向量组，方程组，矩阵，线性空间试题（1-2）

线性相关与线性无关，向量组的秩

1.已知$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性无关，且$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n,\overrightarrow{\beta}$ 线性相关，则$\overrightarrow{\beta}$可由$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性表出.

2.已知$\overrightarrow{\beta}$可由$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性表出，则表示法唯一的充要条件是$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性无关.

3.已知$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性无关，且$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n,\overrightarrow{\beta},\overrightarrow{\gamma}$ 线性相关，证明：要么$\overrightarrow{\beta}$ 或$\overrightarrow{\gamma}$可由$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性表出，要么$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n,\overrightarrow{\beta}$ 与$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n,\overrightarrow{\gamma}$ 等价.

4.已知$\overrightarrow{\alpha}_1\neq 0$，则$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性相关的充要条件是存在$i(2\leq i\leq n)$使得$\overrightarrow{\alpha}_i$可由 $\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_{i-1}$线性表出.

5.设$\mathscr{A}$是线性空间V上的线性变换，如果$\mathscr{A}^{k-1}\xi\neq 0$，但$\mathscr{A}^k\xi=0$，则 $\xi,\mathscr{A}\xi,\cdots,\mathscr{A}^{k-1}\xi(k>0)$线性无关.

6.（1）设$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_r$ 可以被$\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_s$线性表出，且$r>s$，则$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_r$必然线性相关的.

（2）已知$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_r$ 线性无关，且可以被$\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_s$线性表出，则有$s\geq r$.

7.任意$n+1$个n维向量都是线性相关的.

8.向量组I可由向量组II线性表出，则I 的秩小于等于II的秩.

9.已知$\overrightarrow{\beta}_i=a_{1i}\overrightarrow{\alpha}_1+\cdots+a_{ni}\overrightarrow{\alpha}_n(i=1,2,\cdots,n)$，且$A=(a_{ij})$是可逆矩阵，则$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$与 $\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_n$是等价的.

10.已知$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$ 线性无关，$\overrightarrow{\beta}_i=a_{1i}\overrightarrow{\alpha}_1+\cdots+a_{ni}\overrightarrow{\alpha}_n(i=1,2,\cdots,n)$，记$A=(a_{ij})$，则 $\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_n$线性无关的充要条件是$A$可逆.

11.已知$\overrightarrow{\beta}_1=\overrightarrow{\alpha}_2+\overrightarrow{\alpha}_3,\overrightarrow{\beta}_2=\overrightarrow{\alpha}_1+\overrightarrow{\alpha}_3,\overrightarrow{\beta}_3=\overrightarrow{\alpha}_1+\overrightarrow{\alpha}_2$，则$\overrightarrow{\alpha}_1,\overrightarrow{\alpha}_2,\overrightarrow{\alpha}_3$ 与$\overrightarrow{\beta}_1,\overrightarrow{\beta}_2,\overrightarrow{\beta}_3$ 等价.

12.证明当n是奇数时，$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性无关的充要条件是$\overrightarrow{\alpha}_1+\overrightarrow{\alpha}_2,\cdots,\overrightarrow{\alpha}_{n-1}+\overrightarrow{\alpha}_n,\overrightarrow{\alpha}_n+\overrightarrow{\alpha}_1$ 线性无关.

13.设$\overrightarrow{\beta}_1=\overrightarrow{\alpha}_2+\overrightarrow{\alpha}_3+\cdots+\overrightarrow{\alpha}_n,\overrightarrow{\beta}_2=\overrightarrow{\alpha}_1+\overrightarrow{\alpha}_3+\cdots+\overrightarrow{\alpha}_n,,\cdots,\overrightarrow{\beta}_1=\overrightarrow{\alpha}_1+\overrightarrow{\alpha}_2+\cdots+\overrightarrow{\alpha}_{n-1}$，证明:$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$与$\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_n$有相同的秩.

14.已知向量组$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$的秩为$r$，则$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$ 中的任意r个线性无关的向量都构成它的一个极大线性无关组.

15.设向量组$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$的秩为$r$，且$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$ 可由其中r个向量$\overrightarrow{\alpha}_{i_1},\cdots,\overrightarrow{\alpha}_{i_r}$ 线性表出，那么$\overrightarrow{\alpha}_{i_1},\cdots,\overrightarrow{\alpha}_{i_r}$就是$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$的一个极大线性无关组.

16.在n维空间P^n中，n个向量$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$线性无关的充要条件是$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_n$可以线性表出$P^n$中的任一向量.

17.设$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_s$与 $\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_t$是两个秩相同的向量组，且$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_s$可被 $\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_t$线性表出，则$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_s$与 $\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_t$等价.

18.已知向量组$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_r$与 $\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_r,\overrightarrow{\alpha}_{r+1},\cdots,\overrightarrow{\alpha}_n$ 有相同的秩，证明这两个向量组等价.

19.已知A,B是两个同级矩阵，则$r(A+B)\leq r(A)+r(B)$.

20.设$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_r$线性无关，且可以被$\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_n$线性表出，则可以从$\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_n$中选出$r$个向量替换成$\overrightarrow{\alpha}_1,\cdots,\overrightarrow{\alpha}_r$后，得到的新向量组与$\overrightarrow{\beta}_1,\cdots,\overrightarrow{\beta}_n$等价.