1. (20pts) Let C be the curve given by $\mathbf{r}(t) = \sin t\,\mathbf{i} + (1 - \cos t)\mathbf{j} + t^2\mathbf{k}$, for $0 \leq t \leq 2\pi$.

   (a) Find the curvature $\kappa(t)$.

   (b) Find symmetric equations of the tangent line to C at the point $(0, 2, \pi^2)$. At what point does this line intersect the xy-plane?

2. (20pts) Let $f(x, y, z) = xz \cdot e^{yz} - \sin(xy)$ and $P = (1, 0, -2)$.

   (a) What is the maximum rate of change of f at the point P?

   (b) Find the linearization of f and use it to approximate $f(1.03, -0.01, -1.98)$.

3. (20pts) Let S be the hyperboloid $x^2 + 4y^2 - z^2 = 4$.
   (a) Find an equation of the normal line to S at the point $(2, 1, 2)$.
   (b) Find the points on S where the tangent plane is parallel to the plane $2x + 2y + z = 5$.

4. (20pts) Let $f(x,y) = x^3 + xy^2 + 6x^2 + y^2$.

   (a) Find the local maximum, minimum values and saddle points of $f(x,y)$.

   (b) Find the absolute maximum and minimum values of $f(x,y)$ on the disk $x^2 + y^2 \leq 1$.

5. (20pts) Let E be the solid that lies within the sphere $x^2 + y^2 + (z-2)^2 = 4$ and below the cone $z = \sqrt{\dfrac{x^2+y^2}{3}}$. At each point, the density of E is equal to the distance between the point and the z-axis. **Set up** two iterated integrals, each with explicit integrand and explicit limits of integration, to compute the mass of the solid E. **DO NOT EVALUATE** these integrals.

   (a) Use spherical coordinates ρ, θ, ϕ.

   (b) Use rectangular coordinates x, y, z and the order of integration $dzdydx$.

6. (20pts) Given the vector field $\mathbf{F}(x, y, z) = (\sin(\pi x) + \pi x \cos(\pi x))\ \mathbf{i} + \arctan z\ \mathbf{j} + \left(\frac{y}{1+z^2} + 1\right)\ \mathbf{k}$.

   (a) Show that $\mathbf{F}$ is conservative and find a potential function f.

   (b) Use the Fundamental Theorem of line integrals to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is the part of the epicycloid: $\mathbf{r}(t) = (5\cos t - \cos 5t)\mathbf{i} + (5\sin t - \sin 5t)\mathbf{j} + \mathbf{k}$ from $t = 0$ to $t = \pi/2$.

7. (20pts) Let S be the part of the hyperbolic paraboloid $z = y^2 - x^2$ that lies in the cylinder $x^2 + y^2 = 1$.

   (a) Find the surface area of S.

   (b) If S is oriented upward, compute the flux of $\mathbf{F}(x, y, z) = x\mathbf{i} - y\mathbf{j} + (1 - z)\mathbf{k}$ across S.

8. (20pts) Use Green's theorem with the vector field $F(x, y) = -y\mathbf{i} + x\mathbf{j}$ to compute the area of the region enclosed by the hypocycloid $\mathbf{r}(t) = \cos^3 t\ \mathbf{i} + \sin^3 t\ \mathbf{j}$ for $0 \le t \le 2\pi$.

9. (20pts) A particle moves along line segments from the point $(0, 0, 0)$ to points $(0, 2, 0)$, $(1, 1, 1)$ and back to $(0, 0, 0)$ under the influence of the force field $\mathbf{F}(x, y, z) = x\mathbf{i} - e^z\mathbf{j} + z\mathbf{k}$. Use Stokes' Theorem to find the work done.

10. (20pts) Let S be the boundary surface of the solid E that lies within the cylinder $x^2 + y^2 = 1$, below the plane $4x - 4y + z = 0$, and above the plane $z = 0$, with the normal pointing outward. Let S_1 and S_2 be the parts of S that belong to the cylinder and the plane $4x - 4y + z = 0$, respectively. Use the Divergence Theorem to calculate the flux integral $\iint_{S_1 \cup S_2} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F}(x, y, z) = \left(e^z - x^2\right) \mathbf{i} + 3y\mathbf{j} + 2xz\mathbf{k}$.