empty, closed and convex.

- Second, for **Proposition 4.1(b)**, to prove that $f_i(\mathbf{x}_i, \text{avg}(\mathbf{x}))$ is continuously differentiable in $\mathbf{x}$ and convex in $\mathbf{x}_i$ for any fixed $\text{avg}(\mathbf{x})$, we first present the formulation of $f_i(\mathbf{x}_i, \text{avg}(\mathbf{x}))$ as follows:

$$f_i(\mathbf{x}_i, \text{avg}(\mathbf{x})) = -\omega_i^{\text{v2g}} \left(\beta_{1,i} q_i - \beta_{2,i}/2 \, (q_i)^2 + \sum_{d\in\mathcal{D}} \lambda_d w_i^d q_i \right) \tag{5}$$

$$+ \omega_i^{\text{dev}} \left(\frac{\alpha_{1,i}}{2} \|\mathbf{v}_i - \tilde{\mathbf{v}}_i\|^2 + \frac{\alpha_{2,i}}{2} \|\mathbf{w}_i - \tilde{\mathbf{w}}_i\|^2 \right) \tag{6}$$

$$+ \omega_i^{\text{time}} \left(\sum_{e\in\mathcal{E}} t_e(\phi_e(v_e)) \, v_i^e \right) + \left(-\omega_i^{\text{reb}}\right) \left(-\sum_{d\in\mathcal{D}} \kappa_d \varphi_d(\mathbf{w}_d) w_i^d \right) \tag{7}$$

where $\mathbf{x}_i = \mathrm{col}\{(\mathbf{v}_i, \mathbf{w}_i)\}$ and $\mathrm{avg}(\mathbf{x})$ is fixed, i.e., $t_e(\phi_e(v_e))$ and $\varphi_d(\mathbf{w}_d)$ are fixed. Observe that $f_i(\mathbf{x}_i, \mathrm{avg}(\mathbf{x}))$ with fixed $\mathrm{avg}(\mathbf{x})$ is the summation of quadratic (with positive coefficients) and linear terms. Hence, $f_i(\mathbf{x}_i, \mathrm{avg}(\mathbf{x}))$ with fixed $\mathrm{avg}(\mathbf{x})$ is a convex function.
Moreover, a differentiable function f is continuously differentiable if and only if f is of differentiability class C^1, That is, if the first order derivative of f is continuous [**?**]. By Lemma 5.2, the pseudo-gradient of $f_i(\mathbf{x}_i, \mathrm{avg}(\mathbf{x}))$ is $F(\mathbf{x})$:

$$F(\mathbf{x}) := \begin{bmatrix} \omega_i^{\text{dev}}\alpha_{1,i}(\mathbf{v}_i - \tilde{\mathbf{v}}_i) + \omega_i^{\text{time}} t(\phi(\mathbf{v})) \\ \omega_i^{\text{dev}}\alpha_{2,i}(\mathbf{w}_i - \tilde{\mathbf{w}}_i) + \omega_i^{\text{reb}}\kappa_d\varphi_d(\mathbf{w}_d) + \lambda_d q_i \end{bmatrix}_{i=1}^{N} \tag{8}$$

Functions $t(\phi(\mathbf{v}))$ and $\varphi_d(\mathbf{w}_d)$ are continuous, and other linear terms are continuous. Hence, $F(\mathbf{x})$ is continuous and $f_i(\mathbf{x}_i, \mathrm{avg}(\mathbf{x}))$ is continuously differentiable.

- Third, for **Proposition 4.1(c)**, the global set $\mathcal{X}$ in (9) is nonempty and satisfies Slater's constraint qualification [S3].

$$\mathcal{X} := \prod_{i=1}^{N} \Omega_i \bigcap \mathcal{C} \tag{9}$$

where Ω_i is defined in (2) and the coupling constraint set $\mathcal{C}$ is defined as:

$$\mathcal{C} := \mathcal{C}^v \times \mathcal{C}^w, \quad \mathcal{C}^v = \left\{ v \mid \phi_e(\mathbf{v}_e) \le C_e,\ \forall e \in \mathcal{E} \right\}, \quad \mathcal{C}^w = \left\{ w \;\middle|\; \varphi_d(\mathbf{w}_d) \le C_d,\ \forall d \in \mathcal{D} \right\} \tag{10}$$

Slater's constraint qualification is a sufficient condition for strong duality to hold for a convex optimization problem [S4]. Informally, Slater's condition states that the feasible region must have an interior point. It can be deduced that set (9) is non-empty and has an interior point.

References

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