函数插值：三次样条函数插值

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实验要求

实验题目

已知以下数据

x	**0.52**	**3.1**	**8**	**17.95**	**28.65**	**39.62**	**50.65**	**78**	**104.6**	**156.6**
y	5.288	9.4	13.84	20.20	24.90	28.44	31.10	35	36.9	36.6
x	208.6	260.7	312.50	364.4	416.3	468	494	507	520	
y	34.6	31.0	26.34	20.9	14.8	7.8	3.7	1.5	0.2	

解题思路

按照课本所述样条插值算法进行操作。值得注意的是，由于分段线性Hermite插值（两点三次）在本题中是可以使用的，由于

$$S_3(x) = f_i\alpha_i(x) + f_{i+1}\alpha_{i+1}(x) + m_i\beta_i(x) + m_{i+1}\beta_{i+1}(x)$$
$$S_3'(x) = f_i\alpha_i'(x) + f_{i+1}\alpha_{i+1}'(x) + m_i\beta_i'(x) + m_{i+1}\beta_{i+1}'(x)$$
$$S_3''(x) = f_i\alpha_i''(x) + f_{i+1}\alpha_{i+1}''(x) + m_i\beta_i''(x) + m_{i+1}\beta_{i+1}''(x)$$
$$\alpha_i(x) = (1 + 2\frac{x - x_i}{x_{i+1} - x_i})(\frac{x - x_{i+1}}{x_i - x_{i+1}})^2$$
$$\alpha_{i+1}(x) = (1 + 2\frac{x - x_{i+1}}{x_i - x_{i+1}})(\frac{x - x_i}{x_{i+1} - x_i})^2$$
$$\beta_i(x) = (x - x_i)(\frac{x - x_{i+1}}{x_i - x_{i+1}})^2$$
$$\beta_{i+1}(x) = (x - x_{i+1})(\frac{x - x_i}{x_{i+1} - x_i})^2$$
$$\alpha_i'(x) = \frac{2}{x_{i+1} - x_i}(\frac{x - x_{i+1}}{x_i - x_{i+1}})^2 - \frac{2}{x_{i+1} - x_i}(1 + 2\frac{x - x_i}{x_{i+1} - x_i})(\frac{x - x_{i+1}}{x_i - x_{i+1}})$$
$$\alpha_{i+1}'(x) = \frac{2}{x_i - x_{i+1}}(\frac{x - x_i}{x_{i+1} - x_i})^2 - \frac{2}{x_i - x_{i+1}}(1 + 2\frac{x - x_{i+1}}{x_i - x_{i+1}})(\frac{x - x_i}{x_{i+1} - x_i})$$
$$\beta_i'(x) = (\frac{x - x_{i+1}}{x_i - x_{i+1}})^2 + 2\frac{x - x_i}{x_i - x_{i+1}}(\frac{x - x_{i+1}}{x_i - x_{i+1}})$$
$$\beta_{i+1}'(x) = (\frac{x - x_i}{x_{i+1} - x_i})^2 + 2\frac{x - x_{i+1}}{x_{i+1} - x_i}(\frac{x - x_i}{x_{i+1} - x_i})$$
$$\alpha_i''(x) = \frac{8(x - x_{i+1})}{(x_{i+1} - x_i)^3} + \frac{2}{(x_{i+1} - x_i)^2} + \frac{4(x - x_i)}{(x_{i+1} - x_i)^3}$$
$$\alpha_{i+1}''(x) = \frac{8(x - x_i)}{-(x_{i+1} - x_i)^3} + \frac{2}{(x_{i+1} - x_i)^2} + \frac{4(x - x_{i+1})}{-(x_{i+1} - x_i)^3}$$
$$\beta_i''(x) = \frac{4(x - x_{i+1})}{(x_{i+1} - x_i)^2} + \frac{2(x - x_i)}{(x_{i+1} - x_i)^2}$$
$$\beta_{i+1}''(x) = \frac{4(x - x_i)}{(x_i - x_{i+1})^2} + \frac{2(x - x_{i+1})}{(x_i - x_{i+1})^2}$$

其中，m可以有固定边界条件求出，$m_0 = f_0', m_n = f_n'$

$$
A=\begin{bmatrix}
2 & \lambda_1 & & & \\
\mu_2 & 2 & \lambda_2 & & \\
 & \mu_3 & 2 & \ddots & \\
 & & \ddots & 2 & \lambda_{n-2} \\
 & & & \mu_{n-1} & -(2\epsilon+h)
\end{bmatrix}
$$

$$
M=\begin{bmatrix} m_1 \\ m_2 \\ \vdots \\ m_{n-1} \end{bmatrix}
\qquad
D=\begin{bmatrix} d_1-\mu_1 m_0 \\ d_2 \\ \vdots \\ d_{n-2} \\ d_{n-1}-\lambda_{n-1}m_n \end{bmatrix}
$$

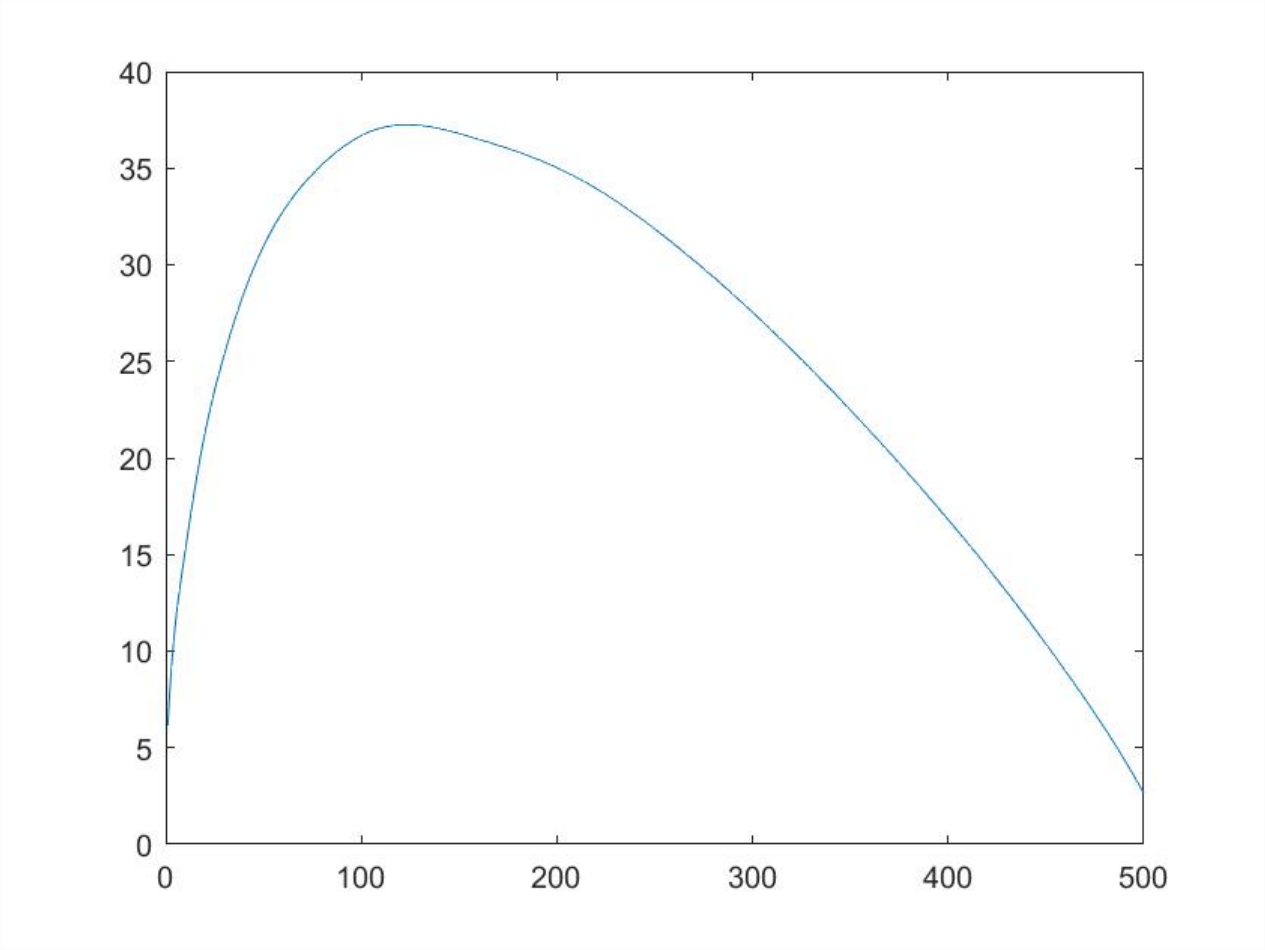
$$
s.t. \qquad AM=D
$$

追赶法解方程即可。由于先前有一个高斯求方程算法，为了节省时间直接调用。原因是这题中n=19，计算量差的上限不超过10^6，在数值计算上表现出来不超过0.01s，因而可以。

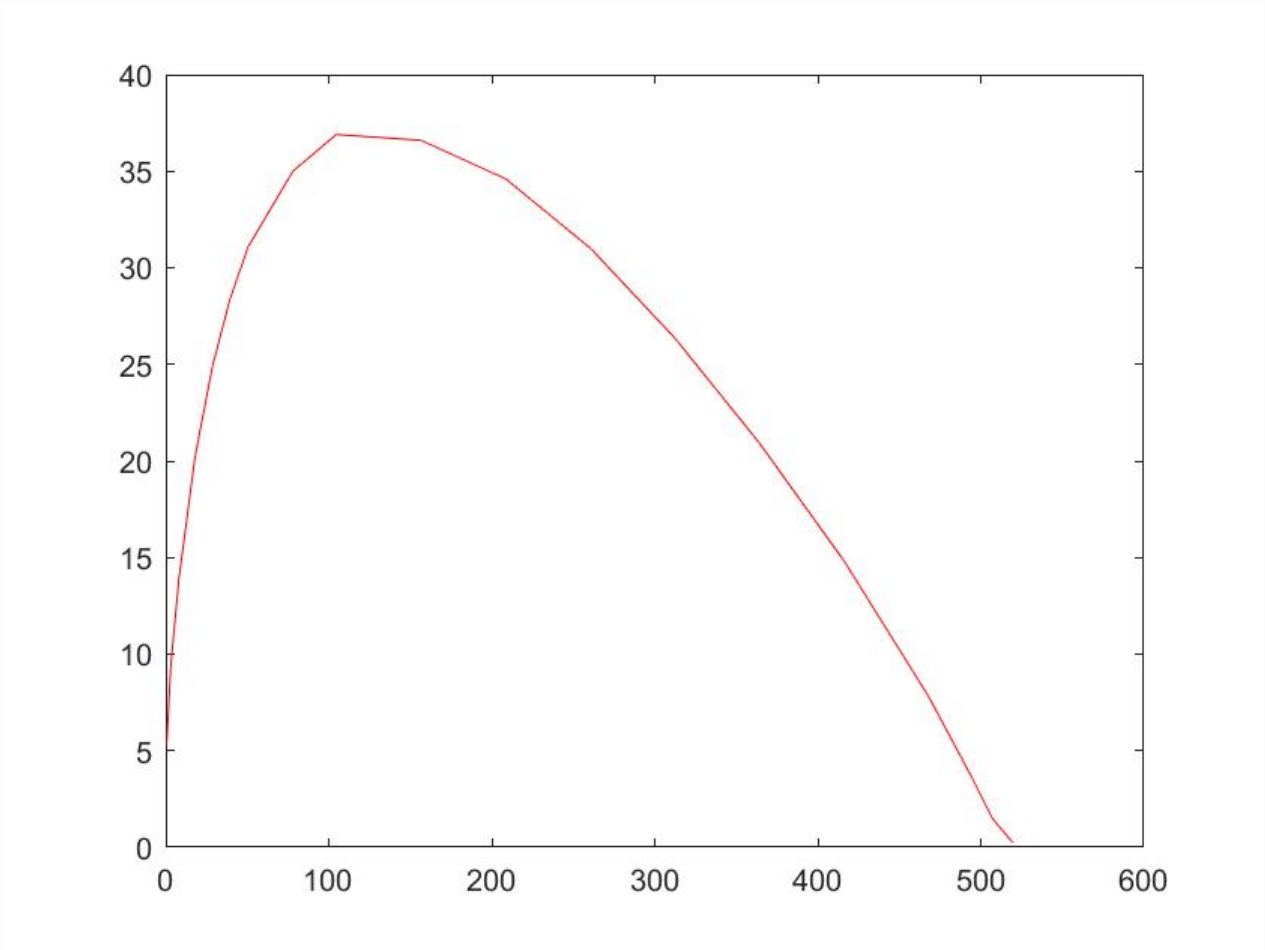
最后，对于所求x，首先判断出它在区间中的位置序列t，随后调用相应的三次函数计算它的函数值、一次导数值、二次导数值。结果在数据中。

插值曲线

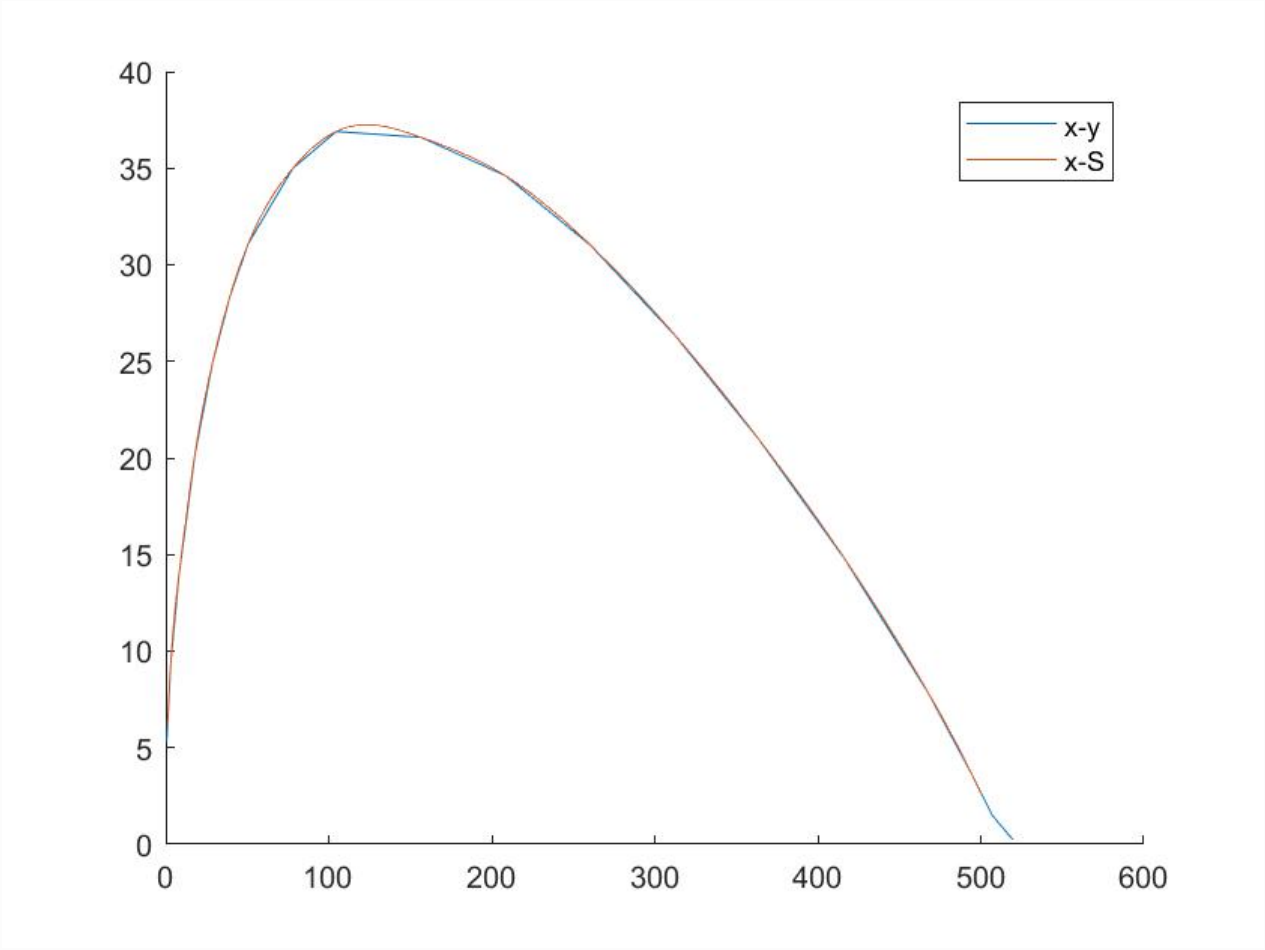
三次样条插值曲线


40
35
30
25
20
15
10
5
0
0
100
200
300
400
500

数据点两两相连


40
35
30
25
20
15
10
5
0
0
100
200
300
400
500
600

两条曲线绘于一图中



实验结果

x	$f(x)$	$f'(x)$	$f''(x)$
2	7.8252	1.5538	-7.8221
30	25.3862	0.3549	-1.5397
130	37.2138	-0.0104	-0.0442
350	22.4751	-0.1078	0.1743
515	0.5427	-0.0899	0.1880

代码

```
function x=gaosixiaoqu(A,b,n)
```

```
for k=1:(n-1)
    for i=(k+1):n
        piv=A(i,k)/A(k,k);
        b(i)=b(i)-piv*b(k);
        for j=k:n
            A(i,j)=A(i,j)-piv*A(k,j);
        end
    end
end
clear i
clear j
clear k

x=zeros(n,1);
x(n)=b(n)/A(n,n);
for i=(n-1):-1:1
        for j=(i+1):n
            x(i)=-x(j)*A(i,j)/A(i,i)+x(i);
        end
        x(i)=x(i)+b(i)/A(i,i);
end
```

```
function [y,y1,y2]=yangtiao(a)

x=
[0.52,3.1,8,17.95,28.65,39.62,50.65,78,104.6,156.6,208.6,260.7,312.5,36
4.4,416.3,468,494,507,520];
f=
[5.288,9.4,13.84,20.2,24.9,28.44,31.1,35,36.9,36.6,34.6,31,26.34,20.9,1
4.8,7.8,3.7,1.5,0.2];

zt=length(x);

for i=1:zt-1
    h(i)=x(i+1)-x(i);
end
for i=1:zt-2
    u(i)=h(i+1)/(h(i+1)+h(i));
    l(i)=1-u(i);
end

for i=1:zt-1
    g(i)=(f(i+1)-f(i))/(x(i+1)-x(i));
end

for i=1:zt-2
    d(i)=3*(u(i)*g(i)+l(i)*g(i+1));
end

m0=1.86548;
```

```
mzt=-0.046115;

A=zeros(zt-2,zt-2);
for i=1:zt-2
    A(i,i)=2;
end

for i=1:zt-3
    A(i+1,i)=u(i+1);
    A(i,i+1)=l(i);
end

b=zeros(zt-2,1);
b(1)=d(1)-u(1)*m0;
b(zt-2)=d(zt-2)-l(zt-2)*mzt;

for i=2:1:zt-3
    b(i)=d(i);
end

M=gaosixiaoqu(A,b,zt-2);
m=zeros(zt,1);
m(1)=m0;
m(zt)=mzt;
for i=1:zt-2
    m(i+1)=M(i);
end

x(zt+1)=a;
x_order=sortrows(x);
t=find(x_order==a);
t=t-1;

y=0;
y=f(t)*(1+2*(a-x(t))/(x(t+1)-x(t)))*((a-x(t+1))/(x(t)-x(t+1)))^2;
y=y+f(t+1)*(1+2*(a-x(t+1))/(x(t)-x(t+1)))*((a-x(t))/(x(t+1)-x(t)))^2;
y=y+m(t)*(a-x(t))*((a-x(t+1))/(x(t)-x(t+1)))^2;
y=y+m(t+1)*(a-x(t+1))*((a-x(t))/(x(t+1)-x(t)))^2;

y1=0;
y1=f(t)*(2/(x(t+1)-x(t))*((a-x(t+1))/(x(t)-x(t+1)))^2-2/(x(t+1)-x(t))*
(1+2*(a-x(t))/(x(t+1)-x(t)))*(a-x(t+1))/(x(t)-x(t+1)));
y1=y1+f(t+1)*(2/(x(t)-x(t+1))*((a-x(t))/(x(t+1)-x(t)))^2-2/(x(t)-
x(t+1))*(1+2*(a-x(t+1))/(x(t)-x(t+1)))*(a-x(t))/(x(t+1)-x(t)));
y1=y1+m(t)*(((a-x(t+1))/(x(t)-x(t+1)))^2+2*(a-x(t))/(x(t)-x(t+1))*(a-
x(t+1))/(x(t)-x(t+1)));
y1=y1+m(t+1)*(((a-x(t))/(x(t+1)-x(t)))^2+2*(a-x(t+1))/(x(t+1)-x(t))*(a-
x(t))/(x(t+1)-x(t)));

y2=0;
y2=f(t)*(8*(a-x(t+1))/(x(t+1)-x(t))^3+2/(x(t+1)-x(t))+4*(a-
x(t))/(x(t+1)-x(t))^3);
```

```
y2=y2+f(t+1)*(8*(a-x(t))/(x(t)-x(t+1))^3+2/(x(t)-x(t+1))+4*(a-
x(t+1))/(x(t)-x(t+1))^3);
y2=y2+m(t)*(4*(a-x(t+1))/(x(t+1)-x(t))^2+2*(a-x(t))/(x(t+1)-x(t))^2);
y2=y2+m(t+1)*(4*(a-x(t))/(x(t)-x(t+1))^2+2*(a-x(t+1))/(x(t)-x(t+1))^2);

% end
```