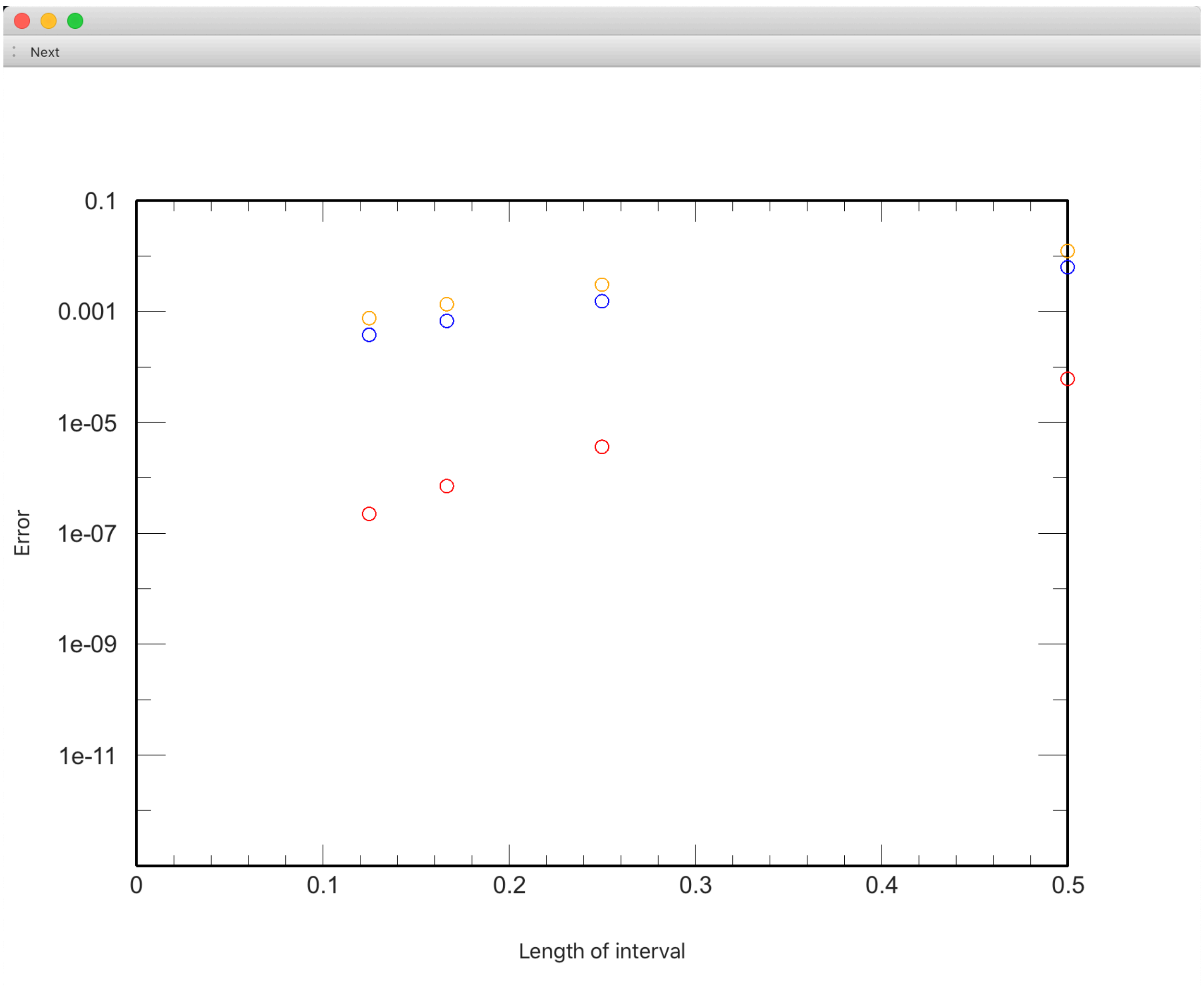
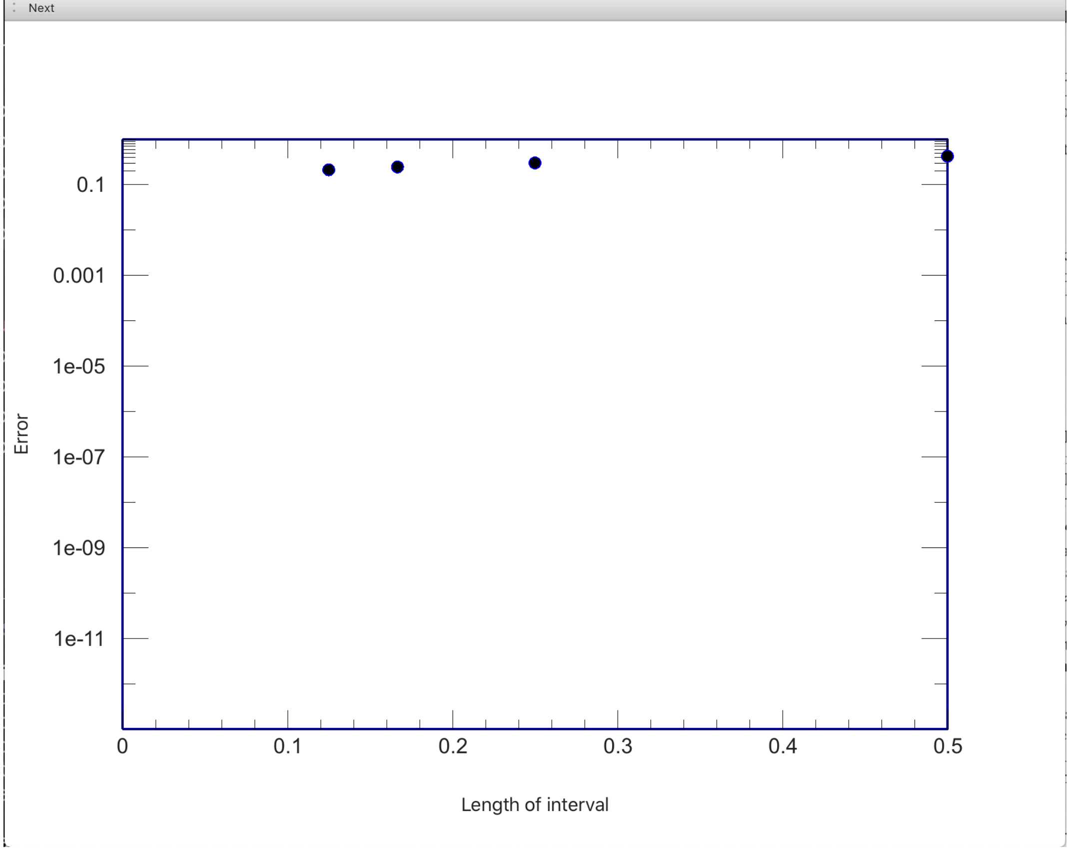
Assignment 5

EX1.

For the integral $\int_0^1 \tanh(x)\mathrm{d}x$, I used *MidpointRule, TrapezoidRule and SimpsonRule* to do the calculation. The results are showed in follow diagram. The *SimpsonRule* (red circle) provide the most accurate result, while the *MidpointRule* (blue circle) is a little better than *TrapezoidRule* (orange circle).

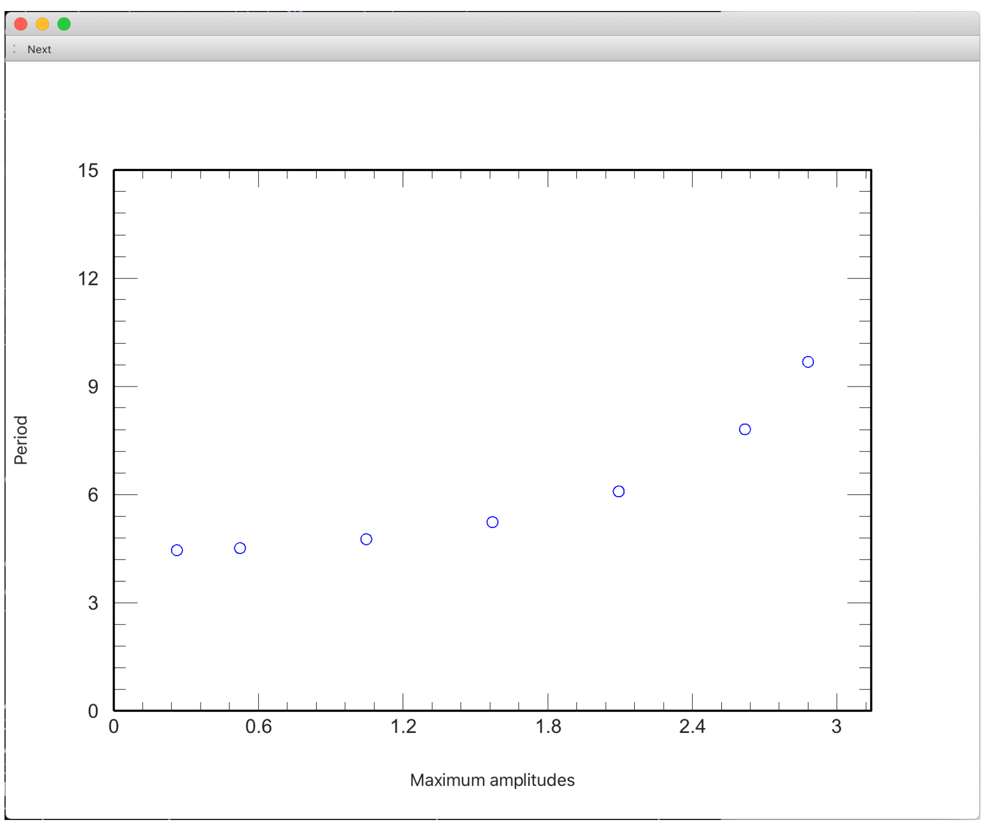


For the integral $\int_0^1 \sqrt{\coth(x)\mathrm{d}x}$, because hyperbolic cotangent divergent at point x=0, we can only use the *MidpointRule,* and the result are showed in follow. Because of the divergence, the error of integral decrease very slowly with the increase of number of interval.



EX2.

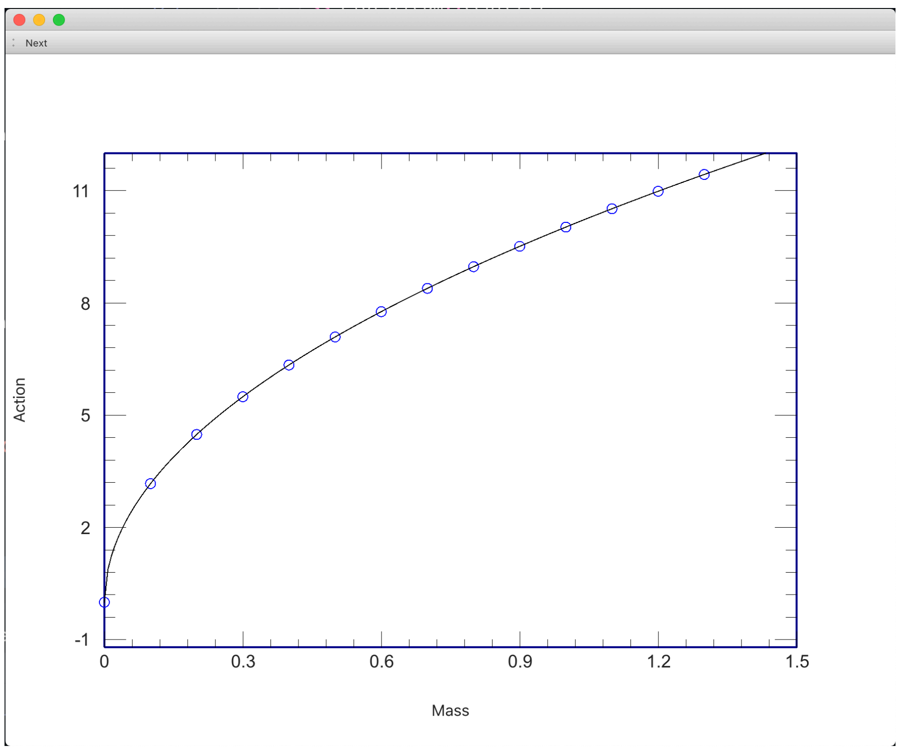
The period of the pendulum is $\sqrt{\frac{2l}{g}} \times \int_{-\theta_{max}}^{\theta_{max}} \frac{1}{\sqrt{\cos(\theta) - \cos(\theta_{\max})}} d\theta$. I chose the *SimpsonRule* to do the integration in this question. The results are showed below. Since I set the $T_0 = \sqrt{\frac{2l}{g}} = 1$, when the angle is small, the period is chose to $\sqrt{2}\pi \sim 4.44$.



EX4.

a) The action $J = 2\int_{x_{min}}^{x_{max}} \sqrt{2m(E - V(x))}dx = (n + 1/2)h$. For bound states, $E < 0$, therefore the number of bound state is determined by $2\int_{x_{min}}^{x_{max}} \sqrt{2m(-V(x))}dx$. It is easy to calculate the integral analytically. We have $J = 4\pi a\sqrt{2mV_0}$. So the number of bound state is around $\frac{4\pi a\sqrt{2mV_0}}{h}$.

b) I calculated the aforementioned integral by using the *SimpsonRule*, and I also plot the analytical solution in the range 0<m<1.5. These two methods fits very well in this region.



EX6.

In this question, I used *GaussHermiteRule* and *GaussIntegrator* to do the integration. The minimum number of points needed in this computation is 6. The out put matrix $A_{ij} = \langle \phi_i(x) | \phi_j(x) \rangle$ is showed in follow:

1	7.28584e-17	8.32667e-17	-1.249e-16	5.41234e-16	1.07553e-16
7.28584e-17	1	-4.16334e-17	1.16573e-15	-8.32667e-17	4.85723e-16
8.32667e-17	-4.16334e-17	1	0	1.83187e-15	-2.77556e-17
-1.249e-16	1.16573e-15	0	1	1.11022e-16	1.4988e-15
5.41234e-16	-8.32667e-17	1.83187e-15	1.11022e-16	1	-8.32667e-17
1.07553e-16	4.85723e-16	-2.77556e-17	1.4988e-15	-8.32667e-17	1