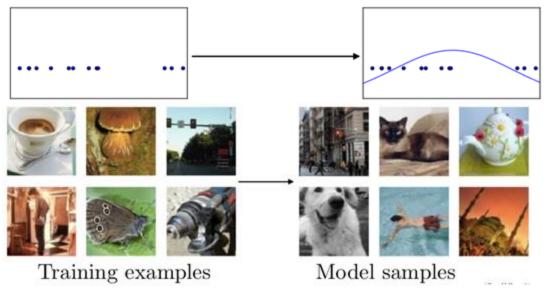
# Introduction to Generative Models (and GANs)

Haoqiang Fan <a href="mailto:fhq@megvii.com">fhq@megvii.com</a>

#### Generative Models: Learning the Distributions

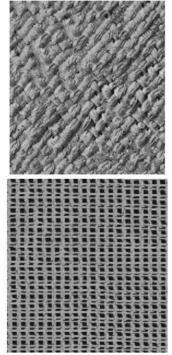
Discriminative: learns the likelihood

Generative: performs Density Estimation (learns the distribution) to allow sampling



#### Texture Synthesis, ICCV 1999

LUT-based, one pixel at a time.



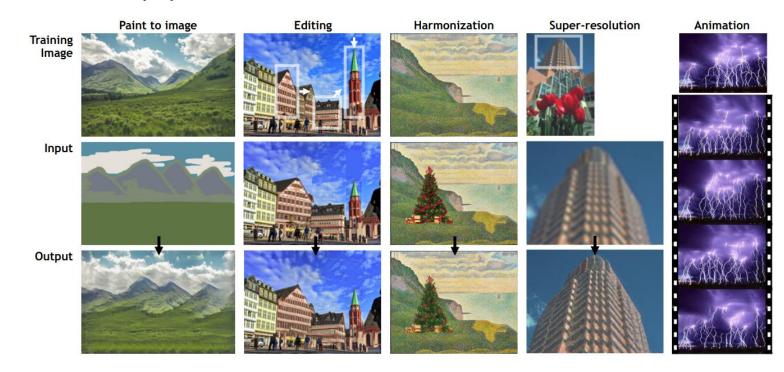






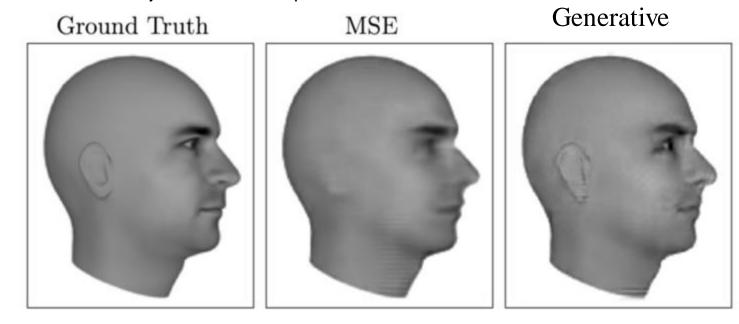
#### SinGAN, ICCV 2019

GAN based, best paper

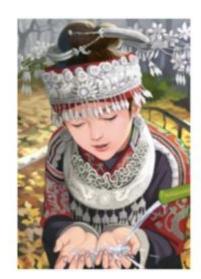


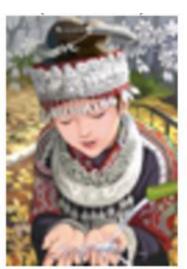
## Loss function for distribution: Ambiguity and the "blur" effect

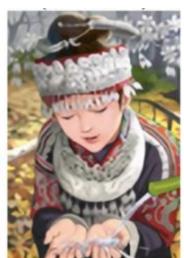
MSE: a Discriminative model just smoothes all possibilities.



#### Ambiguity and the "blur" effect







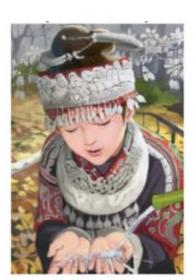
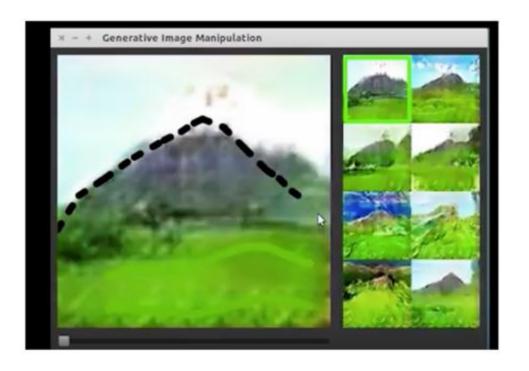


Photo-Realistic Single Image Super-Resolution Using a Generative Adversarial Network

# Example Application of Generative Models

#### Image Generation from Sketch

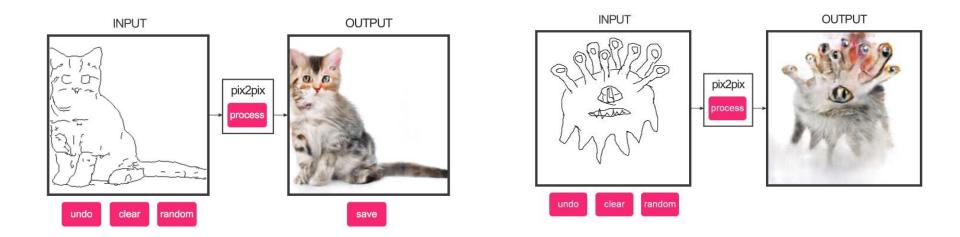


iGAN: Interactive Image Generation via Generative Adversarial Networks

Figures adapted from NIPS 2016 Tutorial Generative Adversarial Networks

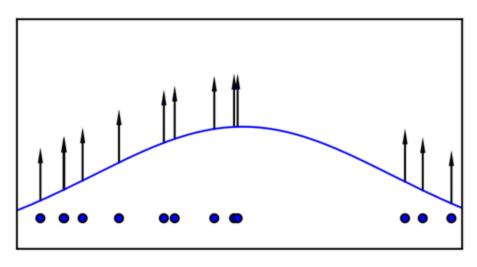
#### Image to Image Translation





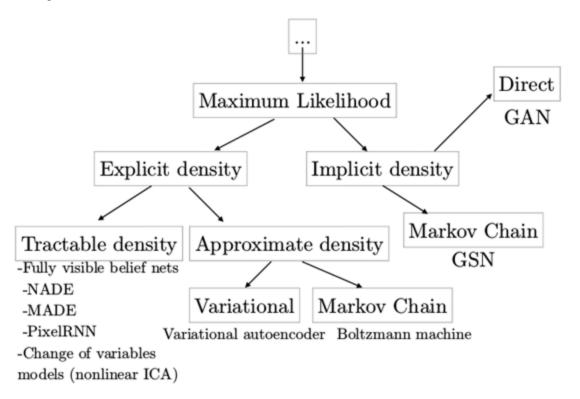
How Generative Models are Trained

#### Learning Generative Models

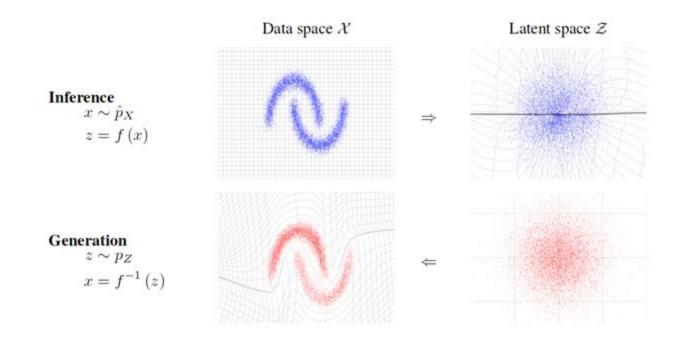


$$\boldsymbol{\theta}^* = \arg\max_{\boldsymbol{\theta}} \mathbb{E}_{x \sim p_{\text{data}}} \log p_{\text{model}}(\boldsymbol{x} \mid \boldsymbol{\theta})$$

#### Taxonomy of Generative Models

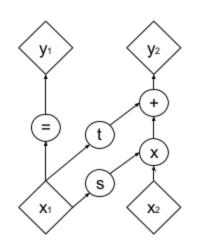


#### Exact Model: NVP (non-volume preserving)

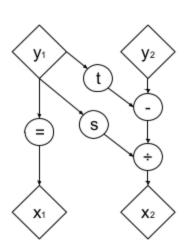


#### Real NVP: Invertible Non-linear Transforms

$$p_x(\mathbf{x}) = p_z(g^{-1}(\mathbf{x})) \left| \det \left( \frac{\partial g^{-1}(\mathbf{x})}{\partial \mathbf{x}} \right) \right|.$$

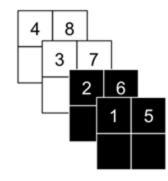


(a) Forward propagation



(b) Inverse propagation





#### Real NVP: Examples



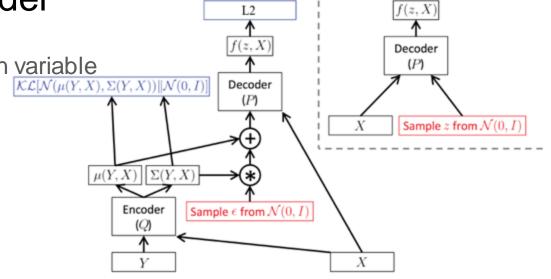


#### Real NVP

Restriction on the source domain: must be of the same as the target.

#### Variational Auto-Encoder

Auto-encoding with noise in hidden variable

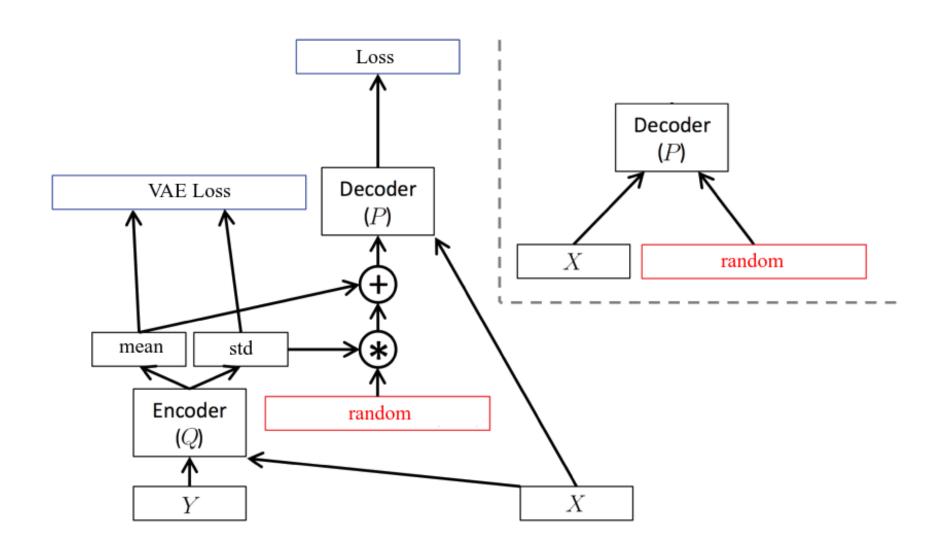


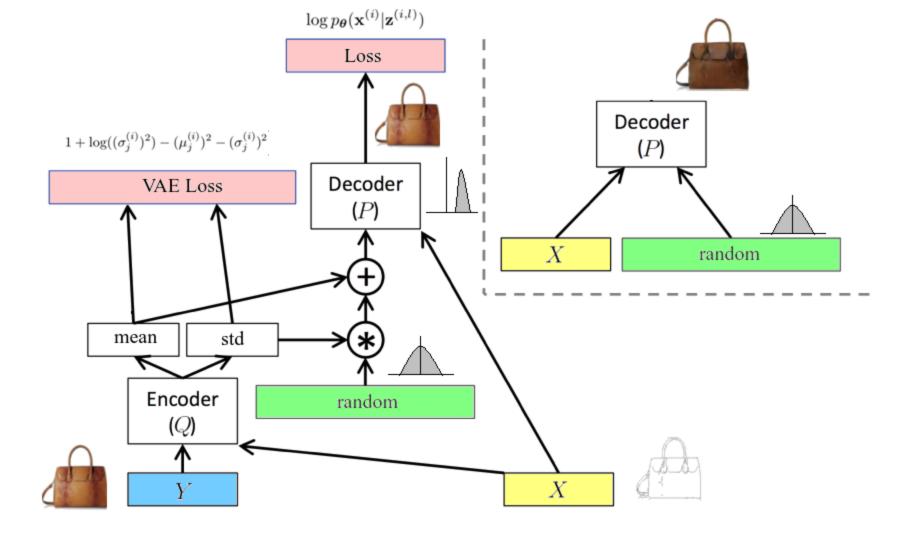
$$\log p_{\boldsymbol{\theta}}(\mathbf{x}^{(i)}) = D_{KL}(q_{\boldsymbol{\phi}}(\mathbf{z}|\mathbf{x}^{(i)})||p_{\boldsymbol{\theta}}(\mathbf{z}|\mathbf{x}^{(i)})) + \mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{x}^{(i)})$$

$$\mathcal{L}(\boldsymbol{\theta}, \boldsymbol{\phi}; \mathbf{x}^{(i)}) \simeq \frac{1}{2} \sum_{i=1}^{J} \left( 1 + \log((\sigma_j^{(i)})^2) - (\mu_j^{(i)})^2 - (\sigma_j^{(i)})^2 \right) + \frac{1}{L} \sum_{l=1}^{L} \log p_{\boldsymbol{\theta}}(\mathbf{x}^{(i)}|\mathbf{z}^{(i,l)})$$

#### Variational Auto-Encoder

$$\begin{split} \mathcal{D}\left[Q(z)\|P(z|X)\right] &= E_{z\sim Q}\left[\log Q(z) - \log P(z|X)\right]. \\ \mathcal{D}\left[Q(z)\|P(z|X)\right] &= E_{z\sim Q}\left[\log Q(z) - \log P(X|z) - \log P(z)\right] + \log P(X). \\ \log P(X) - \mathcal{D}\left[Q(z)\|P(z|X)\right] &= E_{z\sim Q}\left[\log P(X|z)\right] - \mathcal{D}\left[Q(z)\|P(z)\right] \\ \log P(X) - \mathcal{D}\left[Q(z|X)\|P(z|X)\right] &= E_{z\sim Q}\left[\log P(X|z)\right] - \mathcal{D}\left[Q(z|X)\|P(z)\right] \end{split}$$

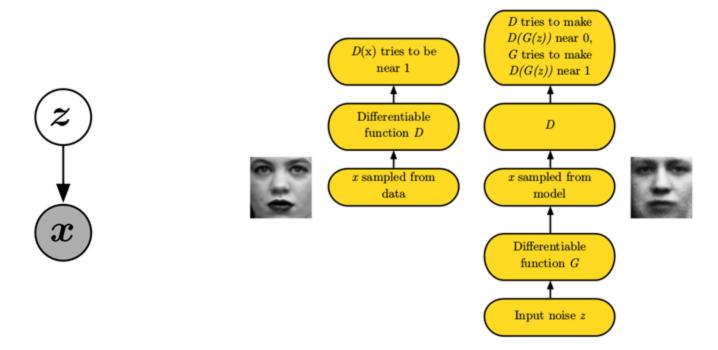




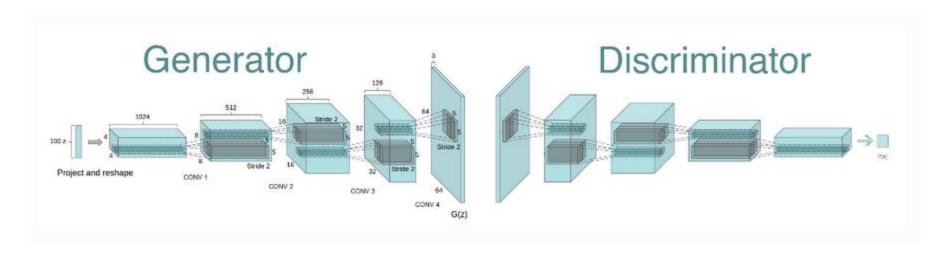
### VAE: Examples



#### Generative Adversarial Networks (GAN)



#### **DCGAN**



### DCGAN: Examples



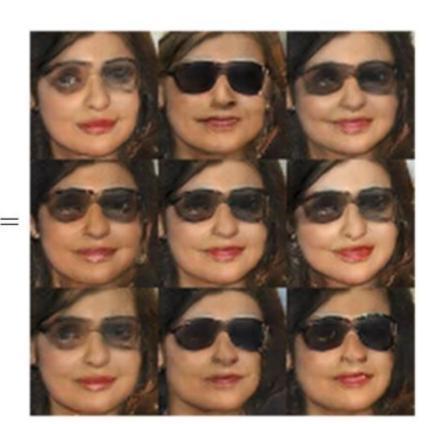
#### DCGAN: Example of Feature Manipulation

Vector arithmetics in feature space









#### Conditional, Cross-domain Generation

#### Generative adversarial text to image synthesis

this small bird has a pink breast and crown, and black almost all black with a red primaries and secondaries.

this magnificent fellow is crest, and white cheek patch.



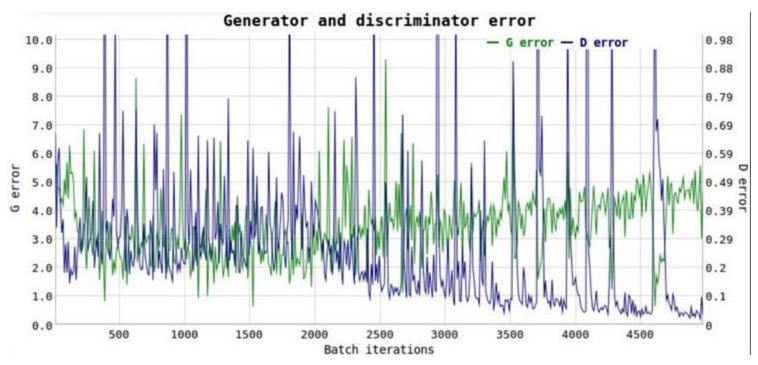
the flower has petals that are bright pinkish purple with white stigma



this white and yellow flower have thin white petals and a round yellow stamen



#### GAN training problems: unstable losses



http://guimperarnau.com/files/blog/Fantastic-GANs-and-where-to-find-them/crazy\_loss\_function.jpg

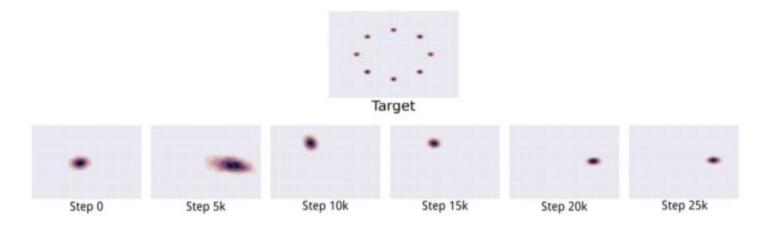
#### GAN training problems: Mini-batch Fluctuation

Differs much even between consecutive minibatches.



#### GAN training problems: Mode Collapse

Lack of diversity in generated results.



#### Improve GAN training: Label Smoothing

Improves stability of training

```
d_on_data = discriminator_logits(data_minibatch)
d_on_samples = discriminator_logits(samples_minibatch)
loss = tf.nn.sigmoid_cross_entropy_with_logits(d_on_data, .9) + \
    tf.nn.sigmoid_cross_entropy_with_logits(d_on_samples, 0.)
```

#### The (once) break-through: WGAN

• The Earth-Mover (EM) distance or Wasserstein-1

$$W(\mathbb{P}_r, \mathbb{P}_g) = \inf_{\gamma \in \Pi(\mathbb{P}_r, \mathbb{P}_g)} \mathbb{E}_{(x,y) \sim \gamma} [\|x - y\|], \qquad (1)$$

where  $\Pi(\mathbb{P}_r, \mathbb{P}_g)$  denotes the set of all joint distributions  $\gamma(x, y)$  whose marginals are respectively  $\mathbb{P}_r$  and  $\mathbb{P}_g$ . Intuitively,  $\gamma(x, y)$  indicates how much "mass" must be transported from x to y in order to transform the distributions  $\mathbb{P}_r$  into the distribution  $\mathbb{P}_g$ . The EM distance then is the "cost" of the optimal transport plan.

#### **Duality**

Again, Theorem 2 points to the fact that  $W(\mathbb{P}_r, \mathbb{P}_{\theta})$  might have nicer properties when optimized than  $JS(\mathbb{P}_r, \mathbb{P}_{\theta})$ . However, the infimum in (1) is highly intractable. On the other hand, the Kantorovich-Rubinstein duality [21] tells us that

$$W(\mathbb{P}_r, \mathbb{P}_{\theta}) = \sup_{\|f\|_L \le 1} \mathbb{E}_{x \sim \mathbb{P}_r}[f(x)] - \mathbb{E}_{x \sim \mathbb{P}_{\theta}}[f(x)]$$
 (2)

#### Improve GAN training: Wasserstein GAN

 $g_{\theta} \leftarrow -\nabla_{\theta} \frac{1}{m} \sum_{i=1}^{m} f_{w}(g_{\theta}(z^{(i)}))$ 

 $\theta \leftarrow \theta - \alpha \cdot \text{RMSProp}(\theta, q_{\theta})$ 

Use linear instead of log

end for

$$W(\mathbb{P}_r, \mathbb{P}_{\theta}) = \sup_{\|f\|_{L} \le 1} \mathbb{E}_{x \sim \mathbb{P}_r}[f(x)] - \mathbb{E}_{x \sim \mathbb{P}_{\theta}}[f(x)]$$
**for**  $t = 0, ..., n_{\text{critic}}$  **do**

$$\text{Sample } \{x^{(i)}\}_{i=1}^m \sim \mathbb{P}_r \text{ a batch from the real data.}$$

$$\text{Sample } \{z^{(i)}\}_{i=1}^m \sim p(z) \text{ a batch of prior samples.}$$

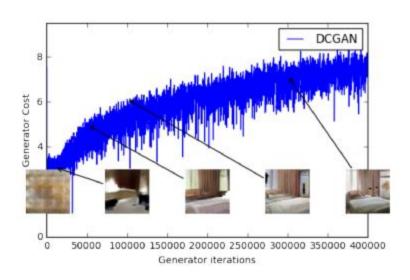
$$g_w \leftarrow \nabla_w \left[\frac{1}{m} \sum_{i=1}^m f_w(x^{(i)}) - \frac{1}{m} \sum_{i=1}^m f_w(g_{\theta}(z^{(i)}))\right]$$

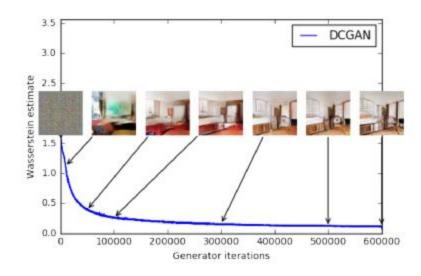
$$w \leftarrow w + \alpha \cdot \text{RMSProp}(w, g_w)$$

$$w \leftarrow \underline{\text{clip}(w, -c, c)}$$
**end for**

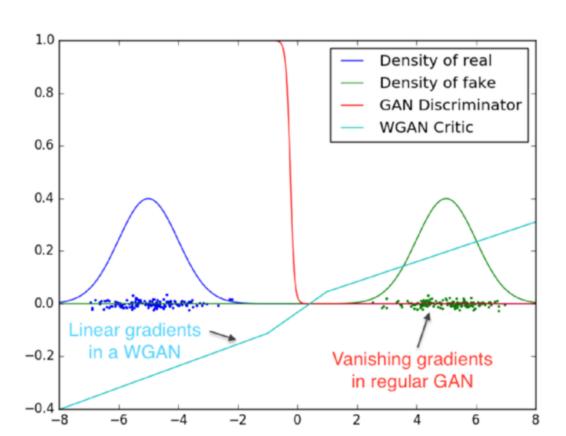
$$\text{Sample } \{z^{(i)}\}_{i=1}^m \sim p(z) \text{ a batch of prior samples.}$$

#### WGAN: Stabilized Training Curve



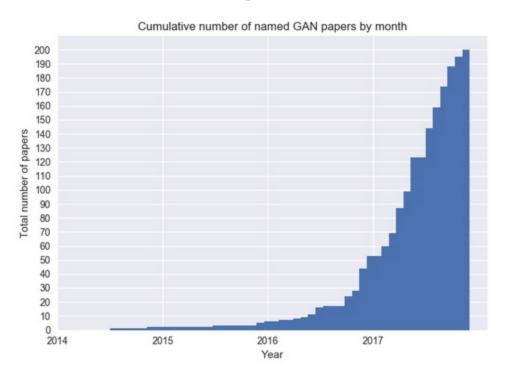


#### WGAN: Non-vanishing Gradient



#### The GAN Zoo

https://github.com/hindupuravinash/the-gan-zoo



 3D-IWGAN - Improved Adversarial Systems for 3D Object Generation and Reconstruction (github) 3D-RecGAN - 3D Object Reconstruction from a Single Depth View with Adversarial Learning (github) ABC-GAN - ABC-GAN: Adaptive Blur and Control for improved training stability of Generative Adversarial Networks (github) AC-GAN - Conditional Image Synthesis With Auxiliary Classifier GANs · acGAN - Face Aging With Conditional Generative Adversarial Networks AdaGAN - AdaGAN: Boosting Generative Models AE-GAN - AE-GAN: adversarial eliminating with GAN AEGAN - Learning Inverse Mapping by Autoencoder based Generative Adversarial Nets AffGAN - Amortised MAP Inference for Image Super-resolution AL-CGAN - Learning to Generate Images of Outdoor Scenes from Attributes and Semantic Layouts ALI - Adversarially Learned Inference AlignGAN - AlignGAN: Learning to Align Cross-Domain Images with Conditional Generative Adversarial Networks. AM-GAN - Activation Maximization Generative Adversarial Nets

3D-GAN - Learning a Probabilistic Latent Space of Object Shapes via 3D Generative-Adversarial Modeling (github)

- AnoGAN Unsupervised Anomaly Detection with Generative Adversarial Networks to Guide Marker Discovery
  - ARAE Adversarially Regularized Autoencoders for Generating Discrete Structures (github) ARDA - Adversarial Representation Learning for Domain Adaptation

    - ARIGAN ARIGAN: Synthetic Arabidopsis Plants using Generative Adversarial Network

    - ArtGAN ArtGAN: Artwork Synthesis with Conditional Categorial GANs

    - b-GAN Generative Adversarial Nets from a Density Ratio Estimation Perspective
    - Bayesian GAN Deep and Hierarchical Implicit Models

    - Bayesian GAN Bayesian GAN

    - BCGAN Bayesian Conditional Generative Adverserial Networks

    - BEGAN BEGAN: Boundary Equilibrium Generative Adversarial Networks
    - BGAN Binary Generative Adversarial Networks for Image Retrieval (github)

- BS-GAN Boundary-Seeking Generative Adversarial Networks C-RNN-GAN - C-RNN-GAN: Continuous recurrent neural networks with adversarial training (github) CaloGAN - CaloGAN: Simulating 3D High Energy Particle Showers in Multi-Layer Electromagnetic Calorimeters with Generative Adversarial Networks (github) CAN - CAN: Creative Adversarial Networks, Generating Art by Learning About Styles and Deviating from Style Norms CatGAN - Unsupervised and Semi-supervised Learning with Categorical Generative Adversarial Networks CausalGAN - CausalGAN: Learning Causal Implicit Generative Models with Adversarial Training CC-GAN - Semi-Supervised Learning with Context-Conditional Generative Adversarial Networks (github) CDcGAN - Simultaneously Color-Depth Super-Resolution with Conditional Generative Adversarial Network CGAN - Conditional Generative Adversarial Nets CGAN - Controllable Generative Adversarial Network Chekhov GAN - An Online Learning Approach to Generative Adversarial Networks CM-GAN - CM-GANs: Cross-modal Generative Adversarial Networks for Common Representation Learning CoGAN - Coupled Generative Adversarial Networks Conditional cycleGAN - Conditional CycleGAN for Attribute Guided Face Image Generation constrast-GAN - Generative Semantic Manipulation with Contrasting GAN
  - Coulomb GAN Coulomb GANs: Provably Optimal Nash Equilibria via Potential Fields Cramèr GAN - The Cramer Distance as a Solution to Biased Wasserstein Gradients

Context-RNN-GAN - Contextual RNN-GANs for Abstract Reasoning Diagram Generation

- crVAE-GAN Channel-Recurrent Variational Autoencoders

- CS-GAN Improving Neural Machine Translation with Conditional Sequence Generative Adversarial Nets
- CVAE-GAN CVAE-GAN: Fine-Grained Image Generation through Asymmetric Training
- CycleGAN Unpaired Image-to-Image Translation using Cycle-Consistent Adversarial Networks (github)

D2GAN - Dual Discriminator Generative Adversarial Nets

BiGAN - Adversarial Feature Learning

 DAN - Distributional Adversarial Networks DCGAN - Unsupervised Representation Learning with Deep Convolutional Generative Adversarial Networks (github) DeliGAN - DeLiGAN: Generative Adversarial Networks for Diverse and Limited Data (github) DiscoGAN - Learning to Discover Cross-Domain Relations with Generative Adversarial Networks DistanceGAN - One-Sided Unsupervised Domain Mapping DM-GAN - Dual Motion GAN for Future-Flow Embedded Video Prediction DR-GAN - Representation Learning by Rotating Your Faces DRAGAN - How to Train Your DRAGAN (github) DSP-GAN - Depth Structure Preserving Scene Image Generation DTN - Unsupervised Cross-Domain Image Generation DualGAN - DualGAN: Unsupervised Dual Learning for Image-to-Image Translation Dualing GAN - Dualing GANs EBGAN - Energy-based Generative Adversarial Network ED//GAN - Stabilizing Training of Generative Adversarial Networks through Regularization EGAN - Enhanced Experience Replay Generation for Efficient Reinforcement Learning ExprGAN - ExprGAN: Facial Expression Editing with Controllable Expression Intensity f-GAN - f-GAN: Training Generative Neural Samplers using Variational Divergence Minimization FF-GAN - Towards Large-Pose Face Frontalization in the Wild Fila-GAN - Synthesizing Filamentary Structured Images with GANs Fisher GAN - Fisher GAN Flow-GAN - Flow-GAN: Bridging implicit and prescribed learning in generative models GAMN - Generative Adversarial Mapping Networks GAN - Generative Adversarial Networks (github) GAN-ATV - A Novel Approach to Artistic Textual Visualization via GAN GAN-CLS - Generative Adversarial Text to Image Synthesis (github) GAN-sep - GANs for Biological Image Synthesis (github)

 GAN-VFS - Generative Adversarial Network-based Synthesis of Visible Faces from Polarimetric Thermal Faces GANCS - Deep Generative Adversarial Networks for Compressed Sensing Automates MRI GANDI - Guiding the search in continuous state-action spaces by learning an action sampling distribution from off-target samples GAP - Context-Aware Generative Adversarial Privacy GAWWN - Learning What and Where to Draw (github) GeneGAN - GeneGAN: Learning Object Transfiguration and Attribute Subspace from Unpaired Data (github) Geometric GAN - Geometric GAN GMAN - Generative Multi-Adversarial Networks GMM-GAN - Towards Understanding the Dynamics of Generative Adversarial Networks GoGAN - Gang of GANs: Generative Adversarial Networks with Maximum Margin Ranking GP-GAN - GP-GAN: Towards Realistic High-Resolution Image Blending (github) GP-GAN - GP-GAN: Gender Preserving GAN for Synthesizing Faces from Landmarks GRAN - Generating images with recurrent adversarial networks (github) IAN - Neural Photo Editing with Introspective Adversarial Networks (github) IcGAN - Invertible Conditional GANs for image editing (github) ID-CGAN - Image De-raining Using a Conditional Generative Adversarial Network iGAN - Generative Visual Manipulation on the Natural Image Manifold (github) Improved GAN - Improved Techniques for Training GANs (github) InfoGAN - InfoGAN: Interpretable Representation Learning by Information Maximizing Generative Adversarial Nets (github) IRGAN - IRGAN: A Minimax Game for Unifying Generative and Discriminative Information Retrieval models IWGAN - On Unifying Deep Generative Models KGAN - KGAN: How to Break The Minimax Game in GAN. I-GAN - Representation Learning and Adversarial Generation of 3D Point Clouds LAGAN - Learning Particle Physics by Example: Location-Aware Generative Adversarial Networks for Physics Synthesis

 LAPGAN - Deep Generative Image Models using a Laplacian Pyramid of Adversarial Networks (github) LD-GAN - Linear Discriminant Generative Adversarial Networks LDAN - Label Denoising Adversarial Network (LDAN) for Inverse Lighting of Face Images . LeakGAN - Long Text Generation via Adversarial Training with Leaked Information LeGAN - Likelihood Estimation for Generative Adversarial Networks LR-GAN - LR-GAN: Layered Recursive Generative Adversarial Networks for Image Generation LS-GAN - Loss-Sensitive Generative Adversarial Networks on Lipschitz Densities LSGAN - Least Squares Generative Adversarial Networks MAD-GAN - Multi-Agent Diverse Generative Adversarial Networks MAGAN - MAGAN: Margin Adaptation for Generative Adversarial Networks MalGAN - Generating Adversarial Malware Examples for Black-Box Attacks Based on GAN MaliGAN - Maximum-Likelihood Augmented Discrete Generative Adversarial Networks MARTA-GAN - Deep Unsupervised Representation Learning for Remote Sensing Images McGAN - McGan: Mean and Covariance Feature Matching GAN MD-GAN - Learning to Generate Time-Lapse Videos Using Multi-Stage Dynamic Generative Adversarial Networks MDGAN - Mode Regularized Generative Adversarial Networks MedGAN - Generating Multi-label Discrete Electronic Health Records using Generative Adversarial Networks MGAN - Precomputed Real-Time Texture Synthesis with Markovian Generative Adversarial Networks (github) MGGAN - Multi-Generator Generative Adversarial Nets MIX+GAN - Generalization and Equilibrium in Generative Adversarial Nets (GANs) MLGAN - Metric Learning-based Generative Adversarial Network MMD-GAN - MMD GAN: Towards Deeper Understanding of Moment Matching Network (github) MMGAN - MMGAN: Manifold Matching Generative Adversarial Network for Generating Images MoCoGAN - MoCoGAN: Decomposing Motion and Content for Video Generation (github) MPM-GAN - Message Passing Multi-Agent GANs . MuseGAN - MuseGAN: Symbolic-domain Music Generation and Accompaniment with Multi-track Sequential Generative Adversarial Networks MV-BiGAN - Multi-view Generative Adversarial Networks

 OptionGAN - OptionGAN: Learning Joint Reward-Policy Options using Generative Adversarial Inverse Reinforcement Learning ORGAN - Objective-Reinforced Generative Adversarial Networks (ORGAN) for Sequence Generation Models PAN - Perceptual Adversarial Networks for Image-to-Image Transformation PassGAN - PassGAN: A Deep Learning Approach for Password Guessing Perceptual GAN - Perceptual Generative Adversarial Networks for Small Object Detection PGAN - Probabilistic Generative Adversarial Networks pix2pix - Image-to-Image Translation with Conditional Adversarial Networks (github) PixelGAN - PixelGAN Autoencoders Pose-GAN - The Pose Knows: Video Forecasting by Generating Pose Futures PPGN - Plug & Play Generative Networks: Conditional Iterative Generation of Images in Latent Space PrGAN - 3D Shape Induction from 2D Views of Multiple Objects PSGAN - Learning Texture Manifolds with the Periodic Spatial GAN PS<sup>2</sup>-GAN - High-Quality Facial Photo-Sketch Synthesis Using Multi-Adversarial Networks RankGAN - Adversarial Ranking for Language Generation RCGAN - Real-valued (Medical) Time Series Generation with Recurrent Conditional GANs RefineGAN - Compressed Sensing MRI Reconstruction with Cyclic Loss in Generative Adversarial Networks RenderGAN - RenderGAN: Generating Realistic Labeled Data ResGAN - Generative Adversarial Network based on Resnet for Conditional Image Restoration RNN-WGAN - Language Generation with Recurrent Generative Adversarial Networks without Pre-training (github) RPGAN - Stabilizing GAN Training with Multiple Random Projections (github) RTT-GAN - Recurrent Topic-Transition GAN for Visual Paragraph Generation RWGAN - Relaxed Wasserstein with Applications to GANs SAD-GAN - SAD-GAN: Synthetic Autonomous Driving using Generative Adversarial Networks SalGAN - SalGAN: Visual Saliency Prediction with Generative Adversarial Networks (github) SBADA-GAN - From source to target and back: symmetric bi-directional adaptive GAN SD-GAN - Semantically Decomposing the Latent Spaces of Generative Adversarial Networks

SEGAN - SEGAN: Speech Enhancement Generative Adversarial Network

 SeGAN - SeGAN: Segmenting and Generating the Invisible SegAN - SegAN: Adversarial Network with Multi-scale L1 Loss for Medical Image Segmentation SegGAN - SegGAN: Sequence Generative Adversarial Nets with Policy Gradient (github) SGAN - Texture Synthesis with Spatial Generative Adversarial Networks SGAN - Stacked Generative Adversarial Networks (github) SGAN - Steganographic Generative Adversarial Networks SimGAN - Learning from Simulated and Unsupervised Images through Adversarial Training SketchGAN - Adversarial Training For Sketch Retrieval SL-GAN - Semi-Latent GAN: Learning to generate and modify facial images from attributes SN-GAN - Spectral Normalization for Generative Adversarial Networks (github) Softmax-GAN - Softmax GAN Splitting GAN - Class-Splitting Generative Adversarial Networks SRGAN - Photo-Realistic Single Image Super-Resolution Using a Generative Adversarial Network SS-GAN - Semi-supervised Conditional GANs ss-InfoGAN - Guiding InfoGAN with Semi-Supervision SSGAN - SSGAN: Secure Steganography Based on Generative Adversarial Networks SSL-GAN - Semi-Supervised Learning with Context-Conditional Generative Adversarial Networks ST-GAN - Style Transfer Generative Adversarial Networks: Learning to Play Chess Differently StackGAN - StackGAN: Text to Photo-realistic Image Synthesis with Stacked Generative Adversarial Networks SteinGAN - Learning Deep Energy Models: Contrastive Divergence vs. Amortized MLE SVSGAN - SVSGAN: Singing Voice Separation via Generative Adversarial Network S^2GAN - Generative Image Modeling using Style and Structure Adversarial Networks TAC-GAN - TAC-GAN - Text Conditioned Auxiliary Classifier Generative Adversarial Network (github) TAN - Outline Colorization through Tandem Adversarial Networks TextureGAN - TextureGAN: Controlling Deep Image Synthesis with Texture Patches . TGAN - Temporal Generative Adversarial Nets TGAN - Tensorizing Generative Adversarial Nets

 TP-GAN - Beyond Face Rotation: Global and Local Perception GAN for Photorealistic and Identity Preserving Frontal View Synthesis Triple-GAN - Triple Generative Adversarial Nets Unrolled GAN - Unrolled Generative Adversarial Networks (github) VAE-GAN - Autoencoding beyond pixels using a learned similarity metric VariGAN - Multi-View Image Generation from a Single-View VAW-GAN - Voice Conversion from Unaligned Corpora using Variational Autoencoding Wasserstein Generative Adversarial Networks VEEGAN - VEEGAN: Reducing Mode Collapse in GANs using Implicit Variational Learning (github) VGAN - Generating Videos with Scene Dynamics (github) VGAN - Generative Adversarial Networks as Variational Training of Energy Based Models (github) ViGAN - Image Generation and Editing with Variational Info Generative Adversarial Networks

TGAN - Tensor-Generative Adversarial Network with Two-dimensional Sparse Coding: Application to Real-time Indoor

 VRAL - Variance Regularizing Adversarial Learning WaterGAN - WaterGAN: Unsupervised Generative Network to Enable Real-time Color Correction of Monocular Underwater Images

Δ-GAN - Triangle Generative Adversarial Networks

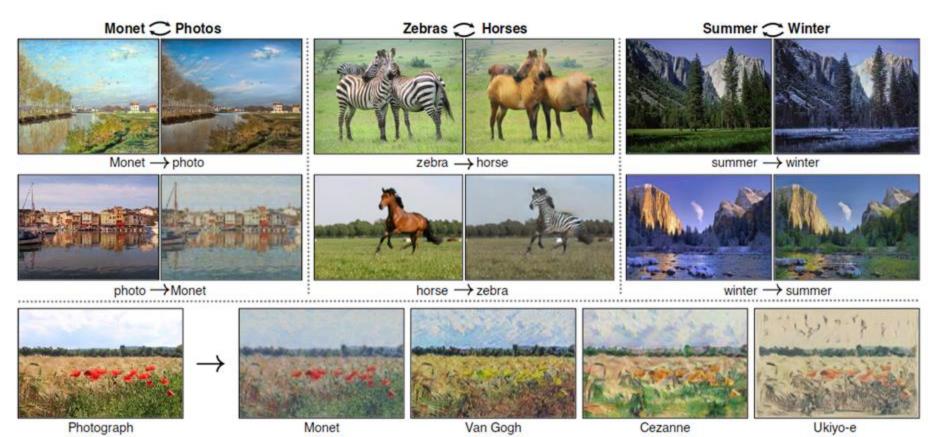
VIGAN - VIGAN: Missing View Imputation with Generative Adversarial Networks

Localization

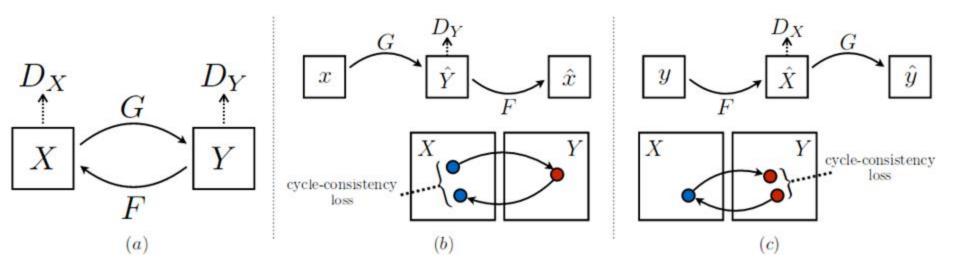
- WGAN Wasserstein GAN (github)
- WGAN-GP Improved Training of Wasserstein GANs (github)

- WS-GAN Weakly Supervised Generative Adversarial Networks for 3D Reconstruction
- ZipNet-GAN ZipNet-GAN: Inferring Fine-grained Mobile Traffic Patterns via a Generative Adversarial Neural Network
- - α-GAN Variational Approaches for Auto-Encoding Generative Adversarial Networks (github)

#### Cycle GAN: Correspondence from Unpaired Data

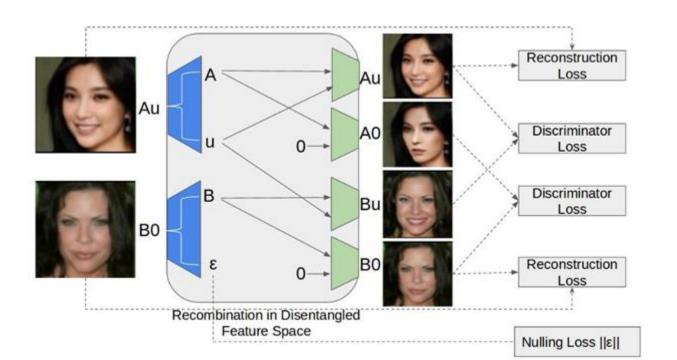


# Cycle GAN



## **GeneGAN**: shorter pathway improves training

Cross breeds and reproductions



## **GeneGAN**: Object Transfiguration

Transfer "my" hairstyle to him, not just a hairstyle.

Slide

**Github** 



## Math behind Generative Models

Those who don't care about math or theory can open their PyTorch now...

#### Formulation of Generative Models

sampling v.s. density estimation

#### **RBM**

$$p(x) = \frac{1}{Z} \sum_{y \in \{0,1\}^n} e^{x^T A y + x^T b + y^T c}$$

$$x \xrightarrow{\text{+b}}$$

#### **RBM**

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$$x \xrightarrow{+b} + b$$

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It is NP-Hard to estimate Z

#### **RBM**

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$$x \xrightarrow{+b} + b$$

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It is NP-Hard to sample from P

## Score Matching

Let L be the likelihood function, score V is:

$$V \equiv V( heta, X) = rac{\partial}{\partial heta} \log \mathcal{L}( heta; X) = rac{1}{\mathcal{L}( heta; X)} rac{\partial \mathcal{L}( heta; X)}{\partial heta}$$

If two distribution's scores match, they also match.

# A Connection Between Score Matching and Denoising Autoencoders

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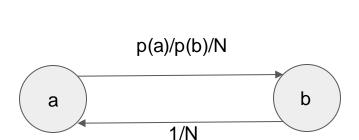
#### Markov Chain Monte Carlo

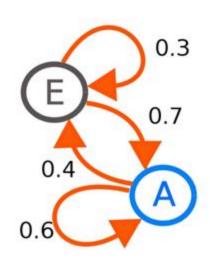
From each node a,

walk to "neighbor" b with probability **proportional** to p(b).

Neighbors must be reciprocal: a <->b

Walk for long enough time to reach equilibrium





#### MCMC in RBM

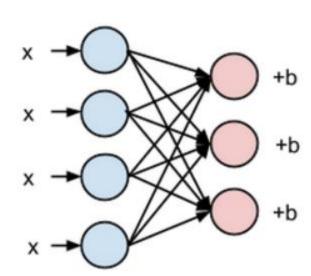
$$p(x) = \frac{1}{Z} \sum_{y \in \{0,1\}^n} e^{x^T A y + x^T b + y^T c}$$

Sample x given y

Sample y given x

Sample x given y

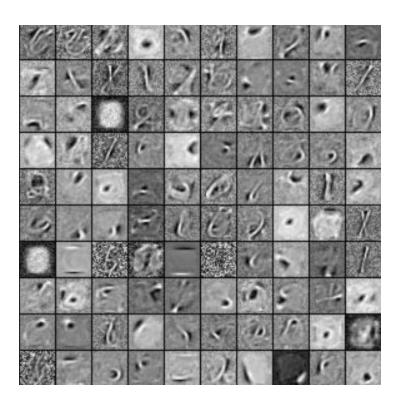
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In theory, repeat for long enough time.

In practice, repeat a few times. ("burnin")

#### RBM: Learned "Filters"



## From Density to Sample

Given density function p(x), can we efficiently black-box sample from it?

No! 
$$p(x) = MD5(x) = 0$$

Unless query  $\Omega(N)$  samples, it is hard to determine.

## From Sample to Density

Given black-box sampler G, can we efficiently estimate the density (frequency) of x?

Naive bound:  $\Omega(\epsilon^{-2})$  absolute,  $\Omega(1/p(x) \epsilon^{-2})$  relative

Cannot essentially do better.

Example: Sample x randomly. Retry iff x=0.

#### What can be done if only samples are available?

Problem: Given black box sampler G, decide if:

- (1) it is uniform
- (2) it is  $\underline{\varepsilon}$ -far from uniform

How to define distance between distributions?

Statistical distance:  $\frac{1}{2}$  sum |p(x)-q(x)| p:G q:Uniform

L2 distance: sum  $(p(x)-q(x))^2$ 

KL divergence: sum q(x)log(q(x)/p(x))

## Uniformity Check using $q(x)\log(q(x)/p(x))$

Impossible to check unless  $\Omega(N)$  samples are obtained.

Consider  $\{1,2,...,N\}^T$  and  $\{1,2,...,N-1\}^T$ . Unbound KL.

Statistical distance = sum max(p(x)-q(x),0)

$$((N-1)/N)^T = 1-o(1)$$
 if  $T=o(N)$ 

Statistical distance is the best distinguisher's advantage over random guess!

advantage = 2\*|Pr(guess correct)-0.5|

#### Uniformity Check using L2 Distance

sum  $(p(x)-q(x))^2 = \text{sum } p(x)^2+q(x)^2-2p(x)q(x) = \text{sum } p(x)^2 - 1/N$ 

 $p(x)^2$ : seeing two x in a row

sum  $p(x)^2$ : counting collisions

Algorithm: Get T samples, count the number of x[i]==x[j] for i < j, divide by C(T,2)

variance calculation:  $O(\epsilon^2)$  is enough!

#### Uniformity Check using L1 Distance

Estimate collision probability to  $1\pm O(\epsilon^2)$ 

 $O(\epsilon^{-4} \operatorname{sqrt}(N))$  samples are enough.

#### Lessons Learned: What We Can Get From Samples

Given samples, some properties of the distribution can be learned, while others cannot.

#### Discriminator based distances

 $\max_{D} E(D(x))_{x\sim p} - E(D(y))_{y\sim q}$ 

0<=D<=1 : Statistical Distance

D is Lipschitz Continuous: Wasserstein Distance

#### Wasserstein Distance

**Duality** 

Earth Mover Distance:

$$W_p(\mu,
u) := \left(\inf_{\gamma \in \Gamma(\mu,
u)} \int_{M imes M} d(x,y)^p \, \mathrm{d}\gamma(x,y) 
ight)^{1/p}$$

Definition using Discriminator:

$$W_1(\mu,
u) = \sup \left\{ \int_M f(x) \operatorname{d}(\mu-
u)(x) \middle| \operatorname{continuous} f: M o \mathbb{R}, \operatorname{Lip}(f) \le 1 
ight\}$$

## Estimating Wasserstein Distance in High Dimension

The curse of dimensionality

There is no algorithm that, for any two distributions P and Q in an n-dimensional space with radius r,

takes poly(n) samples from P and Q and estimates W(P,Q) to precision o(1)\*r w.h.p.

#### Finite Sample Version of EMD

Let  $W_N(P,Q)$  be the expected EMD between N samples from P and Q.

 $W_N(P,Q) >= W(P,Q)$ 

 $W(P,Q) \ge W_N(P,Q) - \min(W_N(P,P), W_N(Q,Q))$ 

#### Projected Wasserstein Distance

The k-dimensional projected EMD: let σ be a random k-dim subspace

$$W^k(P,Q) = \mathcal{E}_{\sigma}W(\sigma(P),\sigma(Q))$$

As a lower bounding approach

$$W(P,Q) \ge \sqrt{n}W^{1}(P,Q) \ge \sqrt{n}(W_{N}^{1}(P,Q) - W_{N}^{1}(P,P))$$

#### Application

Texture Synthesis by matching deep learning features.

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source

previous work



our method



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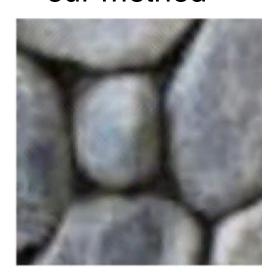
source



previous work



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## Application

#### **GAN** Evaluation



uncertainty caused by sampling

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 $W_1(P,Q)$  3.1  $W_{\text{WGAN}}(P,Q)$   $\leq 0.26$