Supplementary Material: Global Depths for Irregular Observed Multivariate Functional Data

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This supplementary material provides the results of Models 1-4 in the Simulation Studies. First, we present one simulation of all the above models with contamination but no sparseness. Next, we demonstrate the robustness of depths from extracting the median and central region, keeping the rank association for nonoutliers, and simple outlier detection for the above models.

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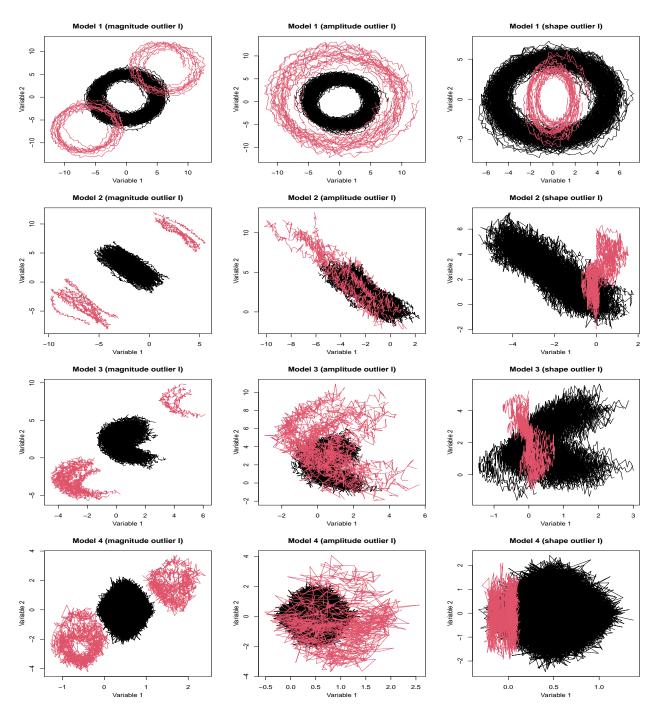


Figure S1: Panels from top to bottom show Models 1-4 contaminated by different types of outliers. Each panel displays the model contaminated by a 10% proportion of magnitude outlier I, amplitude outlier I, and shape outlier I, respectively, from left to right. The non-outlying and outlying curves are colored in black and red, respectively.

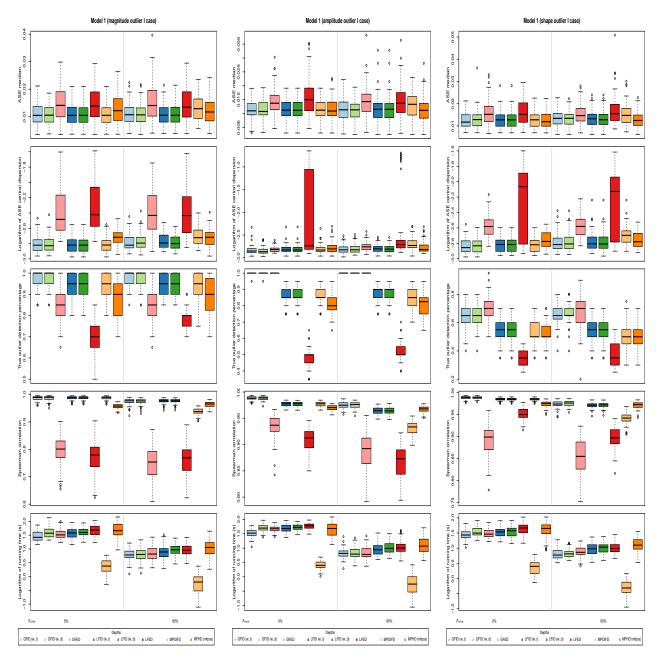


Figure S2: The left to tight shows Model 1 with magnitude outlier I, amplitude outlier I, and shape outlier I, respectively. ASE of the estimated central curve (the first row), of the 50%-dispersion curve (the second row), and the outlier proportion in the lowest 10% depth region (the third row), the Spearman correlation (the fourth row), and the running time (the fifth row). The sparseness type is point sparseness with a dense case (0%) and a high sparseness case (40%-60%).

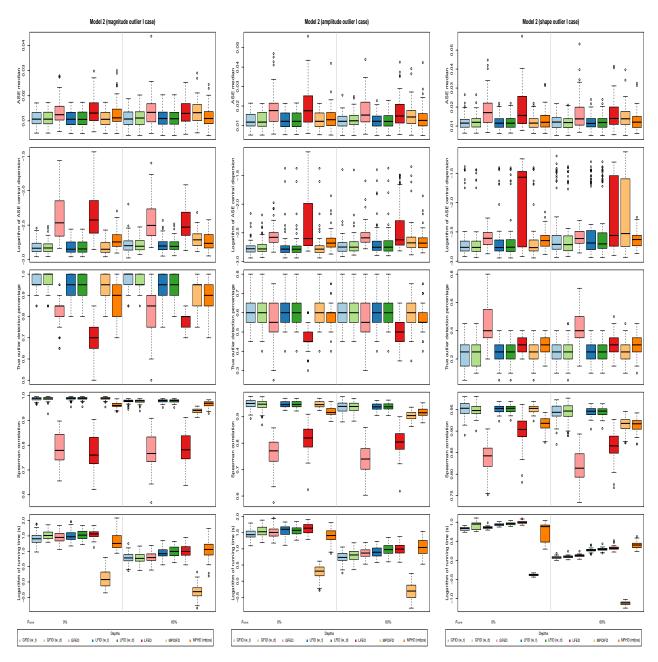


Figure S3: The left to tight shows Model 2 with magnitude outlier I, amplitude outlier I, and shape outlier I, respectively. ASE of the estimated central curve (the first row), of the 50%-dispersion curve (the second row), and the outlier proportion in the lowest 10% depth region (the third row), the Spearman correlation (the fourth row), and the running time (the fifth row). The sparseness type is point sparseness with a dense case (0%) and a high sparseness case (40%-60%).

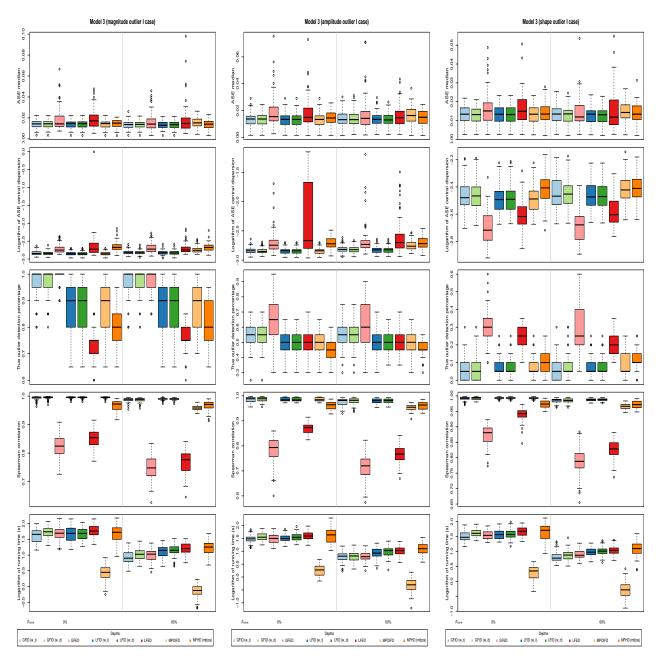


Figure S4: The left to tight shows Model 3 with magnitude outlier I, amplitude outlier I, and shape outlier I, respectively. ASE of the estimated central curve (the first row), of the 50%-dispersion curve (the second row), and the outlier proportion in the lowest 10% depth region (the third row), the Spearman correlation (the fourth row), and the running time (the fifth row). The sparseness type is point sparseness with a dense case (0%) and a high sparseness case (40%-60%).

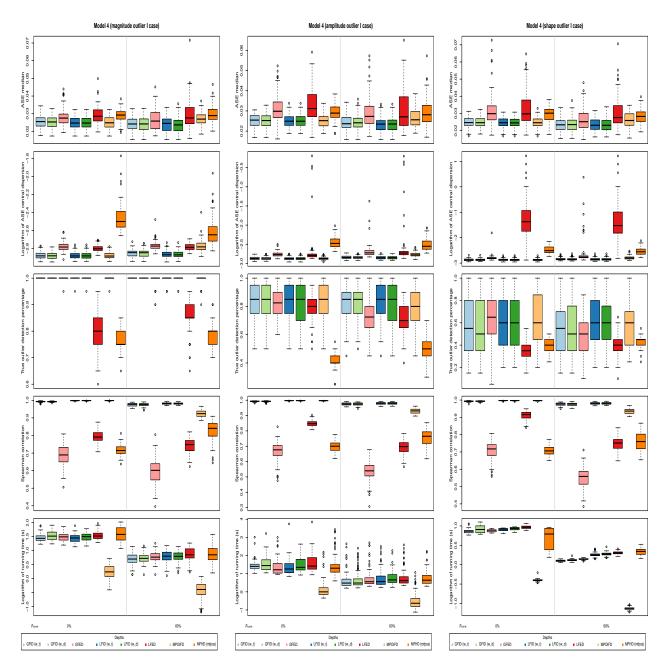


Figure S5: The left to tight shows Model 4 with magnitude outlier I, amplitude outlier I, and shape outlier I, respectively. ASE of the estimated central curve (the first row), of the 50%-dispersion curve (the second row), and the outlier proportion in the lowest 10% depth region (the third row), the Spearman correlation (the fourth row), and the running time (the fifth row). The sparseness type is point sparseness with a dense case (0%) and a high sparseness case (40%-60%).