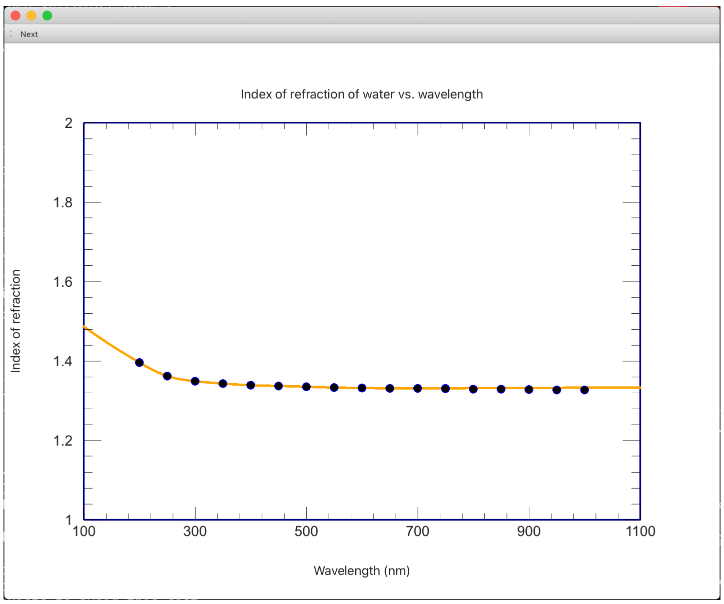
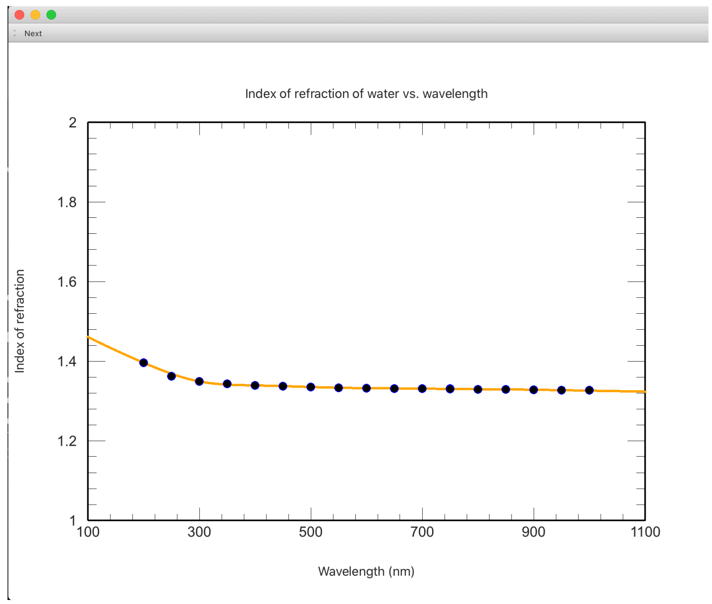
Assignment 6

EX4.7

In this question, I use *CubicSplinePolynomial* to fit the dispersion function of water $n = n(x)$. Notice that the index n approach to 1 when the wavelength x to the infinity. Therefore, for the smooth fitting, I interpolate the ancillary function $y = \ln(n(x) - 1)$. Then I plot the function $n(x) = e^{y(x)} + 1$. And I tried two methods to avoid the failure caused by too many points added into the program. The results are showed below. In the first diagram, I fit the first ten points. In the second diagram, I chose one point from every two nearby point to fit. Both methods work pretty well in this question.





EX5.7

All the results are showed in follow. And it is obvious that $\hat{x}^2 \neq \hat{x^2}$, $\hat{D}^2 \neq \hat{D^2}$.

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The matrix representation for x is
 1.22298e-16     0.707107  1.38778e-17  8.32667e-16 -4.16334e-17
    0.707107    1.249e-16            1 -5.55112e-17  2.22045e-15
 1.38778e-17            1            0      1.22474            0
 8.18789e-16 -8.32667e-17      1.22474  1.11022e-16      1.41421
-5.55112e-17  2.27596e-15            0      1.41421 -2.22045e-16

 The matrix representation for x^2 is
         0.5  4.16334e-17     0.707107 -5.55112e-17  1.63758e-15
 4.16334e-17          1.5  5.55112e-17      1.22474            0
    0.707107  5.55112e-17          2.5            0      1.73205
-5.55112e-17      1.22474            0          3.5  2.22045e-16
 1.63758e-15            0      1.73205            0          4.5
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 The matrix representation for D is
-6.67869e-17      0.707107  3.81639e-16 -6.17562e-16 -4.25875e-16
   -0.707107  1.11022e-16            1  2.77556e-17  1.30451e-15
-1.38778e-17           -1            0      1.22474   1.8735e-16
-8.04912e-16  5.55112e-17     -1.22474 -8.32667e-17      1.41421
 4.16334e-17 -1.55431e-15 -1.66533e-16     -1.41421    2.498e-16

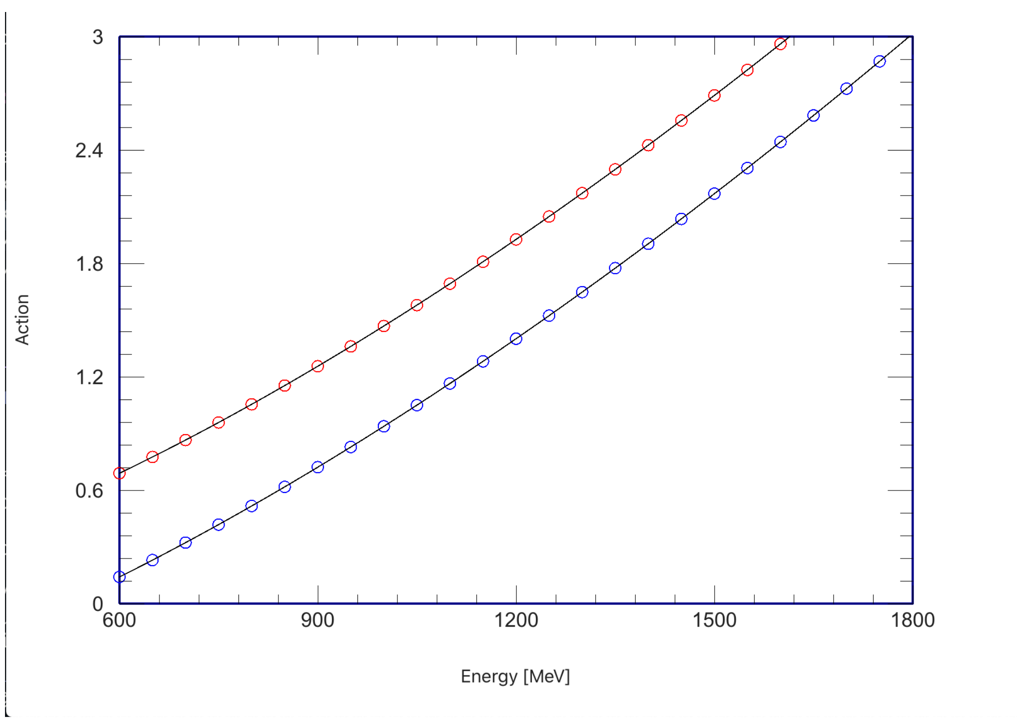
 The matrix representation for D^2 is
        -0.5 -4.33681e-16     0.707107  9.29812e-16 -3.40006e-15
-1.45717e-16         -1.5  7.63278e-17      1.22474  5.55112e-16
    0.707107  5.55112e-17         -2.5 -2.77556e-16      1.73205
 1.11022e-16      1.22474  2.77556e-17         -3.5 -4.44089e-16
 1.11022e-15  2.22045e-16      1.73205 -4.44089e-16         -4.5

 The matrix representation for Hamiltonian is
         0.5  2.37657e-16 -5.55112e-17 -4.92661e-16  2.51882e-15
 9.36751e-17          1.5 -1.04083e-17  3.77476e-15 -2.77556e-16
           0            0          2.5  1.38778e-16  8.65974e-15
-8.32667e-17  1.77636e-15 -1.38778e-17          3.5  3.33067e-16
 2.63678e-16 -1.11022e-16  4.77396e-15  2.22045e-16          4.5
```

The matrix representation for Hamiltonian is a diagonal matrix, which means that the Hermite polynomials are the eigenfunction for this system. And the eigenvalue are $\hbar(n+1/2)$.

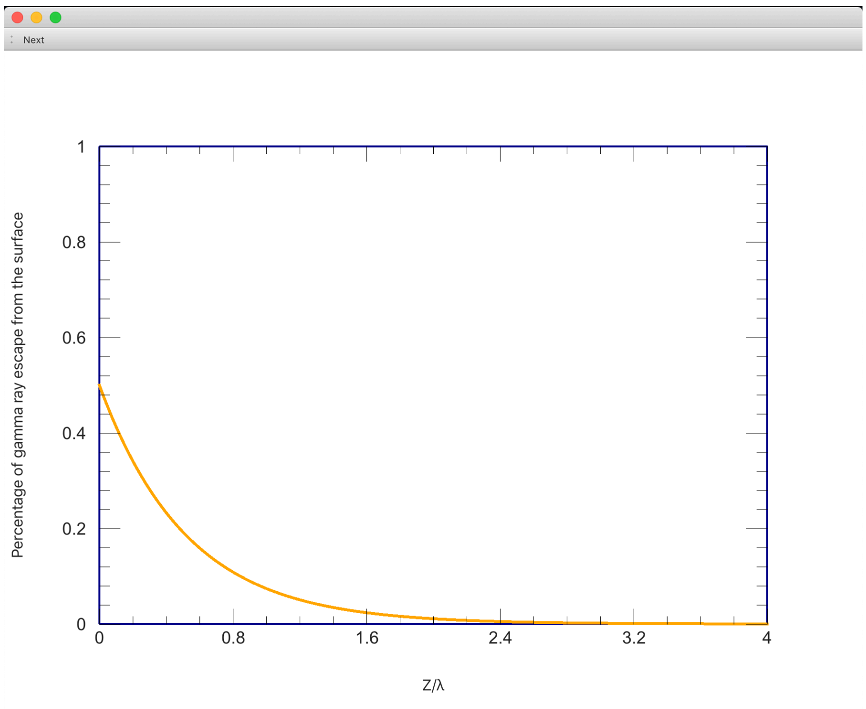
EX5.9

The total potential function for this question is $V(r) = -\alpha\dfrac{4\hbar c}{3r} + \dfrac{r}{\hbar c a^2} + \dfrac{l(l+1)}{2\mu r^2}$. And we have the action $J(E) = \displaystyle\int_{r_{min}}^{r_{max}} \sqrt{2\mu(E - V(r))}\mathrm{d}r$. The results of integral are showed in follow, and I used the *CubicSplinePolynomial* to fit the function.



Now we can read the energy level for the ground state and first two excited state from the diagram. For the classical quantization rule, we have $J = (n + 0.75)\pi\hbar$ when $l = 0$, and $J = (n + 0.5)\pi\hbar$ when $l = 1$. For $l = 0$ (red circle), the energy level are $E_0 = 636\text{MeV}$, $E_1 = 1124\text{MeV}$ and $E_2 = 1520\,\text{MeV}$. For $l = 1$ (blue circle), the energy level are $E_0 = 790\text{MeV}$, $E_1 = 1240\text{MeV}$ and $E_2 = 1620\text{MeV}$.

EX5.12



The calculation are showed in follow:

a) $P(x) = \frac{1}{\lambda}\exp(-x/\lambda)$

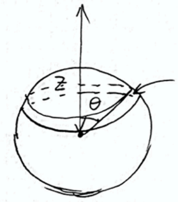
⇒ The probability for a gamma ray to travel more than y meters is :

$$\cancel{\int_0^y (1-P(x))\,dx} \quad 1-\int_0^y P(x)\,dx$$

$$= 1-\int_0^y \frac{e^{-\frac{x}{\lambda}}}{\lambda}\,dx$$

$$= 1+ e^{-\frac{x}{\lambda}}\Big|_0^y$$

$$= e^{-\frac{y}{\lambda}}$$



Total number of gamma ray in this ring is $\frac{N_0 \sin\theta \, d\theta}{2}$, N_0 is the total number of photon emitted in this moument.

The total numbe of photon escap from the surface is:

$$\int_0^{\frac{\pi}{2}} \sin\theta \, \frac{N_0}{2} \, e^{-\frac{z}{\cos\theta \, \lambda}} \, d\theta$$

$$= -\int_1^0 \frac{N_0}{2} \, e^{-\frac{z}{\cos\theta \, \lambda}} \, d\cos\theta$$

$$= \frac{N_0}{2} \int_0^1 e^{-\frac{1}{u}\frac{z}{\lambda}} \, du$$

$$= F\left[\frac{z}{\lambda}\right] \qquad 4.12 \times 10^4$$

b).

The total flux $\Rightarrow \Phi = \int \frac{dn}{dt} F\left(\frac{z}{\lambda}\right) dz \, dA$

$\Rightarrow$ Flux density $\phi = \frac{\Phi}{A} = \int_0^1 \int_0^\infty \frac{dn}{dt} \frac{1}{2} e^{-\frac{z}{u\lambda}} du$

$$\Rightarrow \phi = \frac{1}{2}\frac{dn}{dt} \int_0^1 \int_0^\infty e^{-\frac{z}{u\lambda}} \, dz \, du$$

$$= \frac{1}{2}\frac{dn}{dt} \int_0^1 -u\lambda \, e^{-\frac{z}{u\lambda}} \Big|_0^\infty \, du$$

$$= \frac{1}{2}\frac{dn}{dt} \int_0^1 u\lambda \, du$$

$$= \frac{1}{4}\frac{dn}{dt} \lambda$$

$$= \frac{\lambda n_0 \, e^{-\frac{t}{\tau}}}{4\tau}$$

c). $$\phi = \frac{7.5 \times 10^{-2} \, m \times 8.3 \times 10^{22} \, m^{-3}}{4 \times 3.78 \times 10^{16} s} e^{-\frac{t}{\tau}}$$

$$\overset{\tau \gg 1}{\approx} \frac{7.5 \times 10^{-2} \, m \times 8.3 \times 10^{22} \, m^{-3}}{4 \times 3.78 \times 10^{16} \, s}$$

$$= 4.12 \times 10^4 \, m^{-2} s^{-1} = 4.12 \, cm^{-2} s^{-1}$$