最小二乘法—法方程实现曲线拟合

冯卓尔 计86

实验要求

实验题目

对物理实验中所得下列数据

t_i	**1**	**1.5**	**2**	**2.5**	**3.0**	**3.5**	**4**	
y_i	33.40	79.50	122.65	159.65	189.15	214.15	238.65	
t_i	4.5	5	5.5	6	6.5	7	7.5	8
y_i	252.2	267.55	280.50	296.65	301.65	210.40	218.15	325.15

（1）用公式$y = a + bt + ct^2$做曲线拟合

（2）用指数函数$y = ae^bt$做曲线拟合

（3）比较上述两条拟合曲线，哪条更好?

算法描述

按照课本算法6.2的描述，对于一组给定的基函数$\phi_1(x), \phi_2(x), \dots, \phi_n(x)$和数据点$(x_i, f(x_i)), i = 0, 1, \dots, m$，最小二乘法求$f(x)$的拟合曲线$y = \sum_{i=1}^n a_i\phi_i(x)$中系数$x = [x_1, x_2, \dots, x_n]^T$，使用法方程法

$$A = \begin{bmatrix} \phi_1(t_1) & \phi_2(t_1) & \cdots & \phi_n(t_1) \\ \phi_1(t_2) & \phi_2(t_2) & \cdots & \phi_n(t_2) \\ \vdots & \vdots & \ddots & \vdots \\ \phi_1(t_m) & \phi_2(t_m) & \cdots & \phi_n(t_m) \end{bmatrix}$$

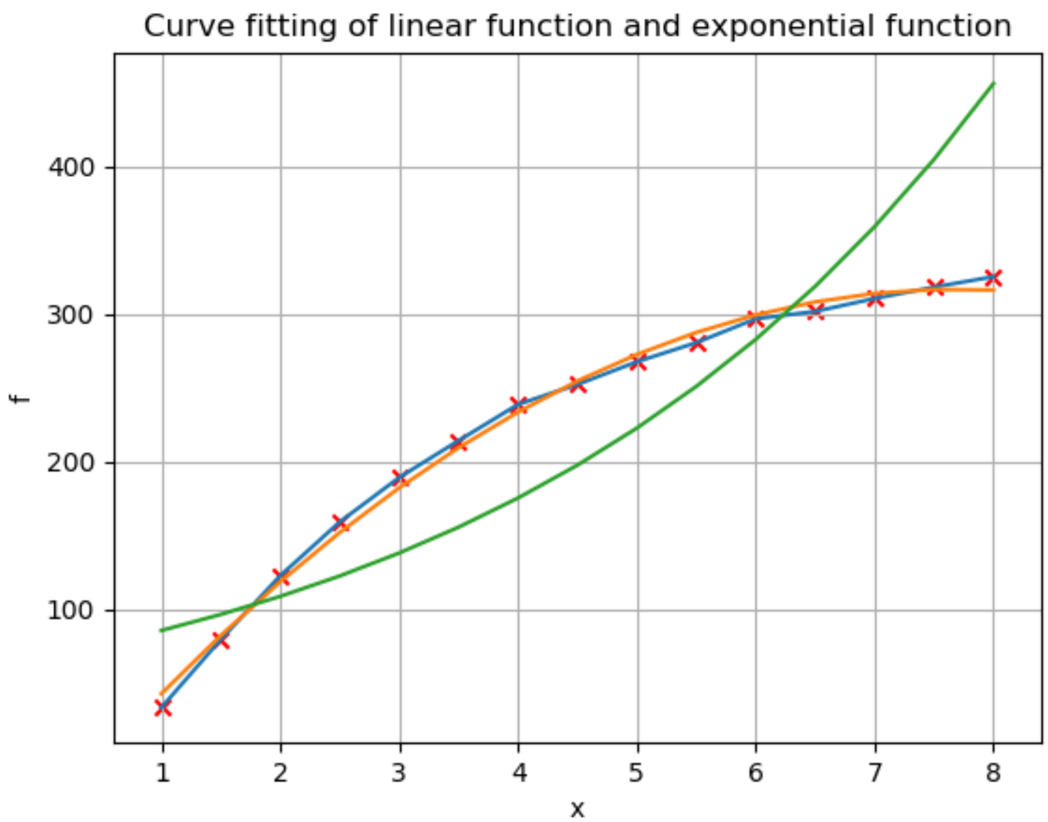
$$
\begin{gathered}
G = A^T A \\
b = A^T f \\
cholesky分解\ G = L^T L \\
L^T L x = b \\
L^T y = b \\
L x = y
\end{gathered}
$$

解得x

（1）取定$\phi_1(x) = 1$，$\phi_2(x) = x$，$\phi_3(x) = x^2$，依照算法得出结果，均方误差为5.683931823476425

（2）先对$y = ae^{bt}$两边取对数，得$\ln(y) = ln(a) + bt$。则现在取$\phi_1(x) = 1$，$\phi_2(x) = x$，f的每一个元素依次取对数。带入算法解得系数$\ln(a)$与b。随后回代计算均方误差$real_f = a * e^{bt}$，$\Delta f = real_f - f$，得到均方误差为56.52224402531076。

（3）画图

- 

其中，红点为数据点，蓝线为插值拟合曲线，橙线为（1）中线性拟合曲线，绿线为（2）中指数函数拟合曲线。

代码

```
import numpy as np
```

```
import matplotlib
import matplotlib.pyplot as plt

# phi 1, t, t^2
A = np.array([
    [1, 1, 1],
    [1, 1.5, 2.25],
    [1, 2, 4],
    [1, 2.5, 6.25],
    [1, 3, 9],
    [1, 3.5, 12.25],
    [1, 4, 16],
    [1, 4.5, 20.25]])
def devide():
    print("-----------------------------------------")

if __name__ == "__main__":
    t = np.array([1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, 6, 6.5, 7, 7.5, 8])
    f = np.array([33.4, 79.5, 122.65, 159.05, 189.15, 214.15, 238.65, 252.2,
267.55, 280.5, 296.65, 301.65, 310.4, 318.15, 325.15])
    f = np.transpose(f)
    t_1 = np.ones(15)
    t_2 = t.copy()
    t_3 = t * t
    a = np.vstack((t_1, t_2, t_3))
    b = np.dot(a, f)
    temp = np.transpose(a)
    G = np.dot(a, temp)
    L = np.linalg.cholesky(G)
    L_inv = np.linalg.inv(L)
    y = np.dot(L_inv, b)
    LT = np.transpose(L)
    LT = np.linalg.inv(LT)

    x = np.dot(LT, y)

    direct_x = np.dot (np.linalg.pinv(np.transpose(a)), f)

    real_f = x[0] * t_1 + x[1] * t_2 + x[2] * t_3
    delta_f = real_f - f
    delta = np.linalg.norm(delta_f) / np.sqrt(15)

    print(a)
    devide()
    print(f)
    devide()
    print(b)
    devide()
    print(G)
```

```
    devide()
    print(L)
    devide()
    print(y)
    devide()
    print(x)
    devide()
    print(direct_x)
    devide()
    print(f)
    devide()
    print(real_f)
    devide()
    print(delta_f)
    devide()
    print(delta)

    fig, ax = plt.subplots()
    ax.plot(t, f)
    ax.plot(t, real_f)
    ax.set(xlabel='x', ylabel='f', title='Curve fitting of linear function and
exponential function')
    ax.grid()
    for i in range(len(t)):
        plt.scatter(t[i], f[i], marker = 'x', color = 'red')

    # fig.savefig("test.png")

    # x - f  x - real_f

    lnf = np.log(f)
    a = np.vstack((t_1, t_2))
    b = np.dot(a, lnf)
    G = np.dot(a, np.transpose(a))
    L = np.linalg.cholesky(G)
    L_inv = np.linalg.inv(L)
    y = np.dot(L_inv, b)
    L = np.transpose(L)
    x = np.dot( np.linalg.inv(L), y)
    numa = np.exp(x[0])
    real_f = numa * np.exp(x[1] * t_2)
    delta_f = real_f -f
    delta = np.linalg.norm(delta_f) / np.sqrt(15)
    ax.plot(t, real_f)

    print(lnf)
    devide()
    print(a)
    devide()
```

```
print(b)
devide()
print(G)
devide()
print(L)
devide()
print(y)
devide()
print(x)
devide()
print(numa)
print(x[1])
devide()
print(f)
devide()
print(real_f)
devide()
print(delta_f)
devide()
print(delta)
plt.show()
```