PowerCurve

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Build Function

We first build the functions of α and β.

$$\alpha(p) = \sum_{m \le k \le n} b(n,p,k)$$
$$\beta(p) = 1 - \sum_{m \le k \le n} b(n,p,k)$$

```
#function of alpha and beta
alpha <- function(m,n,p){
  tmp <- 0
  for (i in m:n) {
    tmp <- tmp + dbinom(i,n,p)
  }
  return(tmp)
}

beta <- function(m,n,p){
  tmp <- 0
  for (i in m:n) {
    tmp <- tmp + dbinom(i,n,p)
  }
  return(1-tmp)
}
```

Find Critical Value

$$\alpha(0.6) = \sum_{m \le k \le 100} b(100,0.6,k) < 0.05$$
$$\Longrightarrow m > 69$$

$$\beta(0.8) = 1 - \sum_{m \le k \le 100} b(100,0.8,k) < 0.05$$
$$\Longrightarrow m < 73$$

```
#calculate critical points
n <- 100
p1 <- 0.6
```

```r
p2 <- 0.8
type1 <- rep(0,100)
type2 <- rep(0,100)

for (i in 1:100) {
  type1[i] <- alpha(i,n,p1)
}
m1 <- min(which(type1<0.05))

for (i in 1:100) {
  type2[i] <- beta(i,n,p2)
}
m2 <- max(which(type2<0.05))

cat("Smallest m to let type-I error < 0.05:",m1,
    "\nLargest m to let type-II error < 0.05:",m2)
```

```
## Smallest m to let type-I error < 0.05: 69
## Largest m to let type-II error < 0.05: 73
```

Plor Curve

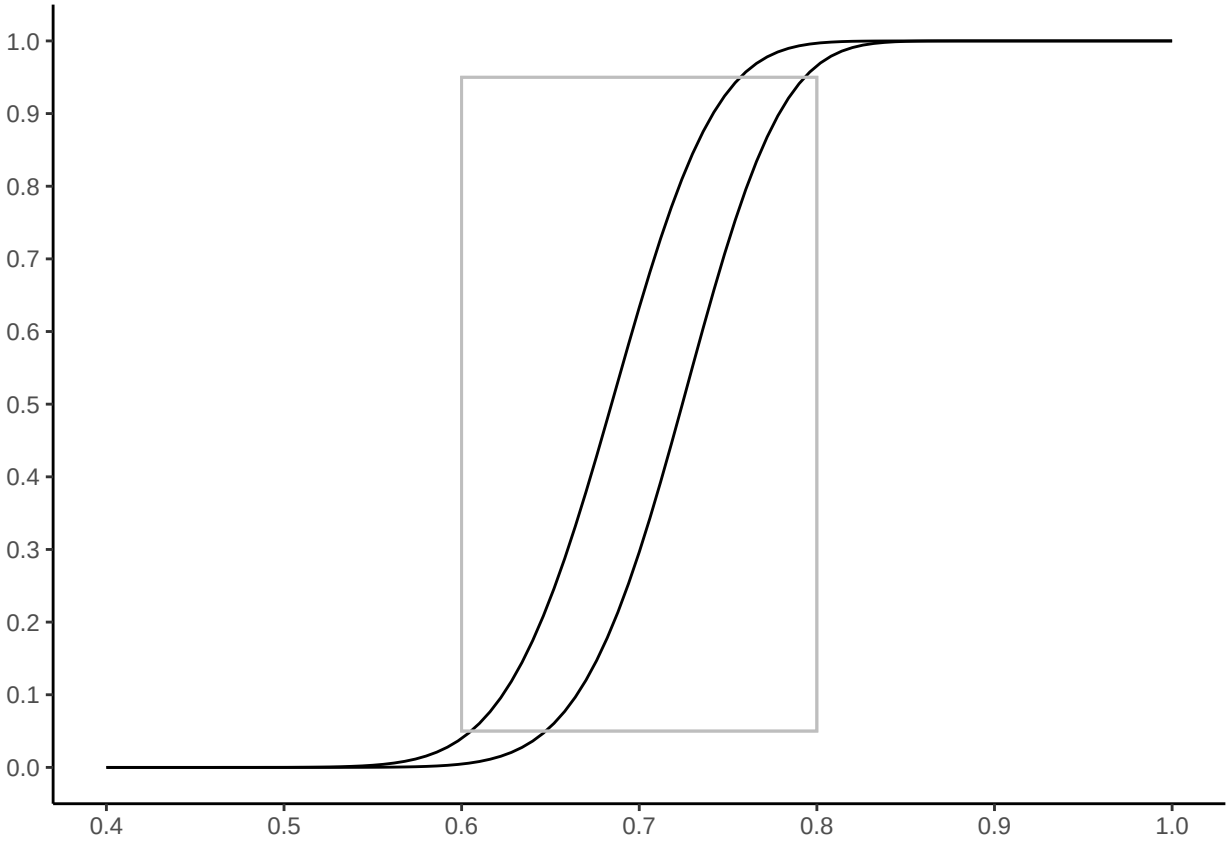
The left curve is using $m = 69$ and the right curve is using $m = 73$ which are the two critical values we get in the previous section. Since we require the type-I error and the type-II error to be smaller than 0.05, the value of these two curves should lie between 0.05 and $1 - 0.05$ for a given range of p. Then, what is this range or value? For type-I error, we choose $p = 0.6$. For type-II error, we choose $p = 0.8$. Therefore, the range should be betweeen 0.6 and 0.8.

```r
#plot
AlphaCurve1 <- function(x){
  sum(dbinom(69:100,100,x))
}

AlphaCurve2 <- function(x){
  sum(dbinom(73:100,100,x))
}

AlphaCurve1<-Vectorize(AlphaCurve1)
AlphaCurve2<-Vectorize(AlphaCurve2)

ggplot(data.frame(x=c(0.4,1)), aes(x=x)) +
  stat_function(fun=AlphaCurve1)+
  stat_function(fun=AlphaCurve2)+
  geom_rect(aes(xmin = 0.6, xmax = 0.8, ymin = 0.05, ymax = 0.95),
              fill = "transparent", color = "grey", size = 0.5)+
  scale_x_continuous(breaks = seq(0.4,1,0.1), minor_breaks = NULL)+
  scale_y_continuous(breaks = seq(0,1,0.1), minor_breaks = NULL)+
  theme_classic()+
  theme(axis.title.x=element_blank(),
        axis.title.y=element_blank())
```


1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
0.4
0.5
0.6
0.7
0.8
0.9
1.0