Problem Chosen	2022 MCM/ICM Summary Sheet	Team Control Number
E		2210081

Less is More: A Model for Optimizing the Forest Management Strategies

Summary

Happiness flutters in the air whilst we rest among the breaths of nature. — Kelly Sheaffer

Climate change has dramatically affected the world adversely in the past decades, causing severe casualties and property damage. Rearrange of forest management plans can optimize the efficiency of carbon storage. To this end, mathematical models are developed to predict forest carbon storage and optimize the management plans of forests.

In this paper, the Carbon Sequestration Model is developed to determine the amount of stored carbon by using linear regression and the relationship between carbon storage and the forest's area. The model also considers the forest's different growth characteristics in 5 stages and quantifies the carbon sequestration rate. Logistic model is employed to estimate the loss rate of the forest products. The spectrum of management plans is given based on the analysis results.

Then, the Comprehensive Decision Model is constructed from six aspects: resource, environment, economy, culture, social population, and policy. Six factors from these aspects were selected: carbon storage, GDP per capita, public attitudes, population size, biological resources, and forest area. Global forest data were collected to establish the dataset used by the model. Afterward, an improved decision model that combined the entropy weight method and TOPSIS was presented to give the optimal dynamic forest management plans.

Next, the application of the model was carried out on the Komi Virgin Forest in Europe to analyze the current situation of the forest. The models also predicted the growth trend of the forest in 100 years from 2010 for the subsequent decision-making process by implementing the grey prediction model. The expected amount of sequestered carbon in the Komi Virgin Forest over 100 years is 2.488 billion tons. The most suitable forest management plans, which included one round of harvest, were made based on the results of the models.

Finally, the sensitivity analysis of the model was conducted. The input values were set to fluctuate by 10% randomly, and the model results remain stable, proving the model's robustness.

Keywords: Forest Sequestration; Carbon Linear Regression; Logistic Model;TOPSIS; Grey Prediction Model; AHP

Contents

1 Introduction

1.1 Problem Background

Since the Industrial Revolution, human has emitted a vast quantity of greenhouse gases (GHGs) such as carbon dioxide (CO_2), methane (CH_4), and chlorofluorocarbons (CFCs) into the atmosphere, causing global climate change and subsequent devastating extreme climate events and natural disasters. Among these GHGs, CO_2 acts as the most effective physical driver of climate change[4].

As a result, reducing the CO_2 content in the atmosphere is of great urgency. Carbon sequestration, using biosphere and by mechanical processes, proves to be promising in the cause of climate change mitigation.

Through photosynthesis, the forest can absorb CO_2 in the atmosphere and sequestrate carbon in the form of organic matter. There have been claims that keeping all the forests uncut can maximize the effect of carbon sequestration. However, considering multiple forestry and socio-economic factors, different forest management strategies may sequestrate carbon more efficiently.

1.2 Restatement of the Problem

Based on the problem statement, major tasks needed to be solved is listed as follows:

- **Task 1:** Develop a carbon sequestration model and determine the amount of sequestered CO_2 as well as the ideal forest management plan.
- **Task 2:** Generalize the model to different forests, forecast the carbon sequestration effect and suppose the optimal management plan, including harvesting.
- **Task 3:** Compose a newspaper article to identify the reasons for including harvesting instead of keeping the forest uncut.

1.3 Our Work

For the sake of convenient and vivid expression, a flowchart (figure 1) of our work and train of thought is presented.

1. Develop the Carbon Sequestration Model using linear regression and logistic model. Find out the relationship between the amount of stored carbon and the growth stage of the forest.
2. Choose the factors from different aspects to develop the Comprehensive Decision Model and give a detailed and practicable forest management plan.
3. Make predictions of the amount of carbon that the forest and its products sequester over 100 years by implementing the Grey Prediction Model and the results obtained by two former models developed above.
4. Write a newspaper article arguing that the timely harvesting of the forests is the best way to optimize the carbon sequestration capability of the forests and their products to convince the community that the way this paper presents is the best for their forest.

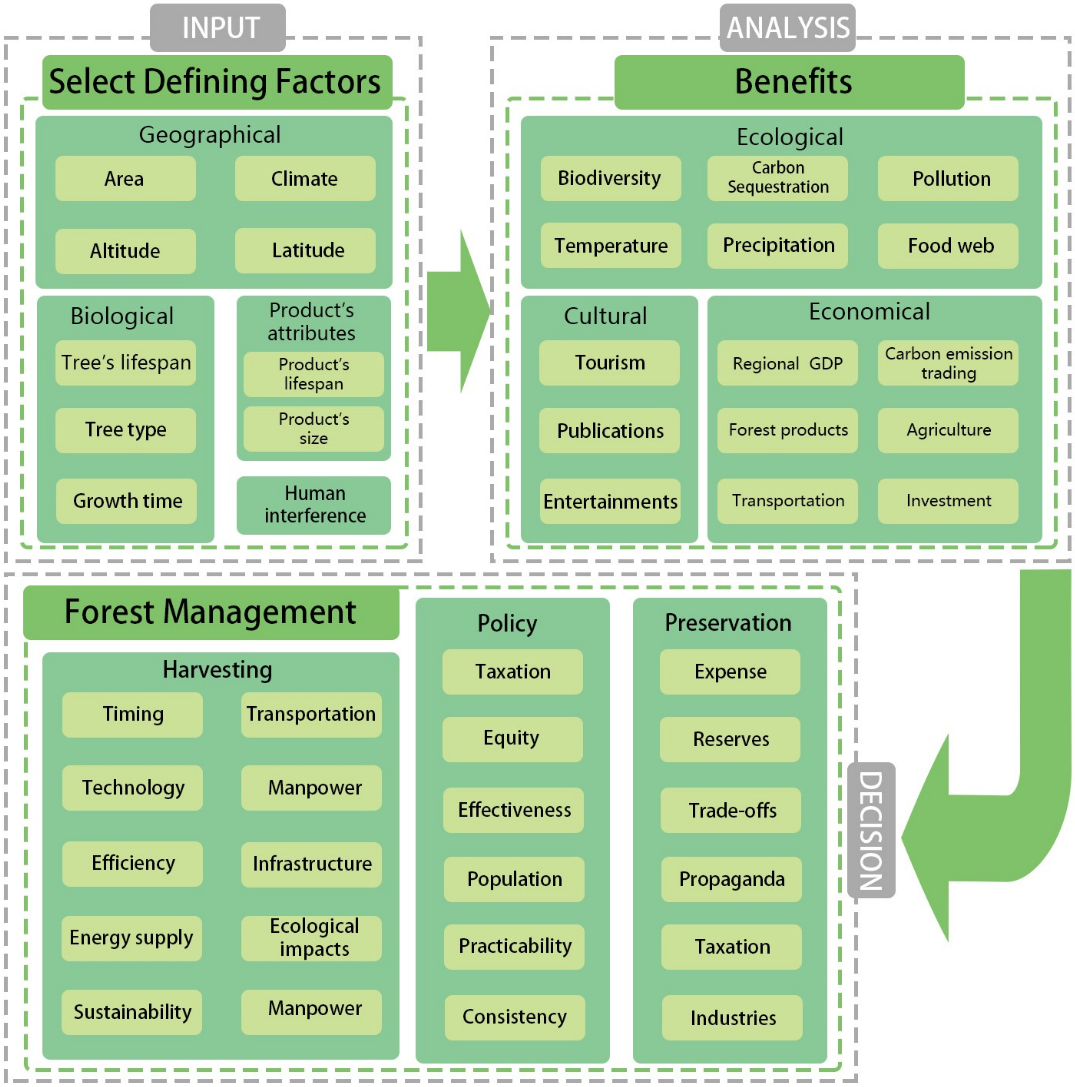

Figure 1: Flowchart of our work

2 Assumptions

1. **Trees have five growth stages, each of which has the same span of 20 years.**
2. **Ages of trees in the same forest distribute uniformly over these stages.**
3. **The products of the forest exclude the products that serve as fuel.**
4. **No significant natural disaster happens, and the climate condition keeps stable during the period considered in this model.**

3 Notations

The mathematical notations used in this paper are listed in Table.

Table 1: Notations used in this paper

Symbol	Description
S_{area}	Area of the forest
Y_h	Amount of carbon storage per hectare or Carbon storage density
t	Growth time of the forest
Y_t	Total amount of sequestered carbon
T_{over}	Carbon storage density during Over-mature Stage
M_{over}	Amount of carbon storage during Over-mature Stage
T_{bos}	Time span before the Over-mature Stage

4 Model Preparation

4.1 Data Collection

Table 2: Data

Data source	Source
Carbon storage density	Literature [5]
Current area of forests	`https://ourworldindata.org/forest-area`
Comprehensive data of forests	`https://data.globalforestwatch.org/`
GDP and History area of Virgin Komi Forest	`https://whc.unesco.org/en/list/719/documents/`

5 Carbon Sequestration Model

To develop the Carbon Sequestration Model (hereafter CSM), estimating the effect of carbon sequestration is a necessity. The fundamental relationship between the density of sequestered carbon in the tree stratum of forest and the tree's attributes[5] is

$$P_c = S \times D \times R \times C \tag{1}$$

where S is the total volume of usable wood in the forest, D is the density of trunk, R is the ratio of trunk's biomass in the total biomass of the trees, C is the proportion of carbon content of trees.

To describe the growth of forest,relevant information is collected. The 5 stages of tree growth is **Young, Middle-aged, Near-mature, Mature and Over-mature Stage** [5], also, as mentioned in **Assumption 1**, each stage is about 25 years. So the estimated lifespan of one generation of forest is about 125 years.

The basic model for carbon sequestration is

$$Y_t = S_{area} \times Y_h \tag{2}$$

where Y_t is the total amount of sequestrated carbon, S_{area} is the area of the forest, Y_h is carbon storage density.

5.1 Case of one round of growth

Consider the case of one round of growth, which is based on the premise that **all the trees in the same forest start growing simultaneously and then grow uniformly.** The feasibility of this assumption lies in the fact that due to the large number of trees in one single forest, deviations of growth rate and biomass of each tree's from the forest's average level is too small to notice when looking at the big picture. Thus, it's reasonable for this version of the model to ignore these minor fluctuations.

Also, **no further planting of young trees or tree seeds is carried out**. Moreover, it is assumed that **in the Over-mature Stage, the growth of forest stalls**. So the 100th year is the turning point of forest growth, after which the increment of forest biomass and carbon sequestration can be ignored.

Data of carbon storage density, which shows the forest's capability of sequestering the carbon from CO_2 in the atmosphere, is shown in Table 3.

Table 3: Carbon storage of different forests[5]

Forest type	Amount of carbon storage per hectare/Mg·ha^{-1}				
	Young	Middle-aged	Near-mature	Mature	Over-mature
Rainforest	30.87	82.95	110.83	134.09	287.50
Broadleaf mixed forest	17.13	36.83	46.51	50.14	79.65
Coniferous and broadleaf mixed forest	13.59	46.44	58.00	86.85	116.48
Coniferous mixed forest	17.57	33.53	46.47	98.87	131.96

Linear Regression is implemented (figure 2)to generate the first half of the continuous piece-wise function (function 3) to determine the relationship between the amount of stored carbon per hectare and time:

$$Y_h = f(t) = \begin{cases} a_i \cdot t + b_i & t \in [0, 100], i = 1, 2, 3, 4. \\ T_i & t \in (100, +\infty), i = 1, 2, 3, 4. \end{cases} \tag{3}$$

where t is the growth time of the forest, a_i and b_i is the coefficient and intercept of linear respectively, T_{over} is the carbon storage density (amount of carbon storage per hectare) during the Over-mature Stage.

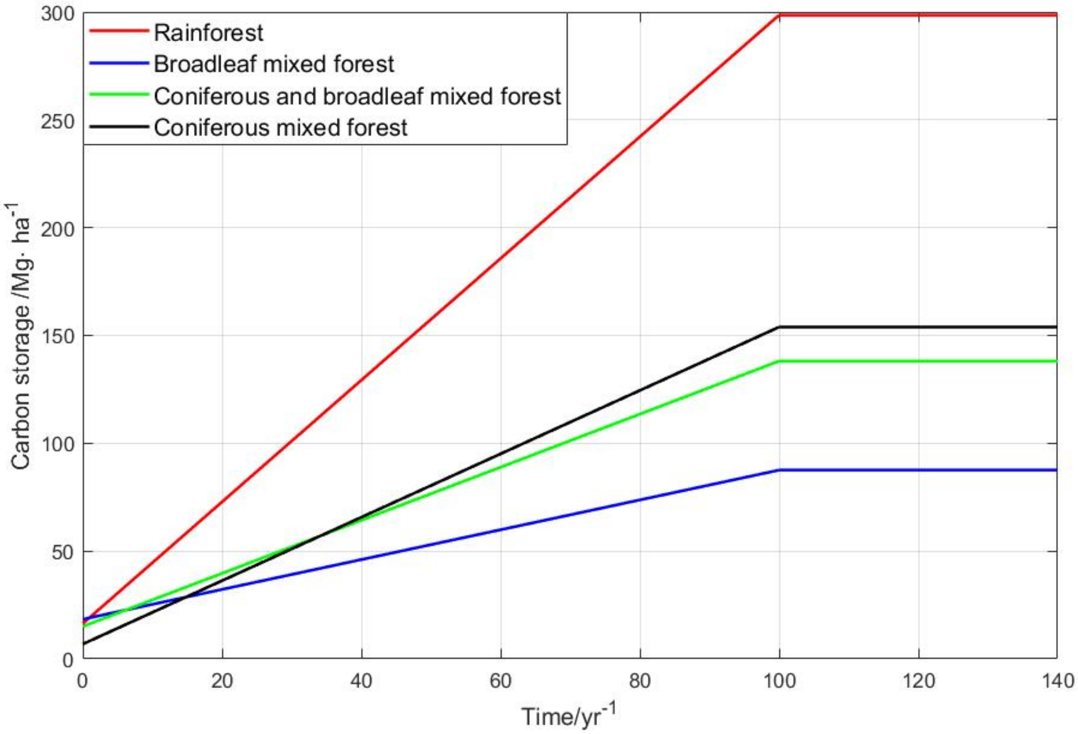

Figure 2: Relationship between amount of carbon storage per hectare and time

Also, the goodness of fit (R^2) of the four curves falls around 0.9, showing that the linear relationship between Y_h and t is significant.

From the graph of Function (3), it can be concluded that regardless of the forest types, carbon storage density (Y_h) reaches its maximum during the Over-Mature Stage.

Consider the durability of wood products made from the harvested over-mature trees. Assume that the erosion of wood products results from the growth of fungus (i.e., mold) and insects (i.e., termite). The growth of these living organisms can be described by the famous Logistic Model:

$$\begin{cases} \frac{\mathrm{d}x}{\mathrm{d}t} = r_0 \left(1 - \frac{x}{x_m}\right) \\ x(t_0) = x_0 \end{cases} \tag{4}$$

where $x(t)$ is the mass of organic matter, r_0 is the initial growth rate of the organism, x_m is the maximum mass of organic matter that the given environment can sustain, and x_0 is the initial mass of organic matter.

The solution of Logistic Model is

$$x(t) = \frac{x_m}{1 + \left(\frac{x_m}{x_0} - 1\right) e^{-r_0(t-t_0)}} \tag{5}$$

where t_0 is the initial time point.

As the fungus and insects assumed above live on the wood products, another assumption suggested is that the increment of the organic matter comes from the corrosion of wood products. Thus, the remaining amount of sequestered carbon can be presented by

$$M_r = M_{over} - \frac{M_{over}}{1 + \left(\frac{M_{over}}{x_0} - 1\right) e^{-r_0(t-t_0)}} \tag{6}$$

where M_r is the remaining amount of sequestered carbon, M_{over} is the initial amount of sequestered carbon in wood products, as well as the amount of carbon storage during the Over-mature Stage. Figure 2 suggest the management plan that **all the trees in the Over-mature Stage are cut and turned into the wood products**. Set the initial time to the point when the forest starts growing (the point where *Time* is 0 in figure 2). Then t_0 is the point that forest enters the Over-mature Stage. The organism's growth rate at the initial time when the wood product is manufactured determines how the growth rate changes when fewer products' material is available due to the constant consumption of the product by the organism. The specific value of r_0 is the function of the wood product's temperature, humidity, organism type, and material attributes.

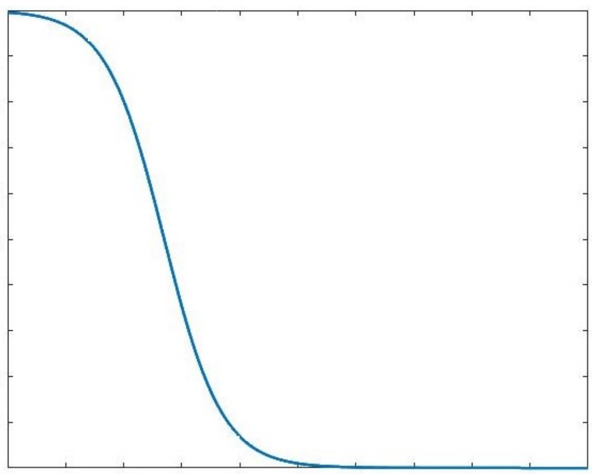

Figure 3: Schematic graph of equation 6

Then, the total amount of sequestered carbon (Y_t) can be described as

$$Y_t = \begin{cases} S_{area} \times (a \cdot t + b) & \text{before Over-mature Stage} \\ S_{area} \cdot T_{over} - \dfrac{S_{area} \cdot T_{over}}{1 + \left(\dfrac{S_{area} \cdot T_{over}}{x_0} - 1\right) e^{-r_0(t-t_0)}} & \text{during the Over-mature Stage} \end{cases} \tag{7}$$

Consequently, **best forest management plan** in the case of one round of growth can be stated as follows:

- During the forest's first four growth stages(Young, Middle-aged, Near-mature, and Mature Stage), keep the forest uncut to maximize the amount of carbon storage and obtain the best sequestration effect.

- Once the forest enters the Over-mature Stage, harvest can be carried out to make room for a new round of forest.

5.2 Case of multiple rounds of growth

Based on the discussions above, an improved version of model 7 is presented:

$$Y_t = \left\{ S_{area} \cdot T_{over} - \frac{S_{area} \cdot T_{over}}{1 + \left(\frac{S_{area} \cdot T_{over}}{x_0} - 1 \right) e^{-r_0(t-t_0)}} \right\} \cdot \left[\frac{t}{T_{bos}} \right] + S_{area} \cdot \left\{ a \cdot \left\{ t - \left[\frac{t}{T_{bos}} \right] \cdot T_{bos} \right\} + b \right\} \tag{8}$$

where the symbol [] means *rounding down*, T_{bos} means time span before the forest entering Over-mature Stage. This model means that people can use the same woodland repeatedly by planting new trees after harvesting the previously planted trees from the last forest growth round.

6 Comprehensive Decision Model

Figure 4: Factors considered in the Decision Model

When making forest management plans, considering only one aspect of the forest is not enough to balance different needs. The consequence of being partial concerning conflicting objectives is horrific, as whatever the decision is, residents or even the whole world will pay for the aftermath of partially-decided choices.

Multiple factors ranging from economy to environment is selected: the amount of sequestered carbon (CO_2), level of local economy measured by Gross Domestic Product (GDP), environmental conditions measured by biodiversity, degree of development indicated by population density and people's views and values on forest measured by cultural characteristics.

The Decision Model is based upon data of 3 forests from different parts of the world:

- Greater Khingan Range Forest in Northeast China

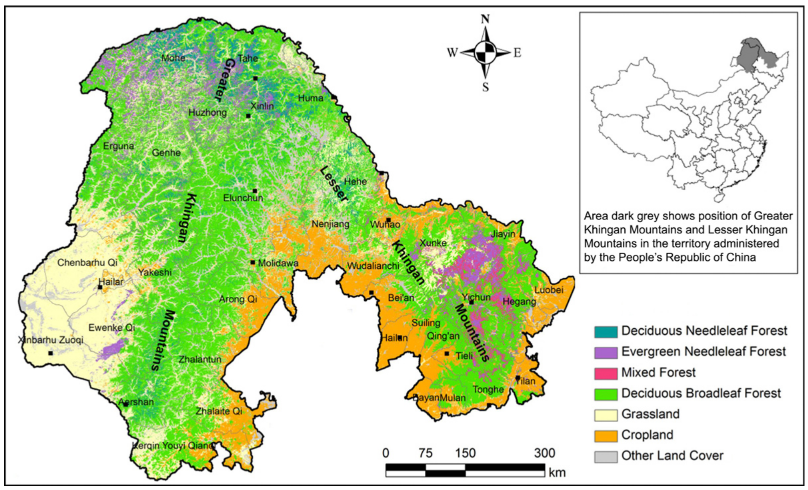

Figure 5: Location of Greater Khingan Range Forest [1]

- Angeles National Forest in Southwestern United States

[1]Source: `https://shanghaibirding.com`

Figure 6: Location of Angeles National Forest[2]

- Amazon Rainforest in Amazon basin of South America

Figure 7: Location of Amazon Rainforest [3]

[2]Source: `https://wiki.radioreference.com`
[3]Source: `https://peru-explorer.com`

6.1 Quantification of factors

All the five factors mentioned above can not be measured intuitively by numeric data, so it's necessary to comb through various data to find useful, relevant indicators. Why and how these factors are built will be discussed respectively below.

6.1.1 Amount of Sequestered Carbon

Global warming, climate change, and hazardous extreme weather events, are increasingly threatening. The only way of getting rid of these disasters is to implement carbon sequestration employing more efficient forest management. The Carbon Sequestration Model describes the forest's capability of stocking the carbon:

$$Y_t = S_{area} \times Y_h \tag{9}$$

A forest is composed of different types of trees, which have different values of Y_h (carbon storage density). To estimate the overall effectiveness of a forest's carbon storage, the area of 3 forests is listed in table 4.

Table 4: Area of 3 forests

Forest	Area/hectare
Greater Khingan Range Forest	327,200
Angeles National Forest	2,833.5
Amazon Rainforest	5,500,000

In equation $Y_t = S_{area} \times Y_h$, the Y_h, which reflects the components of trees a forest comprises, is determined by climate zone and geographical location of the forest. Because the proportion data of components of a forest is hard to obtain, approximation of each component's proportion is used for quantification.

Despite the specific proportion of trees of each forest, the major types of trees are determined by the climate zone the forest is affiliated with. This attribute of the forest has universality across the world. The representative types and corresponding carbon storage densities (Y_h) of the three forests are listed in table 5. As the Amazon Rainforest consists of numerous kinds of trees, its components are hard to estimate. The Amazon Rainforest is taken as a whole by the category of *Tropical Rainforest*.

Table 5: Representative types of 3 forests

Forest	Type	Carbon storage density
Greater Khingan Range Forest	dahurian larch	80.82
Angeles National Forest	cedar	31.33
Amazon Rainforest	tropical rainforest	79.65

Using equation 9, calculate the total amount of sequestered carbon, which shows the capability of carbon sequestration. The outcomes are listed in table 6.

Table 6: Total amount of sequestered carbon in 3 forests

Forest	Amount of sequestered carbon/Mg
Greater Khingan Range Forest	26444304
Angeles National Forest	88774
Amazon Rainforest	438075000

6.1.2 Economic Level & Degree of Development

Generally, the richer a country is, the more money is spent on the cause of environmental preservation, including measures to make use of its forests in an environmentally friendly way. The poorer country is more likely to exploit its forest resources aggressively regardless of the long-term negative ecological impacts. So economic factors to be included to measure the potential and financial ability of the country to preserve its forest while taking advantage of it.

Two monetary indicators: *Gross Domestic Product* (GDP) and *GDP per capita* are used to measure the economic level of the region around the forests. The data collected is listed in table 7.

Table 7: Regional economic data of 3 forests

Forest	Statistical region	GDP/billion U.S. dollars	Population	GDP per capita /U.S. dollars
Greater Khingan Range Forest	Inner Mongolia Autonomous Region,China	272	25340000	10739
Angeles National Forest	Los Angeles, CA, U.S.	711	3898747	182341
Amazon Rainforest	Brazil	1809	190000000	9521

6.1.3 Environmental Conditions & Public Attitudes

The present situation of the forest or environmental conditions can be described by biodiversity. Generally speaking, at a given altitude, a forest of lower latitude has greater biodiversity[2]. As a result, ranks of 3 forests' biodiversity is obtained by comparing the forests' latitude and altitude by implementing *Analytic Hierarchy Process* (AHP), the outcome is listed in table 8.

Public attitudes on the preservation of forests can significantly affect the local government's resolution and financial support concerning forest preservation[1]. Suppose the public identifies the potential benefits of balancing forest development's short-term and long-term interests. In that case, their positive views on environmental protection will make a difference to the implementation of forest policies of the government. AHP is used to evaluate the residents' attitudes and opinions on protective practices on forests, and the results are shown in table 8.

In both factors above, the total number of ranks is 5. A more significant rank number means more positive.

Table 8: AHP results

Forest	Rank of environmental conditions	Public attitudes
Greater Khingan Range Forest	3	4
Angeles National Forest	3	3
Amazon Rainforest	5	3

6.2 Overall Discussion on the Decision Model

The considered factors are listed in table 9:

Table 9: Factors and abbreviations

Factor	Abbreviation	Data unit
Amount of Sequestered Carbon	CO_2	Mg(tonne)
Degree of Development	GDP per capita	USD
Environmental Conditions	Biodiversity	5-level rank
Public Attitudes	Culture	5-level rank
Degree of human Interference	Population	billion USD
Breadth and ecological effect of the forest	Area	hectare

Take table x as a matrix $\mathbf{A}_{n\times m} = (a_{ij})_{m\times n}$, whose rows have subscript i and columns have subscript j. Suppose that w_j is the weight of each field. The greater the weight is, the more contributions it makes to the final decision.

The forests' data selected used to determine the weights are listed in table 10.

Table 10: Forests' data collected

Forest	CO_2	GDP per capita	Biodiversity	Culture	Population	Area
Greater Khingan Range Forest	32.07	10738.8	3	4	25340000	32720000
Angeles National Forest	0.26	182340.6	3	3	3898747	265200
Amazon Rainforest	938.00	9521.1	5	3	190000000	700000000
Changbai Mountain Forest	0.17	5894.1	2	3	31455000	172000
Shennongjia National Forest	0.16	12117.4	3	4	59270000	325300
Sipsongpanna Rainforest	0.32	7556.6	4	4	48580000	242000
Congo Rainforest	116.43	566.6	5	1	89600000	124200000
Valdivia Temperate Rainforest	33.25	14571.5	3	2	3252	733459300

6.2.1 Determination of Weights

To determine the values of weights, *Entropy Weight Method* is implemented. The procedures are as follows.

1. For each a_{ij} in matrix $\mathbf{A}$, calculate the proportions p_{ij}:

$$p_{ij} = \frac{a_{ij}}{\sum_{i=1}^{n} a_{ij}} \qquad i = 1, 2, \ldots, m, \quad j = 1, 2, \cdots, n. \tag{10}$$

2. For each field (column of matrix), calculate the entropy e_j and coefficient of variation $(C_v)_j$:

$$\begin{cases} e_j = -\dfrac{\sum_{i=1}^{n} p_{ij} \ln p_{ij}}{\ln m} \\ (C_v)_j = 1 - e_j \end{cases} \qquad j = 1, 2, \cdots, n. \tag{11}$$

3. For each $(C_v)_j$, calculate the proportion:

$$w_j = \frac{(C_v)_j}{\sum_{i=1}^{n} (C_v)_j} \qquad j = 1, 2, \cdots, n. \tag{12}$$

Now, the vector of weight is generated:

$$W = \left\{ w_1, w_2, \cdots, w_j \right\} \tag{13}$$

The weights are listed in table 11.

Table 11: Weight derived by Entropy Weight Method

Factor	Weight
CO_2	0.302
GDP per capita	0.226
Biodiversity	0.053
Culture	0.037
Population	0.100
Area	0.283

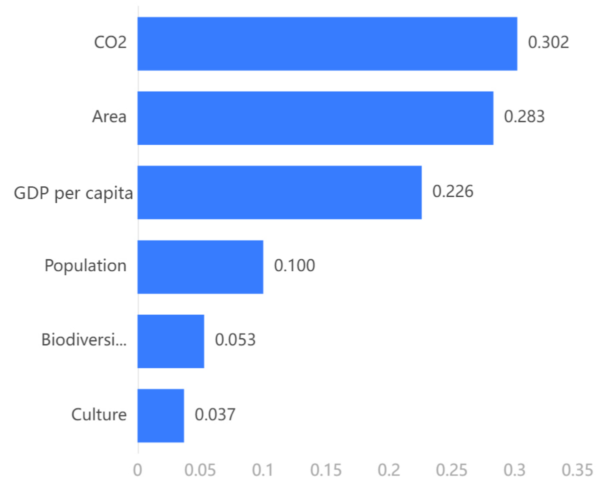

Figure 8: Histogram of weights

6.2.2 Data Standardization and Normalization

As in table 9, the factors have different units, which challenges the analysis of the data. Also, the scales of factor value vary with great range, and it's hard to find a one-size-fits-all standard to evaluate the essence of the data. As a result, the *standardization* and *normalization* is need before further analysis.

The factors in table 9, the greater their values are, the more positive the situation they represent. To standardize these data, several methods of standardization are available. One common method is *0-1 standard transformation*:

$$b_{ij} = \frac{a_{ij} - a_j^{min}}{a_j^{max} - a_j^{min}} \qquad i = 1, 2, \ldots, m, \quad j = 1, 2, \cdots, n. \tag{14}$$

where a_{ij} is the original data item in the matrix, b_{ij} is the transformed item.

Another way of normalization called *Vector Normalization* is also commonly used to standardize the data:

$$b_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \qquad i = 1, 2, \ldots, m, \quad j = 1, 2, \cdots, n. \tag{15}$$

The characteristic of this method is that the sum of square of items under the same field is one, so it's appropriate for calculating and comparing the Euclidean distance on the same scale.

6.2.3 Implementation of TOPSIS and Entropy Weight Method

Then, *Technique for Order of Preference by Similarity to Ideal Solution* (TOPSIS) is applied to get the comprehensive evaluation index for the Decision Model. The main principle of TOPSIS is that by calculating the Euclidean distance between sample point and positive ideal solution, the optimal degree is figured out. But there is one flaw in TOPSIS that the weights of data with different attributes is given arbitrarily or subjectively, which lacks objectivity and consistency. To overcome this fault, a combination of TOPSIS and Entropy Weight Method is used where the weight vector that TOPSIS uses is derived by Entropy Weight Method.

The steps are as follows:

1. Calculate the normalized decision matrix $\mathbf{D}_{normal} = (d_{ij})_{m\times n}$ from original data matrix $\mathbf{A}_{original} = (a_{ij})_{m\times n}$

$$d_{ij} = \frac{a_{ij}}{\sqrt{\sum_{i=1}^{m} a_{ij}^2}} \qquad i = 1, 2, \dots, m, \quad j = 1, 2, \cdots, n. \tag{16}$$

2. Calculate the weighed-normalized matrix $\mathbf{E}_{wn} = (e_{ij})_{m\times n}$:

$$e_{ij} = w_j \cdot d_{ij} \qquad i = 1, 2, \dots, m, \quad j = 1, 2, \cdots, n. \tag{17}$$

Note: the weight vector used is generated by Entropy Weight Method.

3. Calculate the best alternative $(A_b)_j$ and worst alternative $(A_w)_j$:

$$\begin{cases} (A_b)_j = max\left\{e_{ij} | i = 1, 2, \cdots, m\right\} \\ (A_w)_j = min\left\{e_{ij} | i = 1, 2, \cdots, m\right\} \end{cases} \qquad j = 1, 2, \cdots, n. \tag{18}$$

$(A_b)_j$ and $(A_w)_j$ define the ideal best and worst case of in the sample space.

4. Calculate the Euclidean distances between the sample points and best alternative point $(d_b)_i$ and $(d_w)_i$:

$$\begin{cases} (d_b)_i = \sqrt{\sum_{j=1}^{n}(e_{ij} - (A_b)_j)^2} \\ (d_w)_i = \sqrt{\sum_{j=1}^{n}(e_{ij} - (A_w)_j)^2} \end{cases} \qquad j = 1, 2, \cdots, n. \tag{19}$$

5. Calculate the of Comprehensive Evaluation Index I_{CEI}:

$$I = \frac{(d_w)_i}{(d_b)_i + (d_w)_i} \quad j = 1, 2, \cdots, n. \tag{20}$$

The great the I_{CEI} is, the better the situation of the forest is. The result of TOPSIS is listed in table 12.

Table 12: TOPSIS results

Forest	d_b	d_w	I_{CEI}	Rank
Greater Khingan Range Forest	0.885	0.115	0.115	6
Angeles National Forest	0.803	0.477	0.372	2
Amazon Rainforest	0.449	0.814	0.645	1
Changbai Mountain Forest	0.920	0.077	0.077	7
Shennongjia National Forest	0.897	0.135	0.131	5
Sipsongpanna Rainforest	0.903	0.146	0.139	4
Congo Rainforest	0.814	0.217	0.210	3
Valdivia Temperate Rainforest	0.895	0.071	0.073	8

6.3 Forest Management Plans Based on Decision Model

After developing the Comprehensive Decision Model, the discussion of how this model will guide the formulation and implementation of the forest management plans, the spectrum of the management plans contains three significant aspects that have a strong interrelationship: harvesting, policy-making, and preservation. The current situation and attributes of the forest will affect the balancing strategy of the management, say, harvest more to support the local economy or increase finance on preservation.

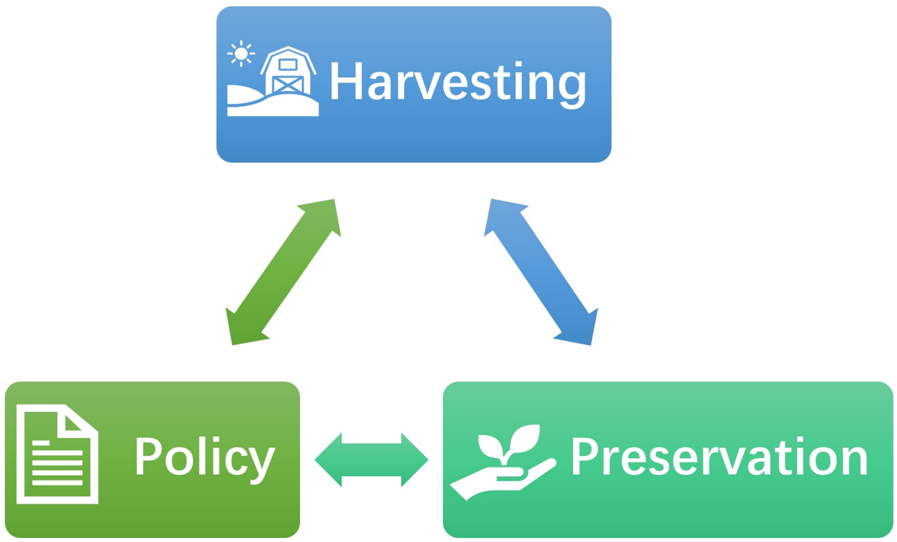

Figure 9: Three aspects of management plan

To determine the balancing strategy between the three aspects, the attributes in table 9 and weights in table 11 will be compared, respectively.

The timing of harvesting includes the time and interval of cutting downing the trees. With different components and configurations, forests may have different growth-stage attributes. So the timing should be tailored to fit the forest's nature and optimize the efficiency of the management.

The intensity of harvesting is the number of trees needed to be harvested. When the forest has entered the Over-mature Stage, when the carbon sequestration rate becomes lower, it's feasible that the forest is harvested and turned into forest products. The policies should be dynamically adjusted to limit or encourage the harvesting of the forest. Policy-makers are supposed to carry out timely investigations on the growing stage and conditions of the forest.

The environmental condition of the forest will affect the intensity of harvesting. If the forest is in good condition, it's reasonable to intensify the harvesting for the local economy. On the contrary, if the forest's biodiversity is seriously damaged, harvesting should be postponed until its recovery and preservation efforts are urgently needed.

The breadth of the forest is also related to the resilience of the forest as the more significant an ecosystem is, the easier it will recover from damages. More extensive forests have a better ability to regrow. More miniature forests are more vulnerable to natural disasters and human interference; thus, policies that strictly limit the exploitation of the forest should be implemented.

Countries with a higher degree of development should be more responsive to preserving the forest. These countries are more likely to have the technology, experts, and equipment essential for preservation. Poorer countries should restraint their urge to exploit their forest aggressively and consider them in the long run. In terms of economy, they should develop characteristic industries such as touring and handcrafting, which may benefit from their well-preserved traditional cultural beliefs and customs.

7 Application of the Models

7.1 Carbon Sequestration Prediction

The Virgin Komi Forest, the largest virgin forest in Europe [3], is selected as the research forest for the application of the models.

To predict the amount of sequestered carbon over 100 years of the forest using equation 9 in Carbon Sequestration Model, history data of S_{area} (forest area) of Virgin Komi Forest is collected and listed in table 13.

Table 13: Information about Virgin Komi Forest

Sequence number	Time	Forest area ($\times 10^7$ hectare)
1	1960s	1.41
2	1980	1.50
3	1990	1.62
4	200	1.65
5	2010	1.74

Grey Prediction Model (GM) is used to predict the trend of S_{area}'s change. This model is applicable to a problem that a modest number of data is available and is often used for short- and medium-term prediction. The original data sequence is:

$$\mathbf{x}^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \cdots, x^{(0)}(n)\right) \tag{21}$$

It's 1-accumulative generating operation sequence is

$$\mathbf{x}^{(1)} = \left(x^{(0)}(1), x^{(0)}(1) + x^{(0)}(2), \cdots, x^{(0)}(1) + x^{(0)}(n)\right) \tag{22}$$

The grey differential equation and corresponding whitening differential equation is

$$\begin{cases} x^{(0)}(k) + az^{(1)}(k) = b \qquad k = 2, 3, \cdots, n. \\ \dfrac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + ax^{(1)}(t) = b \end{cases} \tag{23}$$

The solution is

$$\hat{x}^{(1)}(k+1) = \left(x^{(0)}(1) - \frac{\hat{b}}{\hat{a}}\right) e^{-\hat{a}k} + \frac{\hat{b}}{\hat{a}} \qquad k = 0, 1, 2, \cdots \tag{24}$$

The results of implementing the grey prediction model is shown in figure 10:

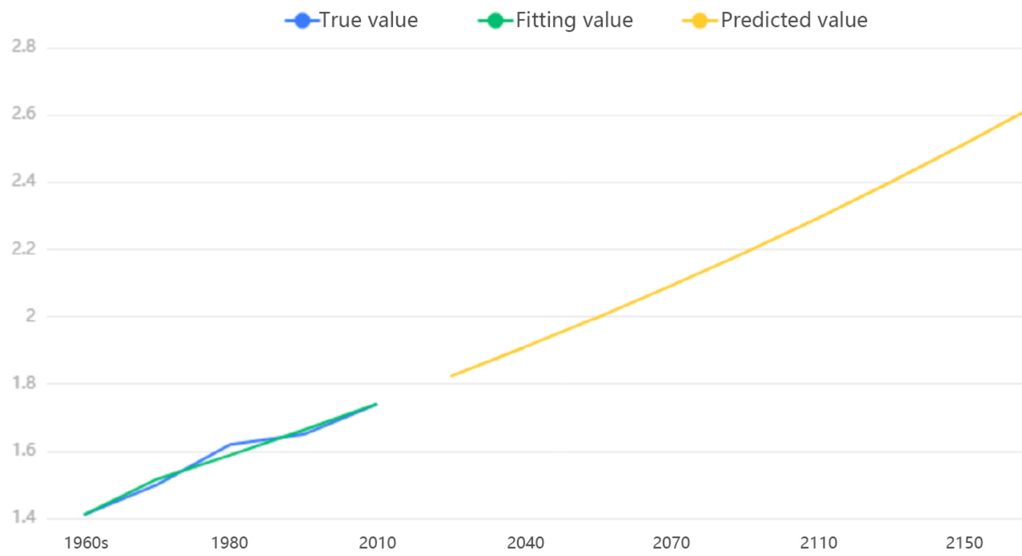

Figure 10: Grey Prediction of S_{area}

From the yellow line in figure 10, prediction is that at the year 2110 (100 years after from the year 2010 when the data of S_{area} is lastly updated), the value of S_{area} is 2.28 $\times 10^7$ hectare. According to the equation 9, supposing that the forest has entered the Over-mature Stage and carbon storage density is a constant. The value (131.96 Mg$\times$hectare^{-1}) of carbon storage density is the value Coniferous mixed forest in table 3. As a result, the predicted amount of sequestered carbon is 3008688000 Mg.

The carbon storage of forest products, which is precisely the one-round -growth case of the Carbon Sequestration Model, the loss of carbon storage due to corrosion is

$$\frac{S_{area} \cdot T_{over}}{1 + \left(\frac{S_{area} \cdot T_{over}}{x_0} - 1\right) e^{-r_0(t-t_0)}} \tag{25}$$

and the outcome is 520,503,024 Mg. Ultimately, the final result of the amount of carbon sequestered by the forest and its products over 100 years is 2,488,184,976 Mg.

7.2 Best Management suggested by the Model

From figure 10, it can be conjectured that the growth trend of Virgin Komi Forests is stable, and there is a constant supply of forest resources. Hence, the main point of forest management plans for Virgin Komi Forests is as follows:

1. The timing of harvesting can be shortened from 100 years to 20-30 years, during which time it's enough for this forest to regrow and re-supply the biological materials.

2. Relatively less restrictive policies on the forest management can be adopted allowing for the economical development of the region the forest belongs to.

3. Due to the proximity of the forest to developed market of th Western Europe, high-end forest products, such as luxurious furniture, can be manufactured preferentially for the purpose of higher financial returns.
4. Timely replanting of new trees is essential, as the latitude of the forest is relatively high and the forest is less resilient to external interference.

8 Sensitivity Analysis

Based on the analysis above, changes of the factor values in the Comprehensive Decision Model are made to evaluate the robustness of the model. By making random fluctuations about 10% on the value of the factor, the result of the model remains stable, as is shown in figure 8. Also, no significant change of forest management plans remains is found.

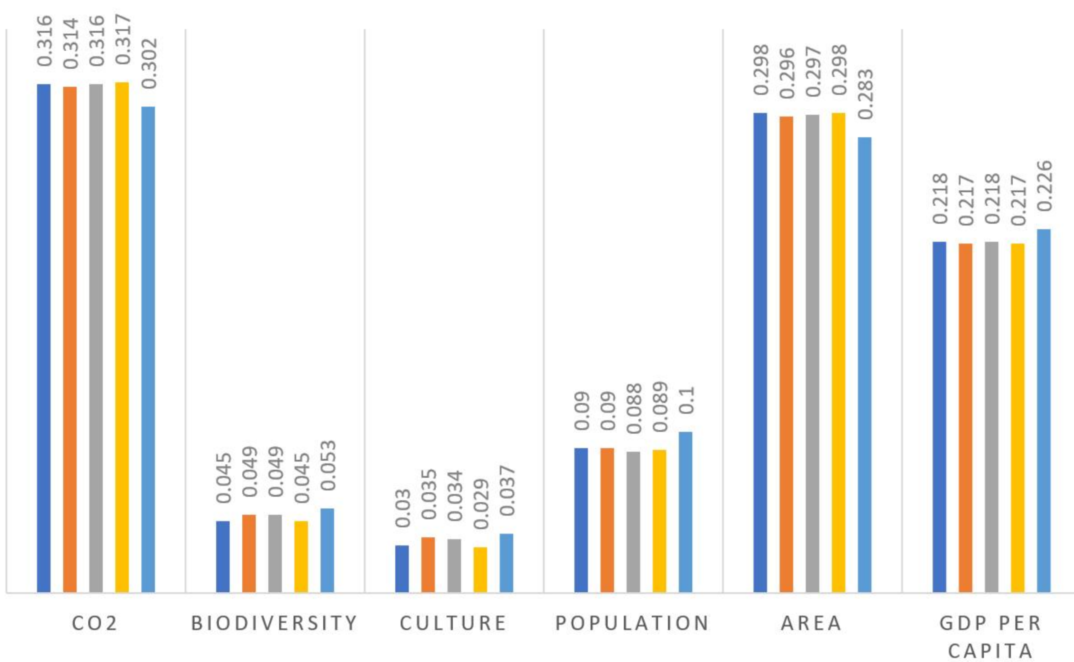

Figure 11: The response of output weights when input data fluctuates for 10%

9 Strengths and Weaknesses

9.1 Strengths

- Various factors are considered to generate the weights the model needs.
- Multiple forest management plans are presented to guide the public and policymakers.
- Several comprehensive decision methods are implemented to obtain the detailed and objective result.

9.2 Weaknesses

- The amount of the data used is relatively small, and the accuracy of linear regression and the grey prediction model is not high enough.
- No accidents such as natural disaster is considered.
- The model is idealistic, and its goodness of fit to the actual situation is not examined.
- Due to the limited distribution of the selected forests, the potential of generalization of the model is modest.

10 Newspaper Article

Together, For the Forest and Our Future

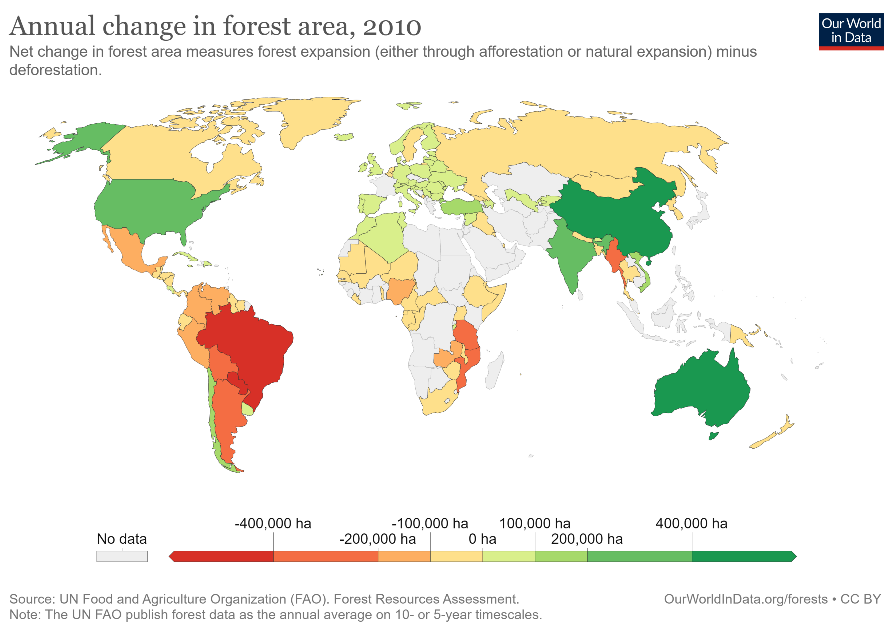

Figure 12: Annual change in forest area, 2010 [4]

In recent years, extreme climate events have increased, causing severe damage to the global economy and human life. In the face of these disasters brought about by climate change, humanity should work together to flatten the curve of global warming and protect people from extreme climate events. So far, countries have signed several international agreements, reflecting the determination of humanity

[4]Source: `https://ourworldindata.org/deforestation`

to deal with risks jointly. We have not done well despite our achievements, and our descendants will live on an increasingly hostile planet. We need to turn our attention to the forest.

Forests have good carbon absorption and storage capacity and sequester carbon dioxide for long periods. But it is not advisable to stop the blind logging of forests. From a CO2 perspective, overripe trees balance supply and demand. The carbon dioxide absorbed through photosynthesis is released through respiration and is no longer sequestered. More carbon can be absorbed if trees are harvested and made into products and new trees are planted in time.

From an economic point of view, it is advantageous to turn trees into products for sale. The raw materials of much furniture are trees. The sale of forest products will also bring economic returns that help subsidize preservations on forests and improve living standards.

From the perspective of biodiversity, good disturbance of the ecosystem is conducive to the reproduction of organisms, forming a more complex food chain and food web, thereby improving the stability of the ecosystem. Considering these three aspects, choosing the appropriate harvest interval will be beneficial. So the idea that keeps the trees untouched can be detrimental in the long run.

Now, human beings still have a long way to go in environmental protection. Everyone has a responsibility to protect our home, and everyone is destined to be a hero of our home. Therefore, we should adopt more reasonable management methods for local forests. So we believe our model must be the best decision for your forest.

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