Shell Investigation

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1 Thin and Thick Shells, Adaptive Meshing

The start of the shell research could be the Discrete Shell[GHDS03]. They use the angle between the faces to depict the energy. However, this kind of thing highly depends on the structure of the mesh. In this case, an independent-mesh shell simulation could be attractive.

Later on, there are several exciting meshing works done by the group at UC Berkeley and Gatech. They did a lot about numerical simulation and discussed the relationship between the mesh and the simulation solution. The papers in this period are Dynamic Local Remeshing for Elastoplastic Simulation[WRK+10], Adaptive Anisotropic Remeshing for Cloth Simulation[NSO12], A finite element method for animating large viscoplastic flow[BWHT07]. Actually, an early adaptive meshing work has been proposed as a rendering framework[ZSS97].

After this, membrane shell deformation work had been actively researched. Such as Fine Wrinkling on Coarsely Meshed Thin Shells[CCK+21], Physical Simulation of Environmentally Induced Thin Shell Deformation[CSvRV18].

Recently, some interesting papers have been published. With the IPC, there is some new work done for the adaptive meshing and in-timestep Remeshing for Contacting Elastodynamics[FSKP23]. There is a multi-grid method-liked topic, has continually published 3 papers, began with Progressive simulation for cloth quasistatics[ZDF+22].

High-order shell simulation also is one possible direction: Second-Order Finite Elements for Deformable Surfaces[LDB+23].

Using the volumetric tetrahedron thin shell, an amazing topics was presented last year: Kirchhoff-Love Shells with Arbitrary Hyperelastic Materials[WB23].

Capturing the wrinkle is still an unsolved topic: Multi-Layer Thick Shells[CXY+23].

1.1 Thin Shell Modeling

1.1.1 Energy Representation

The main focus on the energy should be the wrinkling energy.

Discrete Shell[GHDS03] used three terms to depict the energy of the shell.

$$\begin{cases} E = k_b W_b(x) + k_l W_l(x) + k_A W_A(x) \\ W_b(x) = \sum_e (\tilde{\theta} - \theta)^2 \|\tilde{\theta}\| \\ W_l(x) = \sum_e (1 - \frac{\|e\|}{\|\tilde{e}\|}) \|\tilde{e}\| \\ W_A(x) = \sum_A (1 - \frac{\|A\|}{\|\tilde{A}\|}) \|\tilde{A}\| \end{cases}$$

However, as said in the paper, this method highly depends on the mesh structure.

Later on, a simpler formula was presented in [BMF05], In this paper, they explicitly depicted the velocity and the gradient for the thin shell. With the Newmark scheme, this method is unconditionally stable.

After these, some similar papers were published, but their main ideas are basically the same, just change the energy term or find a shortcut to simplify the implementation. In a nutshell, the state of the old art seems to be the Discrete Shell.

Compared with the thin shell. One can expect to use the volumetric mesh to simulate the thick shell. For the volumetric mesh, one can expect to solve the whole linear system with the basic elastic FEM method. However, this process is slow and therefore people have begun to use the multigrid method to accelerate this process.

1.2 High Order Model and Semi-Volumetric Model

Maybe the self-intersection?

1.3 Progressive Simulation Model

From the textbook [SBS94], there is no guarantee that the non-Poisson problem will converge within a constant iteration rate. Therefore, for a long time, I guess few people tried the multigrid method to solve the energy minimization problem for this reason.

Recently, [ZDF+22] tried to use the multigrid method to solve the thin shell problem. The main ideas behind this paper are safe prolongation and solving locking issues. Both of these use the same pattern, using the finer gird to make the solution more accurate. For safe prolongation, one can expect to first use the finest grid to do CCD to compute the potential collision, then make sure the results from prolongation are collision-free. For the locking issues, the author used the finer grid to measure the energy and then restricted it to the current coarse grid. The second one is not obvious but surprised to know. I think Zhang and Liu have a good connection with each other since they worked on the multigrid shell paper before, which will appear in the next section, and the previous work definitely inspired her in this paper.

For the whole skeleton, the progressive simulation on the thin shell model just uses the current techniques to handle the simulation. The energy consists of membrane bending, stretching energy, IPC friction, and collision energy. In addition, inertial energy will be applied to make the optimization problem more stable.

2 Multigrid Building

The multigrid method is a common strategy to accelerate the converge rate to make the numerical computation quickly. The popular scheme for the iteration would be the V, F, and W schemes. In this section, the main problem I need to figure out is how to build a feasible setting to compute the multigrid system.

2.1 Volumetric Mesh

I found one related paper[ZSTB10] for the volumetric mesh in the Computer Graphics field. In this paper, they used the voxelized mesh to do the simulation. This means, that maybe using the pure multigrid method in shell simulation is possible but needs to have a good mesh decomposition method to extract a good hierarchy mesh structure. I will discuss the mesh decomposition method in the next section. For the multigrid building strategy, they just partitioned the grid roughly.

Another one applied the multigrid method to the tetrahedral mesh [GW06]. This paper did propose a multigrid framework for simulation. However, in this paper, the author used the barycentric interpolation between a fine mesh and a coarse mesh, this method is not bijective and highly depends on the mesh quality. For example, the fine grid vertices could be outside the coarse mesh, making the basis function's coefficient a negative value. Besides, the author assumed a fine mesh and a coarse mesh are given.

2.2 Thin Shell Mesh

Surface multigrid via intrinsic prolongation, an exciting paper [LZBCJ21] has been published recently, which induced more people to restart to focus on the multigrid method. In this paper, the author's main idea is to decimate the mesh and encode the information to help the later mapping operation. Another tiny modification from their previous work is making the self-parameterization become an optimizing problem to reduce the distortion energy, but it seems the result doesn't boost much.

Another similar paper, Bijective projection in a shell, is proposed by Jiang[JSZP20]. This paper also built a bijective and non-self-collision shell mesh without a hierarchy structure. Even the method in Jiang's paper is slower than the previous one. But the mesh property is better than Liu's and the algorithm also can be adaptive to the multigrid method.

2.3 Point Cloud

3 Mesh Decomposition

From the previous section, I realized that mesh decomposition could be a solution to avoid decimation. Also, the mapping is absolutely bijective! However, I need to figure out how to do an approximation to make the integration easier...

4 Numerical Problem and Meshing Cure

Previous Conclusion: Volumetric mesh is better than a thin shell when the number of the shell is larger than xxx. This should be a ratio. Let's dive into this more deeply.

5 Discussion on Bijective Mapping and Polygon Integration

5.1 Discussion on the Mesh Independency

I had a guess. If the object is not too stiff, then we can safely partition the shell system into n subsystems and do a smooth step afterward. The result will not be bad.

5.2 Self-Parameterization

The default method for self-parameterization could be something like LSCM. Also, the self-collision could happen, it highly depends on the mesh.

References

[BMF05] Robert Bridson, Sebastian Marino, and Ronald Fedkiw. Simulation of clothing with folds and wrinkles. In *ACM SIGGRAPH 2005 Courses*, pages 3–es. 2005.

[BWHT07] Adam W Bargteil, Chris Wojtan, Jessica K Hodgins, and Greg Turk. A finite element method for animating large viscoplastic flow. *ACM transactions on graphics (TOG)*, 26(3):16–es, 2007.

[CCK+21] Zhen Chen, Hsiao-Yu Chen, Danny M Kaufman, Mélina Skouras, and Etienne Vouga. Fine wrinkling on coarsely meshed thin shells. *ACM Transactions on Graphics (TOG)*, 40(5):1–32, 2021.

[CSvRV18] Hsiao-Yu Chen, Arnav Sastry, Wim M van Rees, and Etienne Vouga. Physical simulation of environmentally induced thin shell deformation. *ACM Transactions on Graphics (TOG)*, 37(4):1–13, 2018.

[CXY+23] Yunuo Chen, Tianyi Xie, Cem Yuksel, Danny Kaufman, Yin Yang, Chenfanfu Jiang, and Minchen Li. Multi-layer thick shells. In *ACM SIGGRAPH 2023 Conference Proceedings*, pages 1–9, 2023.

[FSKP23] Zachary Ferguson, Teseo Schneider, Danny Kaufman, and Daniele Panozzo. In-timestep remeshing for contacting elastodynamics. *ACM Transactions on Graphics (TOG)*, 42(4):1–15, 2023.

[GHDS03] Eitan Grinspun, Anil N Hirani, Mathieu Desbrun, and Peter Schröder. Discrete shells. In *Proceedings of the 2003 ACM SIGGRAPH/Eurographics symposium on Computer animation*, pages 62–67. Citeseer, 2003.

[GW06] Joachim Georgii and Rüdiger Westermann. A multigrid framework for real-time simulation of deformable bodies. *Computers & Graphics*, 30(3):408–415, 2006.

[JSZP20] Zhongshi Jiang, Teseo Schneider, Denis Zorin, and Daniele Panozzo. Bijective projection in a shell. *ACM Transactions on Graphics (TOG)*, 39(6):1–18, 2020.

[LDB+23] Qiqin Le, Yitong Deng, Jiamu Bu, Bo Zhu, and Tao Du. Second-order finite elements for deformable surfaces. In *SIGGRAPH Asia 2023 Conference Papers*, pages 1–10, 2023.

[LZBCJ21] Hsueh-Ti Derek Liu, Jiayi Eris Zhang, Mirela Ben-Chen, and Alec Jacobson. Surface multigrid via intrinsic prolongation. *arXiv preprint arXiv:2104.13755*, 2021.

[NSO12] Rahul Narain, Armin Samii, and James F O'brien. Adaptive anisotropic remeshing for cloth simulation. *ACM transactions on graphics (TOG)*, 31(6):1–10, 2012.

[SBS94] C Susanne, L Brenner, and LR Scott. The mathematical theory of finite element methods. *Texts in Applied Mathematics*, 15, 1994.

[WB23] Jiahao Wen and Jernej Barbič. Kirchhoff-love shells with arbitrary hyperelastic materials. *ACM Transactions on Graphics (TOG)*, 42(6):1–15, 2023.

[WRK+10] Martin Wicke, Daniel Ritchie, Bryan M Klingner, Sebastian Burke, Jonathan R Shewchuk, and James F O'Brien. Dynamic local remeshing for elastoplastic simulation. *ACM Transactions on graphics (TOG)*, 29(4):1–11, 2010.

[ZDF+22] Jiayi Eris Zhang, Jérémie Dumas, Yun Fei, Alec Jacobson, Doug L James, and Danny M Kaufman. Progressive simulation for cloth quasistatics. *ACM Transactions on Graphics (TOG)*, 41(6):1–16, 2022.

[ZSS97] Denis Zorin, Peter Schröder, and Wim Sweldens. Interactive multiresolution mesh editing. In *Proceedings of the 24th annual conference on Computer graphics and interactive techniques*, pages 259–268, 1997.

[ZSTB10] Yongning Zhu, Eftychios Sifakis, Joseph Teran, and Achi Brandt. An efficient multigrid method for the simulation of high-resolution elastic solids. *ACM Transactions on Graphics (TOG)*, 29(2):1–18, 2010.