Minimization of DFA

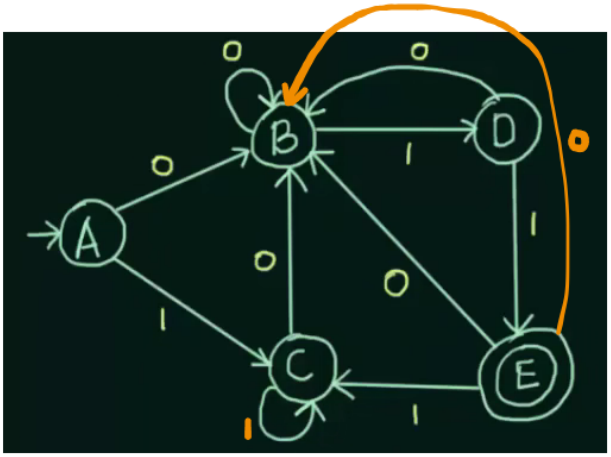
Why is this important?

Say you have a DFA of 5 states, you can minimize it and it would have 4 states by the end.

How?

By combining

Ex: 00000 → 0000



Step 1: Draw the DFA table if not provided

	0	1
→A	B	C
B	B	D
C	B	C
D	B	E
E	B	C

Step 2: Cite the equivalences, always start from 0

NOTE:

In every 0 equivalence, always group each state except the final state together

0 eq.:

{A,B,C,D}{E}

1st eq.:

{A,B,C}{D}{E}

2nd eq.:

{A,C}{B}{D}{E}

3rd eq.:

{A,C}{B}{D}{E}

Stop when the current equivalence is the same as the last one

Step 3: Construct the minimized DFA table based on the last equivalence

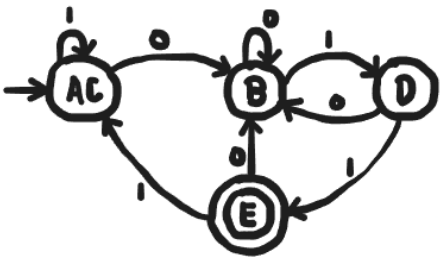
	0	1
→AC	B	AC
B	B	D
D	B	E
*E	B	AC

C is now a combined state

Similar rules with NFA->DFA apply in determining if the combined state is a final or starting state

Bonus Step

Step 4: If asked, construct the newly minimized DFA diagram



Construct a minimum DFA equivalent to the DFA described by

	0	1
→q_0	q_1	q_5
q_1	q_6	q_2
(*)q_2	q_0	q_2
q_3	q_2	q_6
q_4	q_7	q_5
q_5	q_2	q_6
q_6	q_6	q_4
q_7	q_6	q_2

0 equivalence:

$\{q_0, q_1, q_3, q_4, q_5, q_6, q_7\}\ \{q_2\}$

1st equivalence:

$\{q_0, q_4, q_6\}\ \{q_1, q_7\}\ \{q_3, q_5\}\ \{q_2\}$

2nd equivalence:

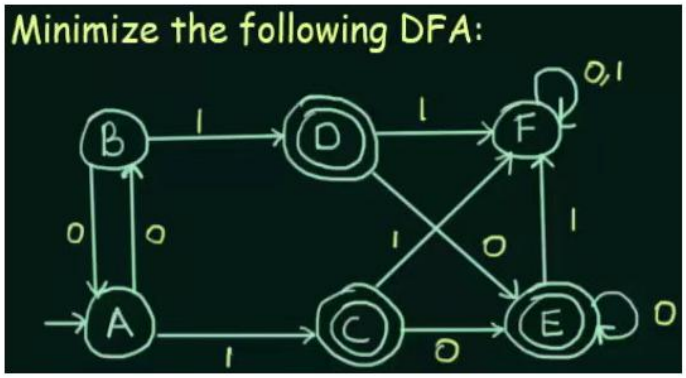
$\{q_0, q_4\}\ \{q_1, q_7\}\ \{q_3, q_5\}\ \{q_6\}\ \{q_2\}$

3rd equivalence:

$\{q_0, q_4\}\ \{q_1, q_7\}\ \{q_3, q_5\}\ \{q_6\}\ \{q_2\}$

	0	1
→q_0q_4	q_1q_7	q_3q_5
q_1q_7	q_6	q_2
q_3q_5	q_2	q_6
q_6	q_6	q_0q_4
*q_2	q_0q_4	q_2

Minimize the following DFA:



	0	1
→A	B	C
B	A	D
(C)	E	F
(D)	E	F
(E)	E	F
F	F	F

NOTE:

If there are multiple final states, combine them

0 equivalence:

$\{A, B, F\}\ \{C, D, E\}$

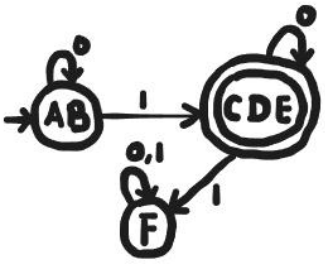
1st equivalence:

$\{A, B\}\ \{F\}\ \{C, D, E\}$

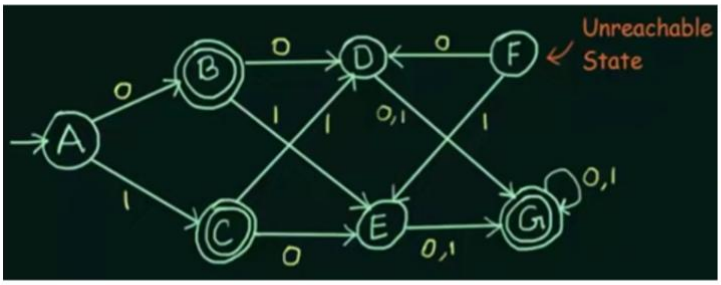
2nd equivalence:

$\{A, B\}\ \{F\}\ \{C, D, E\}$

	0	1
→AB	AB	CDE
F	F	F
*CDE	CDE	F



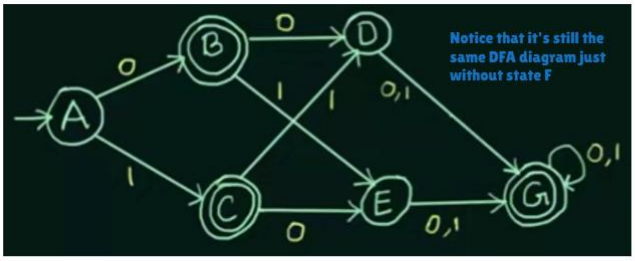
When there are Unreachable States involved



A state is said to be Unreachable if there is no way it can be reached from the Initial State

In a nutshell, unreachable states only have outward edges but no inward edges

Whenever there's an unreachable state, just remove that unreachable state then proceed to usual minimization



Notice that it's still the same DFA diagram just without state F

	0	1
→A	B	C
B	D	E
C	E	D
D	G	G
E	G	G
G	G	G

0 equivalence:

{A, D, E} {B, C, G}

1st equivalence:

{A, D, E} {B, C} {G}

Remember this:
Just because they're final states, doesn't mean they have the same equivalence. So, don't forget to always check

2nd equivalence:

{A} {D, E} {B, C} {G}

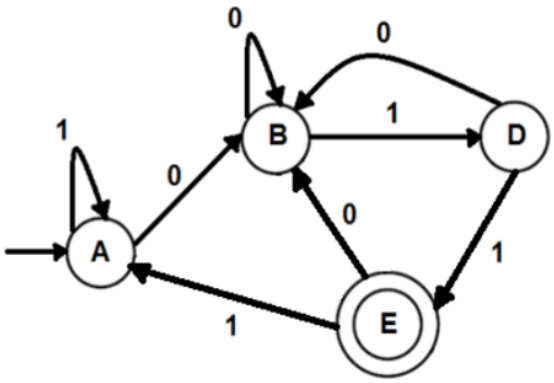
3rd equivalence:

{A} {D, E} {B, C} {G}

	0	1
→A	BC	BC
DE	G	G
*BC	DE	DE
*G	G	G

EXAMPLE

- 1.)



	0	1
→A	B	A
B	B	D
D	B	E
*E	B	A

0-equiv.:
{A,B,D}{E}

1-equiv.:
{A,B}{D}{E}

2-equiv:
{A}{B}{D}{E}

3-equiv:
{A}{B}{D}{E}

	0	1
→A	B	A
B	B	D
D	B	E
*E	B	A

I guess the diagram was already minimized?

MORE EXAMPLE

- 2.)

	a	b
→1	2	6
2	7	3
* 3	1	3
4	3	7
5	8	6
6	3	7
7	7	5
8	7	3

0–equiv.:

{1,2,4,5,6,7,8} {3}

1–equiv.:

{1,5,7} {4,6} {2,8} {3}

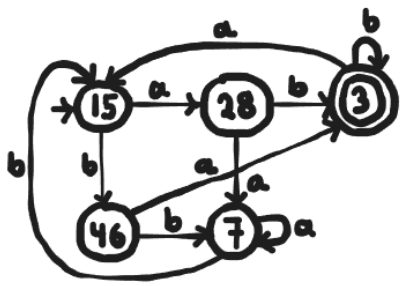
2–equiv.:

{1,5} {7} {4,6} {2,8} {3}

3–equiv.:

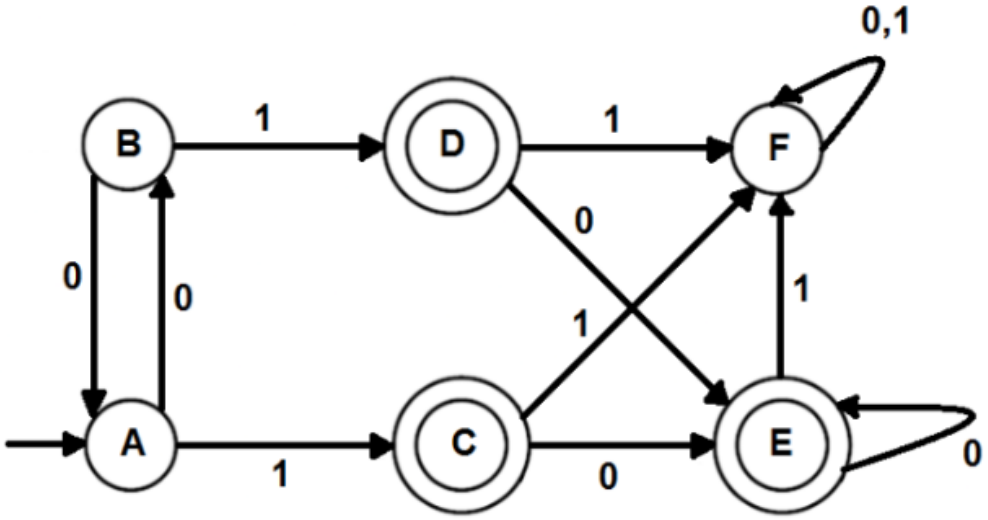
{1,5} {7} {4,6} {2,8} {3}

	a	b
→15	28	46
7	7	15
46	3	7
28	7	3
*3	15	3



MORE EXAMPLE

- 3.) multiple final states



	0	1
→A	B	C
B	A	D
*C	E	F
*D	E	F
*E	E	F
F	F	F

0–equiv.:

{A,B,F}{C,D,E}

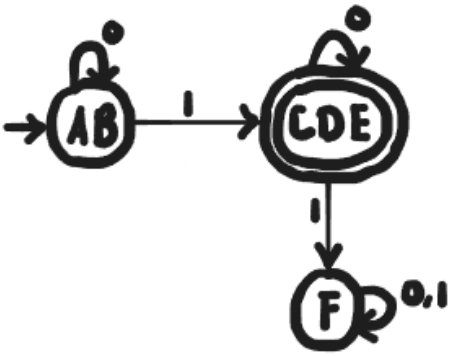
1–equiv.:

{A,B}{F}{C,D,E}

2–equiv.:

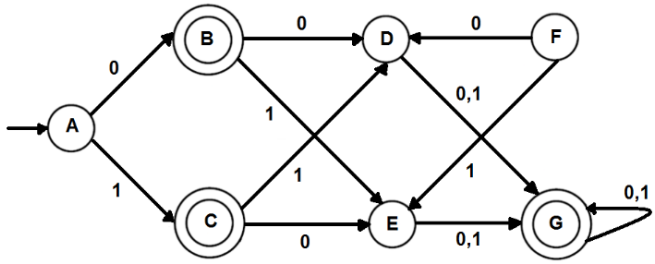
{A,B}{F}{C.D,E}

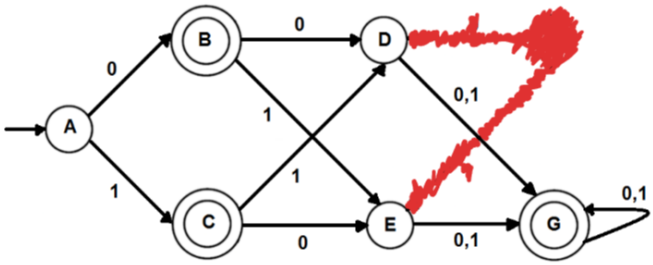
	0	1
→AB	AB	CDE
F	F	F
*CDE	CDE	F



MORE EXAMPLE

- 4.) unreachable state





	0	1
→A	B	C
*B	D	E
*C	E	D
D	G	G
E	G	G
*G	G	G

0-equiv.:
{A, D, E} {B, C, G}
1-equiv.:
{A, D, E} {B, C} {G}
2-equiv.:
{A} {D, E} {B, C} {G}
3-equiv.:
{A} {D, E} {B, C} {G}

	0	1
→A	BC	BC
DE	G	G
*BC	DE	DE
*G	G	G