shortest-paths-algorithms

February 14, 2024

```
[55]: import random
import networkx as nx
import matplotlib.pyplot as plt
from itertools import combinations, groupby
```

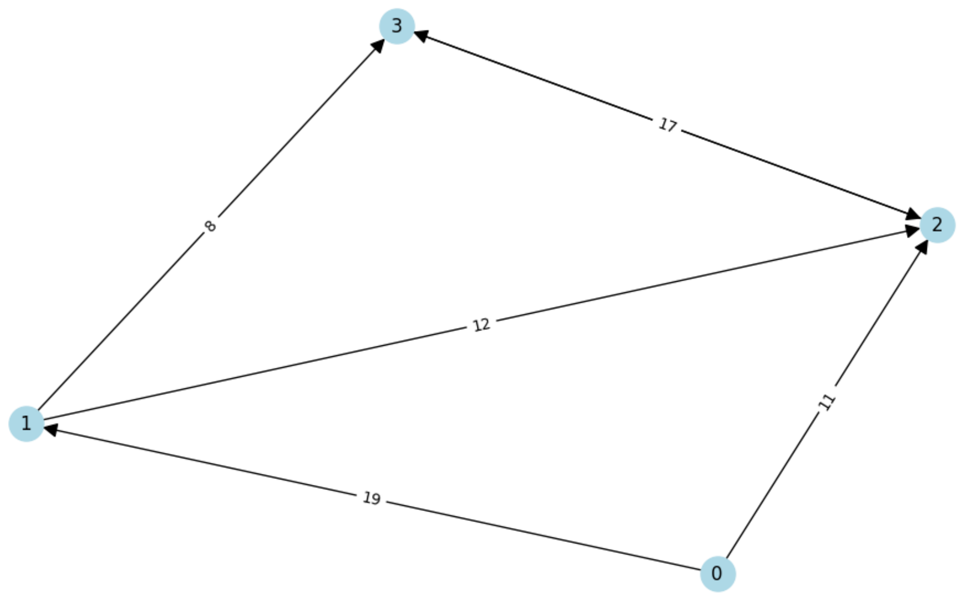
0.1 Generating graph

```
[56]: # You can use this function to generate a random graph with 'num_of_nodes' nodes
# and 'completeness' probability of an edge between any two nodes
# If 'directed' is True, the graph will be directed
# If 'draw' is True, the graph will be drawn
def gnp_random_connected_graph(num_of_nodes: int,
                               completeness: int,
                               directed: bool = False,
                               draw: bool = False):
    """
    Generates a random graph, similarly to an Erdős-Rényi
    graph, but enforcing that the resulting graph is conneted (in case of␣
 ↪undirected graphs)
    """


    if directed:
        G = nx.DiGraph()
    else:
        G = nx.Graph()
    edges = combinations(range(num_of_nodes), 2)
    G.add_nodes_from(range(num_of_nodes))

    for _, node_edges in groupby(edges, key = lambda x: x[0]):
        node_edges = list(node_edges)
        random_edge = random.choice(node_edges)
        if random.random() < 0.5:
            random_edge = random_edge[::-1]
        G.add_edge(*random_edge)
        for e in node_edges:
            if random.random() < completeness:
                G.add_edge(*e)

    for (u,v,w) in G.edges(data=True):
        w['weight'] = random.randint(-5, 20)

    if draw:
        plt.figure(figsize=(10,6))
        if directed:
            # draw with edge weights
            pos = nx.arf_layout(G)
            nx.draw(G,pos, node_color='lightblue',
                    with_labels=True,
                    node_size=500,
                    arrowsize=20,
                    arrows=True)
            labels = nx.get_edge_attributes(G,'weight')
            nx.draw_networkx_edge_labels(G, pos,edge_labels=labels)

        else:
            nx.draw(G, node_color='lightblue',
                with_labels=True,
                node_size=500)

    return G
```

0.2 Bellman-Ford algorithm

[57]:
```
from networkx.algorithms import bellman_ford_predecessor_and_distance
```

[58]:
```
G = gnp_random_connected_graph(4, 0.5, True, True)
```



[71]:
```
# pred is a dictionary of predecessors, dist is a dictionary of distances
def nx_bellman_ford(G, node):
    try:
        _, dist = bellman_ford_predecessor_and_distance(G, node)
        sorted_keys = sorted(dict(dist).keys())
        return {i: dict(dist)[i] for i in sorted_keys}
    except:
        return "Negative cycle detected"
print(nx_bellman_ford(G, 0))
```

```
{0: 0, 1: 19, 2: 11, 3: 25}
```

[60]:
```
def bellman_ford(G, node):
    distances = [float('inf')] * G.number_of_nodes()
    distances[list(G.nodes)[node]] = 0
    for _ in range(G.number_of_nodes() - 1):
        for i, k ,w in G.edges(data=True):
            if distances[i] != float('inf') and distances[i] + w['weight'] <␣
↪distances[k]:
                distances[k] = distances[i] + w['weight']
    for i, k, w in G.edges(data=True):
        if distances[i] != float('inf') and distances[i] + w['weight'] <␣
↪distances[k]:
            return "Negative cycle detected"
    return distances
```

```
print(bellman_ford(G, 0))
```

```
[0, 19, 11, 25]
```

0.3 Floyd-Warshall algorithm

[61]:
```
from networkx.algorithms import floyd_warshall_predecessor_and_distance
```

[62]:
```
# pred is a dictionary of predecessors, dist is a dictionary of distances␣
 ↪dictionaries
def nx_floyd_warshall(G):
    distances = []
    _, dist = floyd_warshall_predecessor_and_distance(G)
    for value in dist.values():
        sorted_nodes = sorted(dict(value).keys())
        distances.append(list({i: dict(value)[i] for i in sorted_nodes}.
 ↪values()))
    for j in range(len(distances)):
        if distances[j][j] < 0:
            return "Negative cycle detected"
    return distances
print(nx_floyd_warshall(G))
```

```
[[0, 19, 11, 25], [inf, 0, 12, 8], [inf, inf, 0, 14], [inf, inf, 17, 0]]
```

[63]:
```
def floyd_warshall(G):
    n = G.number_of_nodes()
    diction = {(x[0], x[1]):x[2]['weight'] for x in G.edges(data=True)}
    matrix = [[0 for _ in range(n)] for _ in range(n)]
    for i, line in enumerate(matrix):
        for k in range(len(line)):
            if i == k:
                matrix[i][k] = 0
            elif (i, k) in G.edges():
                matrix[i][k] = diction[(i, k)]
            else:
                matrix[i][k] = float('inf')
    for a in range(n):
        for b in range(n):
            for c in range(n):
                if matrix[b][c] > matrix[b][a] + matrix[a][c]:
                    matrix[b][c] = matrix[b][a] + matrix[a][c]
    for j in range(n):
        if matrix[j][j] < 0:
            return "Negative cycle detected"
    return matrix
print(floyd_warshall(G))
```

```
[[0, 19, 11, 25], [inf, 0, 12, 8], [inf, inf, 0, 14], [inf, inf, 17, 0]]
```

0.4 Testing

0.4.1 1. Testing correctness

We need to check whether the algorithms return the same data with the same input.

Let's test them on graphs of different sizes.

[78]:
```
def test(name):
    for n in range(1, 7):
        n = 2 ** n
        print(">>> Graph size:", n)
        for j in range(1, 5):
            G = gnp_random_connected_graph(n, 1 / j, True, False)
            if name == 'f':
                nx_result = nx_floyd_warshall(G)
                our_result = floyd_warshall(G)
                assert nx_result == our_result
            if name == 'b':
                for k in range(n):
                    nx_result = nx_bellman_ford(G, k)
                    if isinstance(nx_result, dict):
                        nx_result = list(nx_bellman_ford(G, k).values())
                    our_result = bellman_ford(G, k)
                    if isinstance(our_result, list) and float('inf') in␣
 ↪our_result:
                        our_result = [x for x in our_result if x !=␣
 ↪float('inf')]
                    assert nx_result == our_result
```

[79]:
```
print("Test Bellman-Ford implementation")
test('b')
print("Test Floyd-Warshall implementation")
test('f')
```

```
Test Bellman-Ford implementation
>>> Graph size: 2
>>> Graph size: 4
>>> Graph size: 8
>>> Graph size: 16
>>> Graph size: 32
>>> Graph size: 64
Test Floyd-Warshall implementation
>>> Graph size: 2
>>> Graph size: 4
>>> Graph size: 8
>>> Graph size: 16
>>> Graph size: 32
>>> Graph size: 64
```

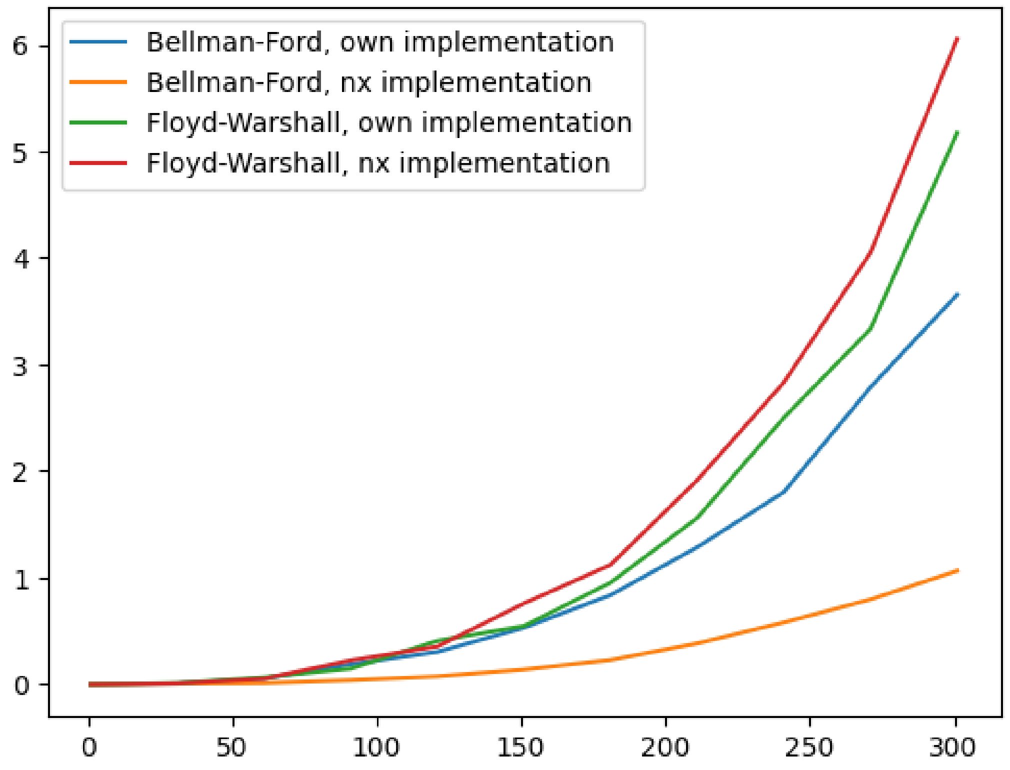
0.4.2 Time tests

Testing time performance of algorithms

[80]:
```
import time
def plot_time_size(algorithms: dict[str, callable], max_nodes: int,␣
↪completeness: float, step: str):

    algorithms_performance = {algorithm: [] for algorithm in algorithms}

    x = list(range(1, max_nodes+step, step))

    for i in x:
        g = gnp_random_connected_graph(i, completeness, True, False)

        for algorithm, algorithm_function in algorithms.items():
            if algorithm_function in [bellman_ford,␣
↪bellman_ford_predecessor_and_distance]:
                start = time.perf_counter()
                try:
                    algorithm_function(g, 0)
                except:
                    pass
                end = time.perf_counter()
            else:
                start = time.perf_counter()
                try:
                    algorithm_function(g)
                except:
                    pass
                end = time.perf_counter()
            algorithms_performance[algorithm].append(end-start)

    for algorithm, performance in algorithms_performance.items():
        plt.plot(x, performance, label=algorithm)
    plt.legend()

    plt.show()
```

[84]:
```
algorithms = {
    'Bellman-Ford, own implementation': bellman_ford,
    "Bellman-Ford, nx implementation": bellman_ford_predecessor_and_distance,
    "Floyd-Warshall, own implementation": floyd_warshall,
    "Floyd-Warshall, nx implementation": floyd_warshall_predecessor_and_distance
}
```

```
plot_time_size(algorithms, 300, 0.5, 30)
```



Conclusions

As can bee seen, we implemented Bellman-Ford and Floyd-Warshall algorithms that return correct matrix (see Testing correctness stage).

Floyd-Warshall algorithm analysis

Our realization of Floyd-Warshall algorithm works almost as fast as nx realization for all tested combinations, so can be used in both cases.

Bellman-Ford algorithm analysis

Our implementation of Bellman-Ford algorithm works faster than nx one. This implementation can be used on bigger graphs. As the number of nodes increases, the performance difference also increases.

Overall analysis

As can be seen from graphs both Bellman-Ford and Floyd-Warshall algorithms work as good as nx implementation on a small number of nodes. But as the number of nodes increases, the difference becomes noticeable.