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Chapter 10
Linear Regression

Outline

- Introduction
- linear regression model
  - linear model
  - least square method --> basic linear regression
  - ridge regression(岭回归)
  - lasso regression

Regression

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- The output value of a classification model is discrete.
- The output value of a regression model is continuous.

gender

house price

crop yield

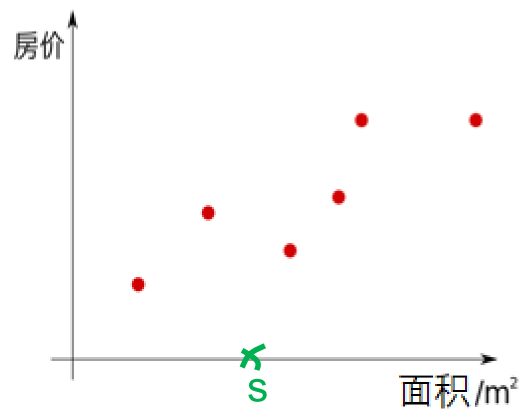
height

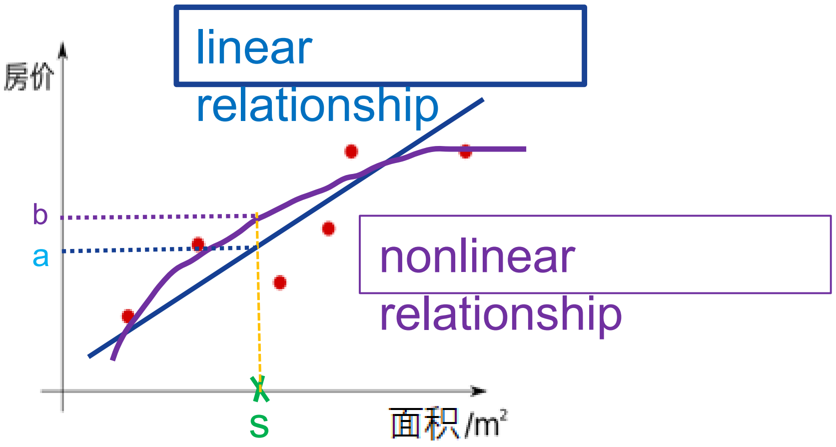
movie type

watermelon:good or bad

regression	classification
house price	gender:male or female
crop yield	movie type
height	good or bad watermelon
...	...

Regression





- Regression estimates the relationship between input values and output values, and establishes a mathematical model in order to accurately predict the output value of new sample.
- Regression is a supervised learning question.

Regression

Regression

the number of input values

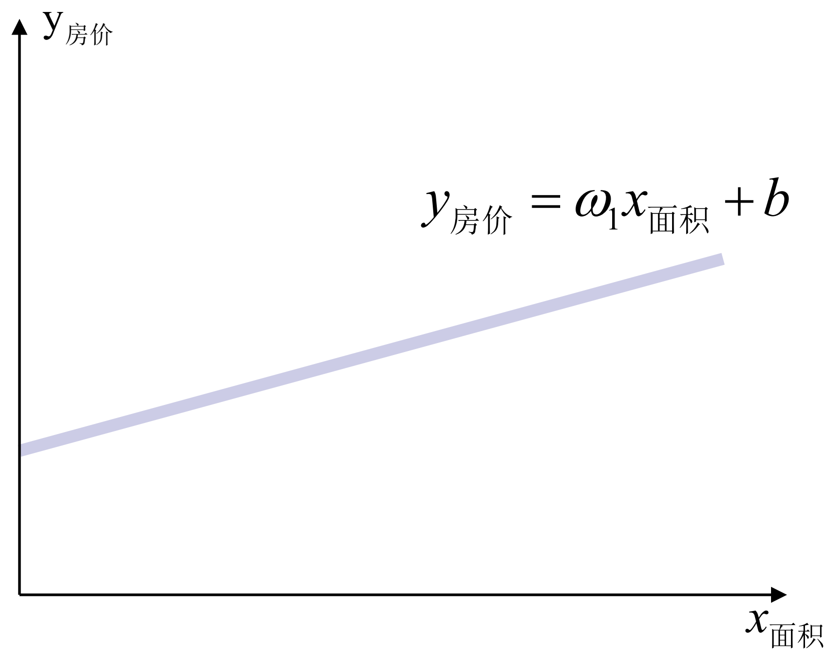
- single regression(单元回归)
- multiple regression(多元回归)

Regression

the relationship of input values and output value

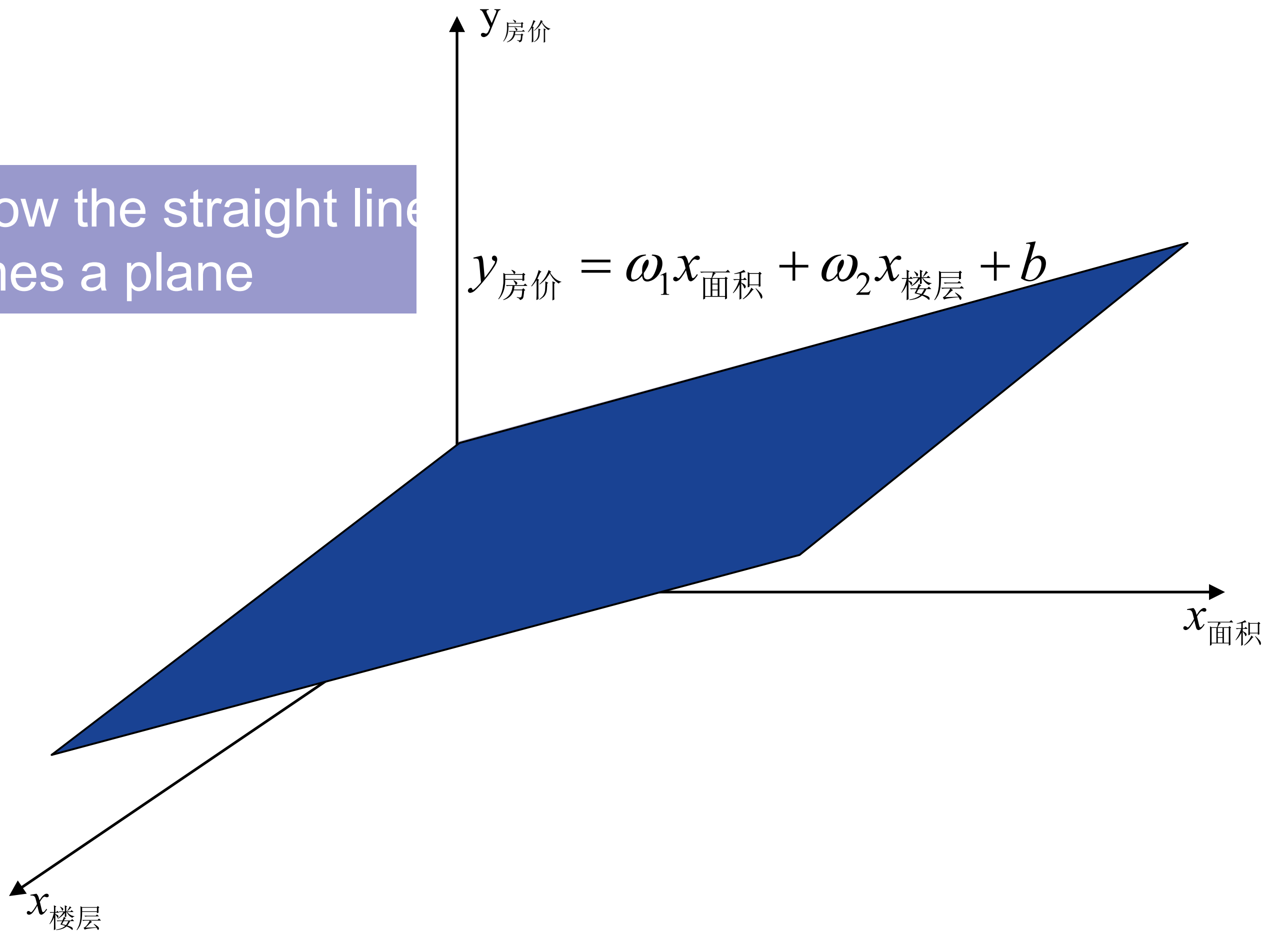
- linear regression
- nonlinear regression

Linear Regression Model



Linear Regression Model

Note how the straight line becomes a plane



Linear Regression Model

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general model: $$\boxed{f(\boldsymbol{x}) = w_1x_1 + w_2x_2 + \ldots + w_dx_d + b}$$

$x_1, x_2, \ldots, x_d$: the feature of sample

$w_1, w_2, \ldots, w_d$: weight, represent the importance of corresponding feature

$$f_{\text{房价}}(x) = 0.2 \cdot x_{\text{楼层}} + 0.5 \cdot x_{\text{地段}} + 0.3 \cdot x_{\text{面积}} + 1$$

importance: $x_{\text{地段}} > x_{\text{面积}} > x_{\text{楼层}}$

Linear Model's Vector Representation

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general model: $f(\boldsymbol{x}) = w_1 x_1 + w_2 x_2 + \ldots + w_d x_d + b$

vector formal:

$$f(\boldsymbol{x}) = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{x} + b$$

$$\boldsymbol{w} = (w_1; w_2; \ldots; w_d)$$

$$\boldsymbol{x} = (x_1; x_2; \ldots; x_d)$$

Linear Model's Vector Representation

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training set:

S1: 楼层=6，地段=5，面积=120。房价=100
S2: 楼层=3，地段=1，面积=100。房价=200
S3: 楼层=16，地段=8，面积=200。房价=120

$$\begin{cases} 6\omega_1 + 5\omega_2 + 120\omega_3 = 100 \\ 3\omega_1 + \omega_2 + 100\omega_3 = 200 \\ 16\omega_1 + 8\omega_2 + 200\omega_3 = 120 \end{cases} \Rightarrow \begin{array}{l} \omega_1 = a \\ \omega_2 = b \\ \omega_3 = c \end{array}$$

$$f_{\text{房价}}(x) = a \cdot x_{\text{楼层}} + \mathrm{b} \cdot x_{\text{地段}} + \mathrm{c} \cdot x_{\text{面积}}$$

- Solve weigt vector w according to training set.

Which model is better?

$$f_{\text{房价}}(x) = 0.2 \cdot x_{\text{楼层}} + 0.5 \cdot x_{\text{地段}} + 0.3 \cdot x_{\text{面积}} + 1$$

$$g_{\text{房价}}(x) = 0.1 \cdot x_{\text{楼层}} + 0.4 \cdot x_{\text{地段}} + 0.5 \cdot x_{\text{面积}} + 3$$

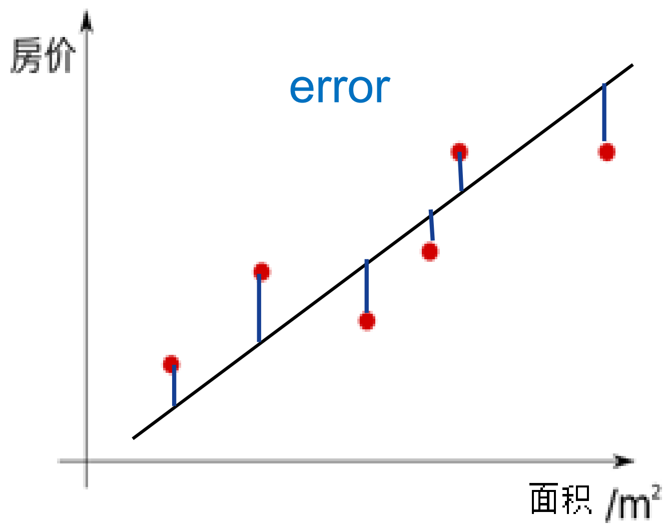
a known data:楼层=6，地段=5，面积=120，房价=100
model f predictive value: 0.2*6+0.5*5+0.3*120+1=40.7
model g predictive value: 0.1*6+0.4*5+0.5*120+3=65.6

model f error: 100-40.7=59.3
model g error: 100-65.6=34.4
34.4<59.3
model g is better

•the model which can minimize error between model predictive value and true value.

Least Square Method

- In regression task, the least square method is often used measuring error between model predictive value and true value.



$$\text{error}=\sum_{i=1}^{m}\left(f\left(x_i\right)-y_i\right)^2$$

predictive value

true value

least square method:

$$min\sum_{i=1}^{m}\left(f\left(x_i\right)-y_i\right)^2$$

Basic Linear Regression

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- solve w and b

single linear regression:

model: $f(x_i) = wx_i + b$

loss function:

$$E_{(w,b)} = \sum_{i=1}^{m} \left(f\left(x_i\right) - y_i\right)^2$$

$$= \sum_{i=1}^{m} (y_i - wx_i - b)^2$$

$$\boxed{min\ E_{(w,b)}}$$

Basic Linear Regression

MIMA

- solve w and b

single linear regression: $E_{(w,b)} = \sum_{i=1}^{m}(y_i - wx_i - b)^2$

$$\frac{\partial E_{(w,b)}}{\partial w} = 2\left(w\sum_{i=1}^{m}x_i^2 - \sum_{i=1}^{m}(y_i - b)\,x_i\right) = 0$$

$$\frac{\partial E_{(w,b)}}{\partial b} = 2\left(mb - \sum_{i=1}^{m}(y_i - wx_i)\right) = 0$$

new sample

$$w = \frac{\sum_{i=1}^{m} y_i(x_i - \bar{x})}{\sum_{i=1}^{m} x_i^2 - \frac{1}{m}\left(\sum_{i=1}^{m} x_i\right)^2}$$

model: $f(x_i) = wx_i + b$

$$b = \frac{1}{m}\sum_{i=1}^{m}(y_i - wx_i) \qquad \bar{x} = \frac{1}{m}\sum_{i=1}^{m} x_i$$

dataset: $D = \{(x_i, y_i)\}_{i=1}^{m}$

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Basic Linear Regression

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multiple linear regression:

model : $f(\boldsymbol{x}_i) = w_1 x_{i1} + w_2 x_{i2} + \ldots + w_d x_{id} + b$

vector model: $f(\boldsymbol{x}_i) = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}_i + b$

$* m$

$f(\boldsymbol{x}) = \mathbf{X}\hat{\boldsymbol{w}}$

$\hat{\boldsymbol{w}} = (\boldsymbol{w}; b)$ $(d+1)$ column vector

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \ldots & x_{1d} & 1 \\ x_{21} & x_{22} & \ldots & x_{2d} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{m1} & x_{m2} & \ldots & x_{md} & 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{x}_1^{\mathrm{T}} & 1 \\ \boldsymbol{x}_2^{\mathrm{T}} & 1 \\ \vdots & \vdots \\ \boldsymbol{x}_m^{\mathrm{T}} & 1 \end{pmatrix}$$ $m \times (d+1)$ matrix

$\boldsymbol{y} = (y_1; y_2; \ldots; y_m)$ m column vector,true values of samples

Basic Linear Regression

MIMA

- solve $\hat{\boldsymbol{w}}$

multiple linear regression:

model: $f(\boldsymbol{x}) = \mathbf{X}\hat{\boldsymbol{w}}$

least square method

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})$

$$\hat{\boldsymbol{w}} = (\boldsymbol{w}; b)$$

$$\mathbf{X} = \begin{pmatrix} x_{11} & x_{12} & \dots & x_{1d} & 1 \\ x_{21} & x_{22} & \dots & x_{2d} & 1 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ x_{m1} & x_{m2} & \dots & x_{md} & 1 \end{pmatrix} = \begin{pmatrix} \boldsymbol{x}_1^{\mathrm{T}} & 1 \\ \boldsymbol{x}_2^{\mathrm{T}} & 1 \\ \vdots & \vdots \\ \boldsymbol{x}_m^{\mathrm{T}} & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 2 & 3 & 4 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix} = 1^2 + 2^2 + 3^2 + 4^2$$

$$\boldsymbol{y} = (y_1; y_2; \dots; y_m)$$

Basic Linear Regression

MIMA

- solve $\hat{\boldsymbol{w}}$

multiple linear regression:

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})$

$$= y^T y - y^T X \hat{w} - \hat{w}^T X^T y + \hat{w}^T X^T X \hat{w}$$

$$- 2X^T y + 2X^T X \hat{w}$$

向量求导公式

$$\frac{\partial (X\theta)}{\partial \theta} = X^T$$

$$\frac{\partial (\theta^T X)}{\partial \theta} = X$$

$$\frac{\partial (\theta^T X \theta)}{\partial \theta} = (X^T + X)\theta$$

令 $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})$, 对 $\hat{\boldsymbol{w}}$ 求导得到

$$\frac{\partial E_{\hat{\boldsymbol{w}}}}{\partial \hat{\boldsymbol{w}}} = 2\,\mathbf{X}^{\mathrm{T}} (\mathbf{X}\hat{\boldsymbol{w}} - \boldsymbol{y}) = 0$$

$$\Downarrow$$

analytical solution: 解析解

$$\hat{\boldsymbol{w}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}} \boldsymbol{y}$$

Basic Linear Regression

- multiple linear regression:

dataset: $D = \{(\boldsymbol{x}_1, y_1), (\boldsymbol{x}_2, y_2), \ldots, (\boldsymbol{x}_m, y_m)\}$

1. substitute

$$\hat{\boldsymbol{w}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\boldsymbol{y}$$

2. get

model: $f(\boldsymbol{x}) = \mathbf{X}\hat{\boldsymbol{w}}$

new sample

Least square method's drawback

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- 最小二乘法的损失函数有可能会造成模型过拟合。
- 解析解中 $\mathbf{X}^{\mathrm{T}}\mathbf{X}$ 有可能不是满秩矩阵，逆矩阵不存在
- 样本的特征数远远超过样本数， 这样, $\hat{\boldsymbol{w}}$ 会有多个解。

ridge regression & lasso regression

model: $f(\boldsymbol{x}_i) = w_1 x_{i1} + w_2 x_{i2} + \ldots + w_d x_{id} + b$

vector model: $f(\boldsymbol{x}_i) = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}_i + b$

$$f(\boldsymbol{x}) = \mathbf{X}\hat{\boldsymbol{w}}$$

loss function:

basic linear regression: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})$

ridge regression: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda \|\hat{\boldsymbol{w}}\|_2^2 \quad \lambda > 0$

lasso regression: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda \|\hat{\boldsymbol{w}}\|_1 \quad \lambda > 0$

ridge 回归和lasso回归可以防止模型过拟合，也可以解决样本的特征数远超过样本数的问题。

Norm

Norm
范数

- Vector norm
- Matrix norm

•to measure the size of vector and matrix
•Norm is a value.

Vector Norm

$\boldsymbol{w} \epsilon \boldsymbol{R}^d$

L_0-norm: the number of non-zero elements in the vector

L_1-norm: $\|\boldsymbol{w}\|_1 = |w_1| + |w_2| + \ldots + |w_d| = \sum_{i=1}^{d} |w_i|$

L_2-norm: $\|\boldsymbol{w}\|_2 = \sqrt{w_1^2 + w_2^2 + \ldots + w_d^2} = (\sum_{i=1}^{d} w_i^2)^{\frac{1}{2}}$

L∞- $\|\boldsymbol{w}\|_\infty = \mathbf{max}\{|w_1|, |w_2|, \ldots, |w_d|\} = \mathbf{max}_{1 \le i \le d}\{|w_i|\}$

$\boldsymbol{w} = \begin{bmatrix} 1 & 2 & -2 \end{bmatrix}$

$||\boldsymbol{w}||_0 = 3 \quad ||\boldsymbol{w}||_1 = 5$

$||\boldsymbol{w}||_2 = 3 \quad ||\boldsymbol{w}||_\infty = 2$

ridge regression

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model : $f(\boldsymbol{x}_i) = w_1 x_{i1} + w_2 x_{i2} + \ldots + w_d x_{id} + b$

$$\Updownarrow$$

$$f(\boldsymbol{x}_i) = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}_i + b$$

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda \|\hat{\boldsymbol{w}}\|_2^2 \quad \lambda > 0$

$$= (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda \hat{\boldsymbol{w}}^{\mathrm{T}} \hat{\boldsymbol{w}}$$

$$= (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda \hat{\boldsymbol{w}}^{\mathrm{T}} \boldsymbol{I} \hat{\boldsymbol{w}}$$

$\boldsymbol{I}$:identity matrix

向量求导公式:

$$\frac{\partial (X\theta)}{\partial \theta} = X^T$$

$$\frac{\partial (\theta^T X)}{\partial \theta} = X$$

$$\frac{\partial (\theta^T X \theta)}{\partial \theta} = (X^T + X)\theta$$

$$\frac{\partial E_{\hat{\boldsymbol{w}}}}{\partial \hat{\boldsymbol{w}}} = 2\,\mathbf{X}^{\mathrm{T}} (\mathbf{X}\hat{\boldsymbol{w}} - \boldsymbol{y}) + \lambda \boldsymbol{I} \hat{\boldsymbol{w}} = 0$$

$$\Downarrow$$

analytical solution:
解析解

$$\hat{\boldsymbol{w}} = (\boldsymbol{X}^T \boldsymbol{X} + \lambda \boldsymbol{I})^{-1} \boldsymbol{X}^T \boldsymbol{y}$$

ridge regression

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basic linear regression:

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})$

analytical solution: $\hat{\boldsymbol{w}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\boldsymbol{y}$

ridge regression:

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda\|\hat{\boldsymbol{w}}\|_2^2 \quad \lambda > 0$

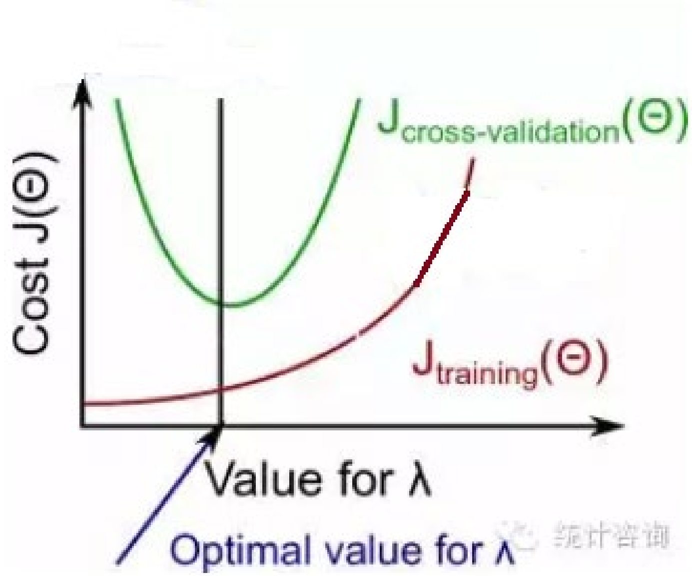
analytical solution: $\hat{\boldsymbol{w}} = (\boldsymbol{X}^T\boldsymbol{X} + \lambda\boldsymbol{I})^{-1}\boldsymbol{X}^T\boldsymbol{y}$

$(\boldsymbol{X}^T\boldsymbol{X} + \lambda\boldsymbol{I})$ is full rank matrix, $\hat{\boldsymbol{w}}$ can get analytical solution.

可以解决特征数大于样本数的问题

ridge regression

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda \|\hat{\boldsymbol{w}}\|_2^2 \ \ \lambda > 0$



lasso regression

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- Least Absolute Shrinkage and Selection Operator,最小绝对收敛算子

model : $f(\boldsymbol{x}_i) = w_1 x_{i1} + w_2 x_{i2} + \ldots + w_d x_{id} + b$

$\Updownarrow$

vector model: $f(\boldsymbol{x}_i) = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}_i + b$

$\Updownarrow$

$f(\boldsymbol{x}) = \mathbf{X}\hat{\boldsymbol{w}}$

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}}(\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda\|\hat{\boldsymbol{w}}\|_1 \quad \lambda > 0$

$$\|\boldsymbol{w}\|_1 = |w_1| + |w_2| + \ldots + |w_d| = \sum_{i=1}^{d} |w_i|$$

lasso regression

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loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}}(\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda\|\hat{\boldsymbol{w}}\|_1 \quad \lambda > 0$

- 函数在某点可导条件：
- 1）函数在该点连续
- 2）函数在该点左右两侧导数都存在并且相等。

$$y = |\omega|, \omega \in R$$
$$y'(0^-) = -1, y'(0^+) = 1$$

lasso regression

MIMA

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda \|\hat{\boldsymbol{w}}\|_1 \quad \lambda > 0$

•Solving loss function

Due to $\|w\|_1$aving not derivative(导数), we use proximal gradient descent method(近端梯度下降法) to minimize the loss function of lasso regression.

analytical solution:

$$w_{k+1}^i = \begin{cases} z^i - \lambda/L, \lambda/L < z^i; \\ 0, |z^i| \leq \lambda/L; \\ z^i + \lambda/L, z^i < -\lambda/L, \end{cases} \qquad L > 0 \text{ 的常数}$$

$$\boldsymbol{z} = \boldsymbol{w}_k - 1/L \nabla f(\boldsymbol{w}_k)$$

可以解决特征数大于样本数的问题

$$\min_x f(x) + \lambda \cdot g(x),$$

根据利普希茨连续性，对于任意 x,y一定存在常数 L使得满足

$$|f'(y) - f'(x)| \le L|y - x|.$$

用 x^{(k)} 来表示x的第 k 次更新后的结果，则对于 x逼近 x^{(k)}时， f(x)近似可以用 x 和 x^{(k)} 的函数来表示：

$$\hat{f}(x, x^{(k)}) = f(x^{(k)}) + \nabla f^T(x^{(k)})(x - x^{(k)}) + \frac{L}{2}\left\|x - x^{(k)}\right\|^2$$
$$= \frac{L}{2}[x - (x^{(k)} - \frac{1}{L}\nabla f(x^{(k)}))]^2 + \mathrm{CONST}$$

$$x^{(k+1)} = \mathrm{argmin}_x\{f(x) + \lambda \cdot g(x)\},$$

$$x^{(k+1)} = \mathrm{argmin}_x\{\hat{f}(x, x^{(k)}) + \lambda \cdot g(x)\}$$
$$= \mathrm{argmin}_x\{\frac{L}{2}[x - (x^{(k)} - \frac{1}{L}\nabla f(x^{(k)}))]^2 + \mathrm{CONST} + \lambda \cdot g(x)\}.$$

Given $g(x) = \|x\|_1$, and let $z = x^{(k)} - \frac{1}{L}\nabla f(x_k)$, we have

$$x^{(k+1)} = \mathrm{argmin}_x\{\frac{L}{2}\|x - z\|^2 + \lambda\|x\|_1\}.$$

$$F(x) = \frac{L}{2}\sum_{j=1}^p (x_j - z_j)^2 + \lambda\sum_{j=1}^p |x_j|.$$

$$\frac{\partial F(x)}{\partial x_j} = L(x_j - z_j) + \lambda \cdot \mathrm{sgn}(x_j) = 0,$$

$$z_j = x_j + \frac{\lambda}{L}\mathrm{sgn}(x_j)$$

$$x_j = \mathrm{sgn}(z_j)(|z_j| - \frac{\lambda}{L})_+$$
$$= \mathrm{sgn}(z_j) \cdot \max\{|z_j| - \frac{\lambda}{L},\ 0\}$$

lasso regression

- Lasso regression is easier to get sparse solution

sparse solution(稀疏解):

- $\boldsymbol{w}$ contains less non-zero elements.
- It can use for feature selection.

$w_d = 0$ means the feature x_{id} is not important to the task.

lasso regression

MIMA

model : $f(\boldsymbol{x}_i) = w_1 x_{i1} + w_2 x_{i2} + \ldots + w_d x_{id} + b$

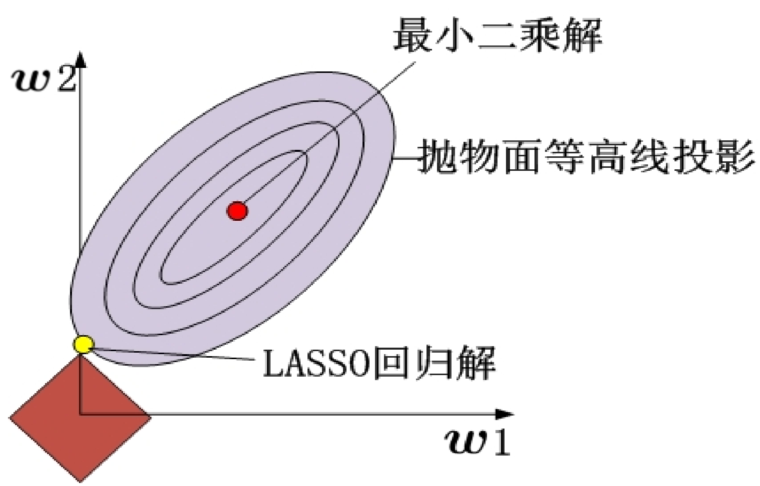
$$\Updownarrow$$

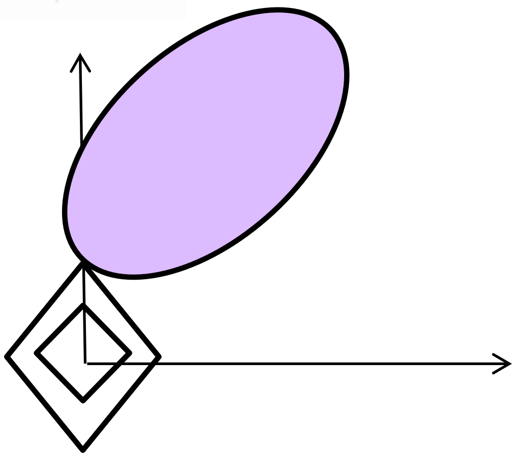
$$f(\boldsymbol{x}_i) = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}_i + b$$

loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda \|\hat{\boldsymbol{w}}\|_1 \quad \lambda > 0$

$$\|\boldsymbol{w}\|_1 = |w_1| + |w_2| + \ldots + |w_d|$$

basic linear regression: $E_{(w,b)} = \sum_{i=1}^{m} (f(\boldsymbol{x}_i) - y_i)^2$





lasso vs ridge regression

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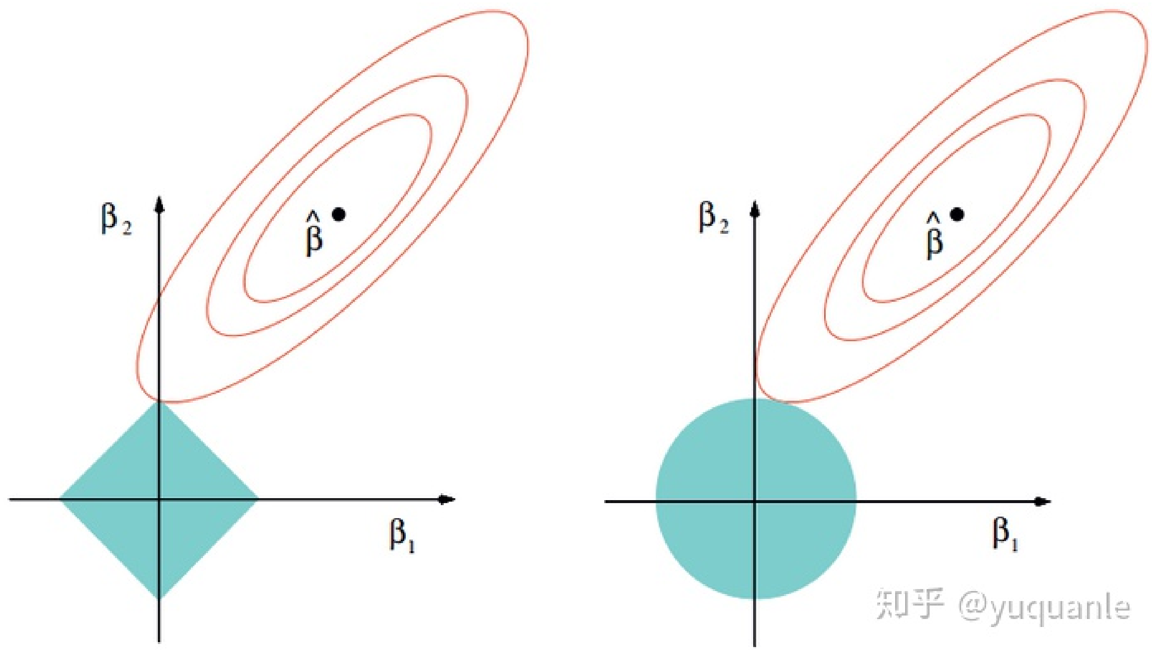

Lasso和岭回归的区别很好理解，在优化过程中，最优解为函数等值线与约束空间的交集，正则项可以看作是约束空间。可以看出二范的约束空间是一个球形，一范的约束空间是一个方形，这也就是二范会得到很多参数接近0的值，而一范会尽可能非零参数最少。

lasso vs ridge regression

- Prevent model overfitting.
- solve the problem that the number of features is larger than the number of samples.

ridge regression

lasso regression

It is easier to get sparse solution.

Summary

MIMA

- linear regression model
  - linear model

    model: $f(\boldsymbol{x}_i) = w_1 x_{i1} + w_2 x_{i2} + \ldots + w_d x_{id} + b$

    vector model: $f(\boldsymbol{x}_i) = \boldsymbol{w}^{\mathrm{T}} \boldsymbol{x}_i + b$

    $* m$

    $f(\boldsymbol{x}) = \mathbf{X}\hat{\boldsymbol{w}}$

  - least square method --> basic linear regression

    loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}} (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})$

    analytical solution: $\hat{\boldsymbol{w}} = (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1} \mathbf{X}^{\mathrm{T}} \boldsymbol{y}$

Summary

- linear regression model
  - ridge regression(岭回归)

    loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}}(\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda\|\hat{\boldsymbol{w}}\|_2^2 \quad \lambda > 0$

    analytical solution $\hat{\boldsymbol{w}} = (\boldsymbol{X}^T\boldsymbol{X} + \lambda\boldsymbol{I})^{-1}\boldsymbol{X}^T\boldsymbol{y}$

  - lasso regression

    loss function: $E_{\hat{\boldsymbol{w}}} = (\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}})^{\mathrm{T}}(\boldsymbol{y} - \mathbf{X}\hat{\boldsymbol{w}}) + \lambda\|\hat{\boldsymbol{w}}\|_1 \quad \lambda > 0$

    Lasso regression is easier to get sparse so$\hat{\boldsymbol{w}}$tion

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Thank You !

Any Question?