Statistics 110: Introduction to Probability | Section 2 Solutions, Fall 2018

Conditional Probability

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Practice Problems and Brainteasers

Example 1. Recruiting.
Suppose the Harvard Consulting, Investment, and Tech Group™currently consists of two freshmen and some number of upperclassmen. A new student joins the group, but she forgot to indicate what year she was in! At the next club meeting, a recruiter from BainBookSachs™comes in and plucks a lucky student to join their ranks. Given that the student is a freshman, what is the probability that the student that just joined was a freshman? Suppose that freshmen and upperclassmen are equally likely to join HCITG.

Solution
Let there be u upperclassmen in the group, A be the event that the new student was a freshman, and B be the probability that the selected student was the freshman. We are clearly interested in

$$P(A|B) = \frac{P(B|A)P(A)}{P(B)}$$

Here, $P(B|A) = \frac{3}{3+u}$, as we are conditioning that a freshman joined the group. Next, we can calculate $P(B)$ using LOTP.

$$P(B) = P(B|A)P(A) + P\left(B|A^c\right)P\left(A^c\right) = \frac{1}{2} \cdot \frac{3}{3+u} + \frac{1}{2} \cdot \frac{2}{3+u}$$

Plugging everything in, we find that $P(A|B) = \frac{3}{5}$

Example 2. Russian Roulette.
In a game of Russian Roulette, you find a revolver with **six** chambers containing two real bullets **side-by-side** and four empty chambers. You spin the chamber and point the gun at yourself... Click. No bullet. It is your turn again, but do you want to spin the barrel again or just pull the trigger?

Solution
The probability that you are shot if you spin the barrel again is simply $\frac{1}{3}$, as there are still two bullets and four empty chambers. To calculate the probability of being shot if the interviewer pulls the trigger right away, we can use conditional probability! Let B be the event that the previous shot was not a bullet, and let A be the event that the next adjacent shot in the chamber is a bullet. We know that $P(B) = \frac{2}{3}$ and $P(A \cap B) = \frac{1}{6}$, as there is only one way for the previous shot to not be a bullet while the next shot is a bullet. Therefore,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{1}{4}$$

So you're better off taking the next shot without spinning the chamber!