

*Digital Integrated Circuits*

# ***Inverter***

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# CMOS Inverter

- At a glance
- Static behavior
- Dynamic behavior
- ***Power, Energy, and Energy Delay***
- Perspective tech.

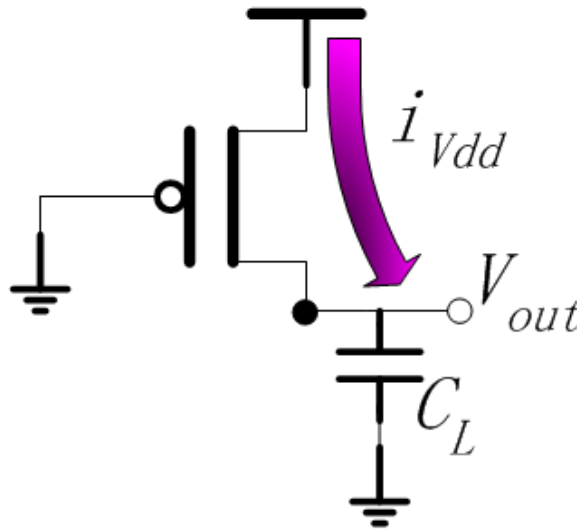
# Dynamic Power Consumption

- (dis)charge process
  - $C_L$  is charged through pMOS on-resistance
  - $C_L$  is discharged through nMOS on-resistance
- Power distribution
  - Charge processing: One part of Supply power is dissipated in the pMOS transistor, another part is dissipated in the charge  $C_L$
  - Discharge processing: all dissipated in the nMOS transistor

# Precise measure of dynamic power consumption

$$E_{V_{dd}} = \int_0^{\infty} i_{V_{dd}}(t) V_{dd} dt = V_{dd} \int_0^{\infty} C_L \frac{dv_{out}}{dt} dt$$

$$= C_L V_{dd} \int_0^{V_{dd}} dv_{out} = C_L V_{dd}^2$$

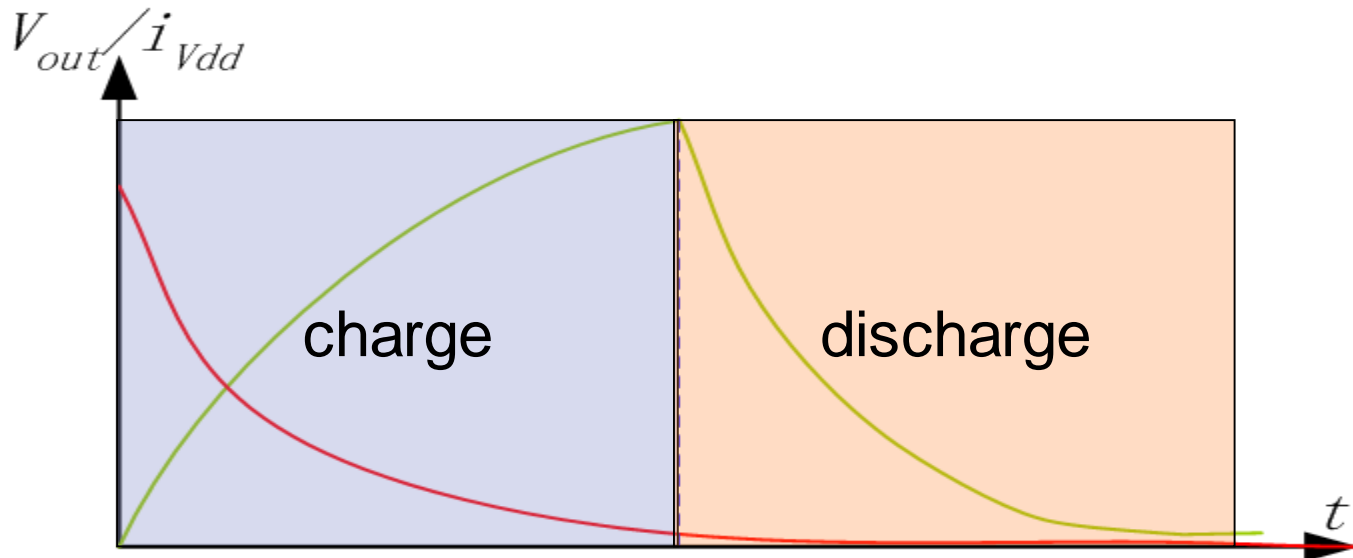


$$E_C = \int_0^{\infty} i_{V_{dd}}(t) v_{out} dt = \int_0^{\infty} C_L \frac{dv_{out}}{dt} v_{out} dt$$

$$= C_L \int_0^{V_{dd}} v_{out} dv_{out} = \frac{C_L V_{dd}^2}{2}$$

# Output voltages and supply current during (dis)charge of $C_L$

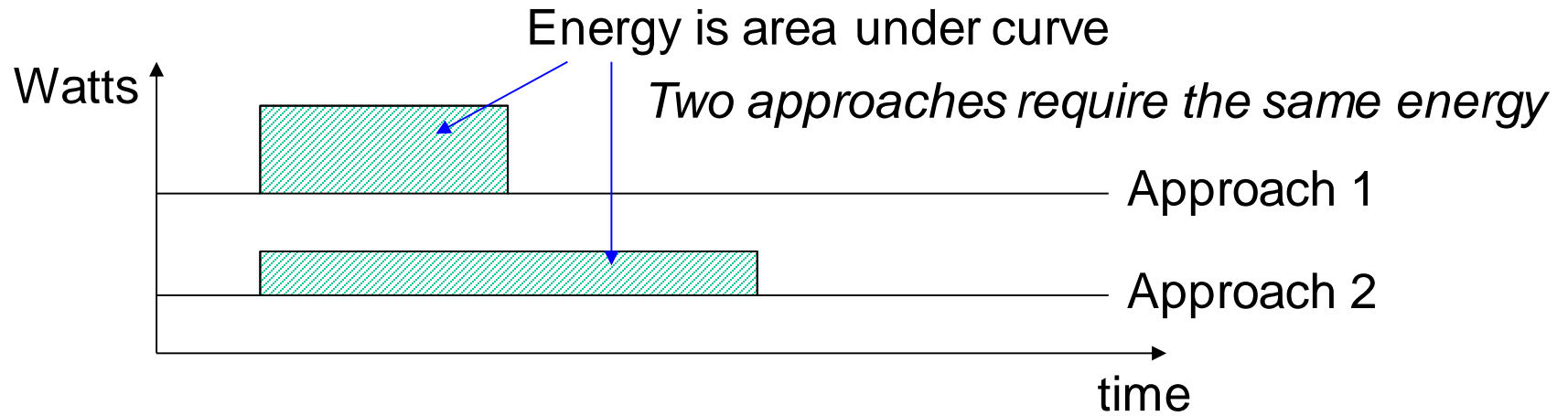
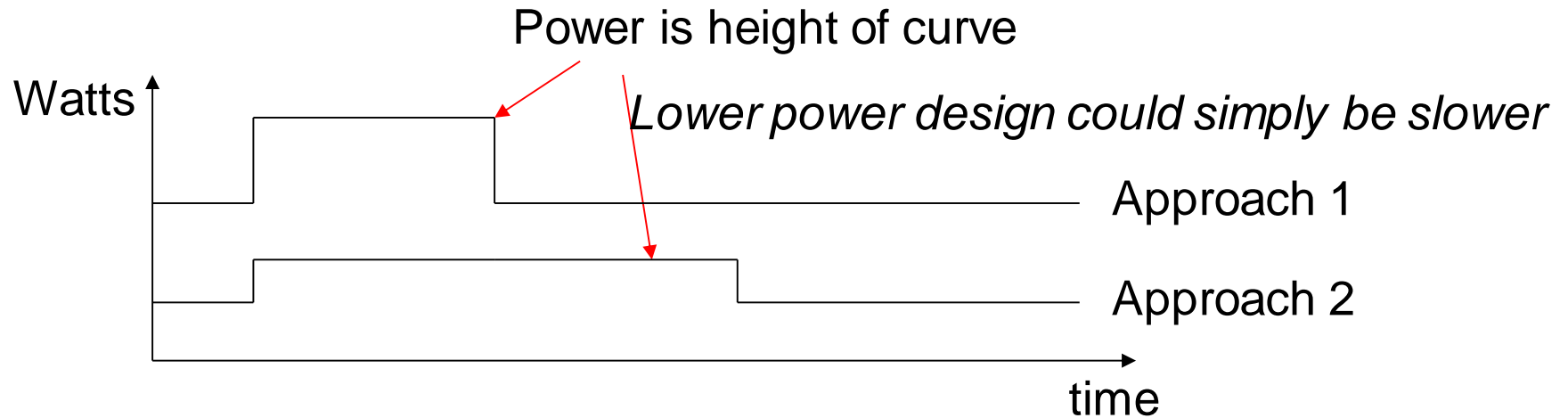
- Energy dissipation is independent of the size
- Power consumption is dependent of device switching number.



# Power and Energy Figures of Merit

- **Power** consumption in **Watts**
  - determines battery life in hours
- Peak power
  - determines power ground wiring designs
  - sets packaging limits
  - impacts signal noise margin and reliability analysis
- **Energy** efficiency in **Joules**
  - rate at which power is consumed over time
- **Energy = power \* delay**
  - **Joules = Watts \* seconds**
  - lower energy number means less power to perform a computation at the same frequency

# Power versus Energy

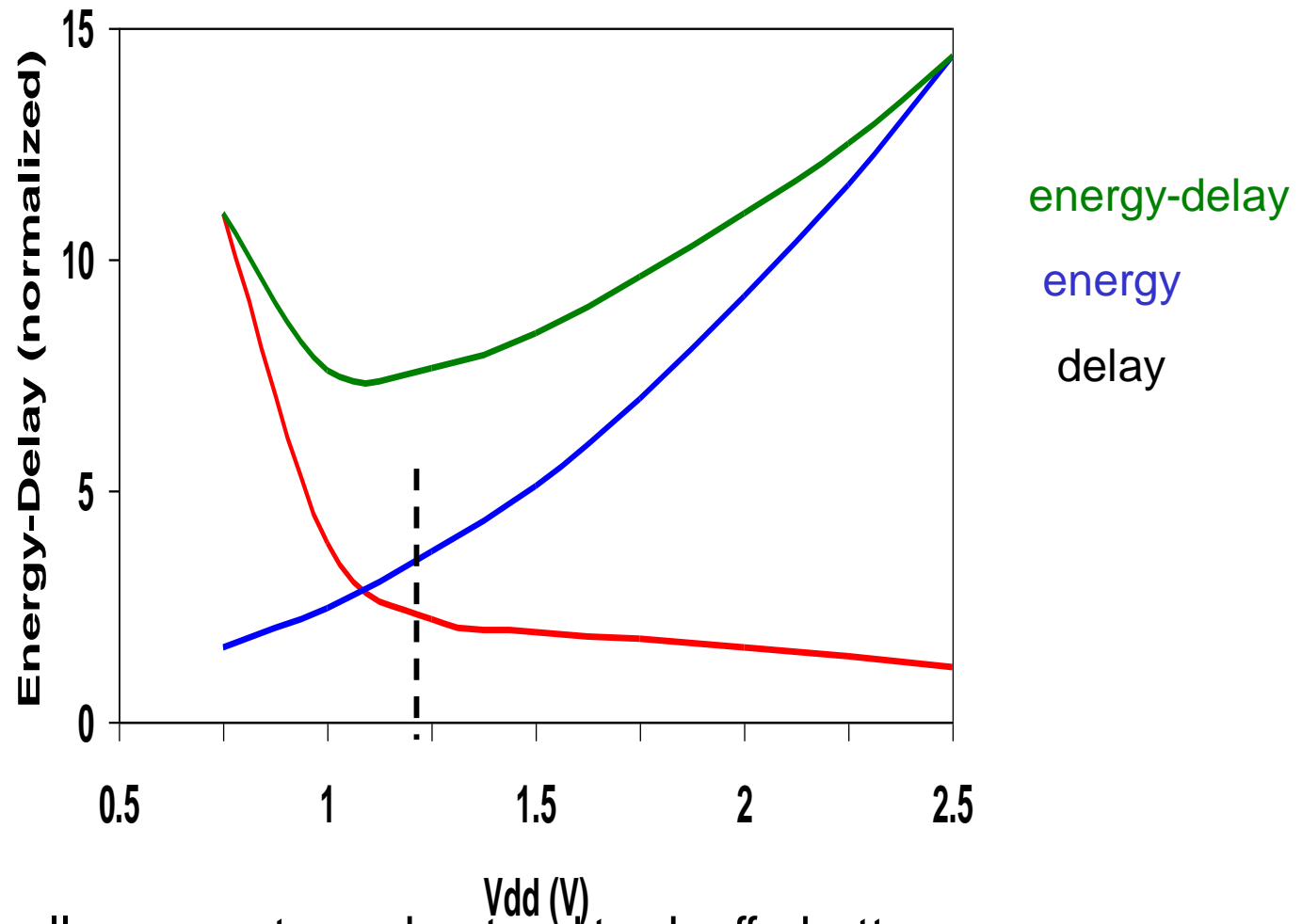


# PDP and EDP

- Power-delay product (PDP) =  $P_{av} * t_p = (C_L V_{DD}^2)/2$ 
  - PDP is the average **energy** consumed per switching event (Watts \* sec = Joule)
  - **lower power design could simply be a slower design**
- Energy-delay product (EDP) =  $PDP * t_p = P_{av} * t_p^2$ 
  - EDP is the average energy consumed multiplied by the computation time required
  - takes into account that one can trade increased delay for lower energy/operation(e.g., via supply voltage scaling that increases delay, but decreases energy consumption)



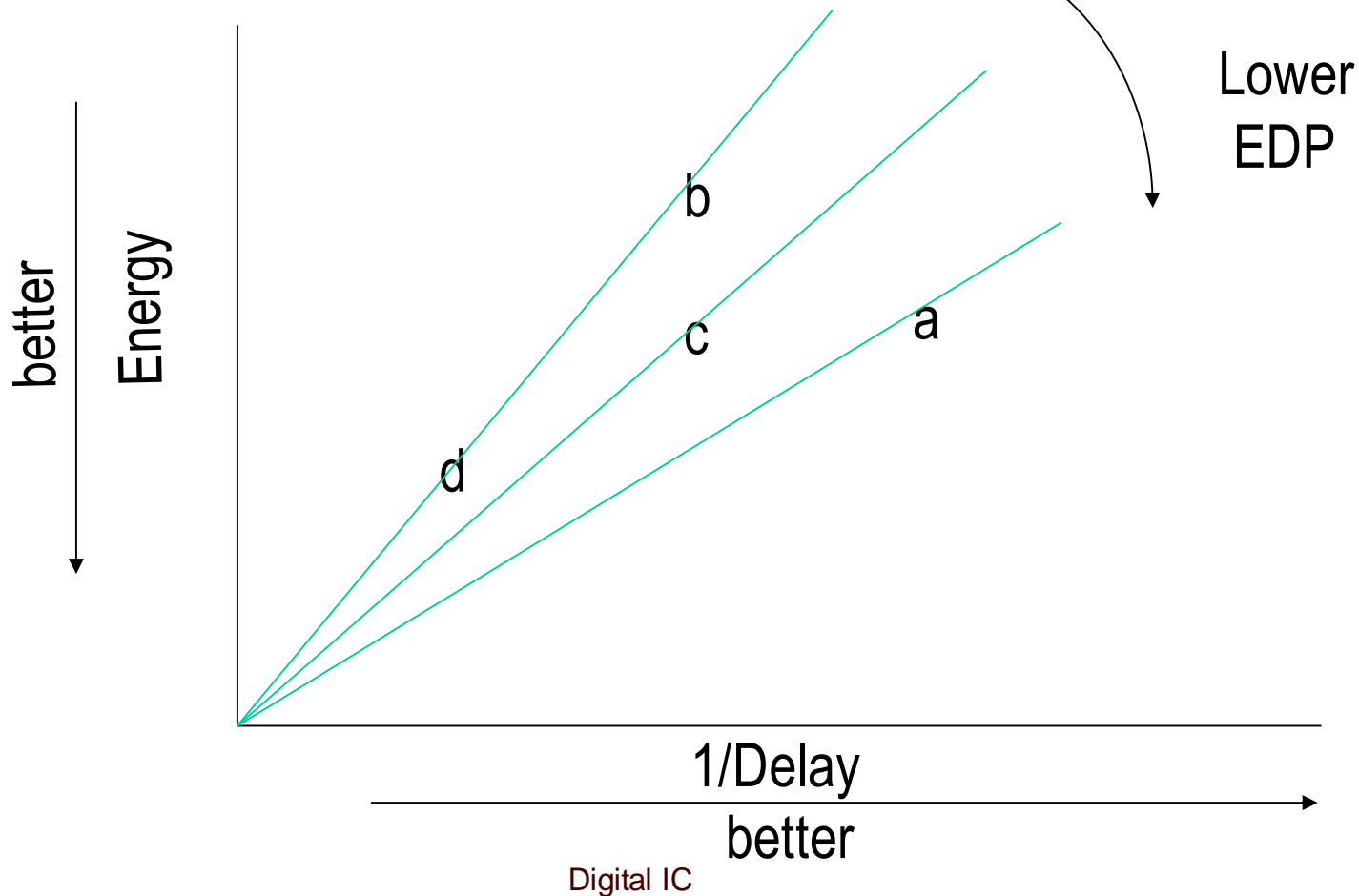
# PDP and EDP



allows one to understand tradeoffs better

# Understanding Tradeoffs

- Which design is the “best” (fastest, coolest, both) ?



# Where Does Power Go in CMOS?

- Dynamic Power Consumption
  - Charging and Discharging Capacitors
- Short Circuit Currents
  - Short Circuit Path between Supply Rails during Switching
- Leakage
  - Leaking diodes and transistors

# CMOS Energy & Power Equations

$$E = C_L V_{DD}^2 P_{0 \rightarrow 1} + t_{sc} V_{DD} I_{peak} + V_{DD} I_{leakage} 1/f_{clock}$$

$$f_{0 \rightarrow 1} = P_{0 \rightarrow 1} * f_{clock}$$

$$P = C_L V_{DD}^2 f_{0 \rightarrow 1} + t_{sc} V_{DD} I_{peak} f + V_{DD} I_{leakage}$$

*Dynamic power*

*Short-circuit  
power*

*Leakage power*

# Lowering Dynamic Power

**Capacitance:**  
*Function of fan-out, wire length, transistor sizes*

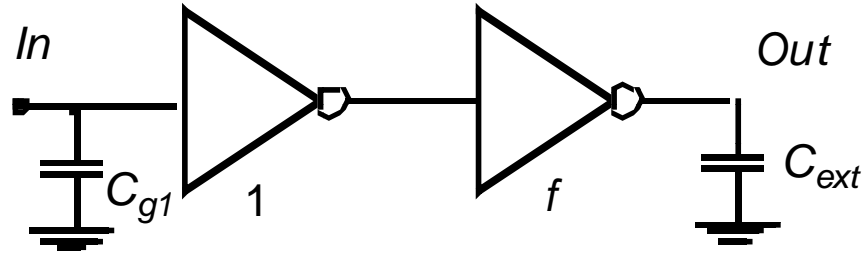
**Supply Voltage:**  
*Has been dropping with successive generations*

$$P_{\text{dyn}} = C_L V_{DD}^2 P_{0 \rightarrow 1} f$$

**Activity factor:**  
*How often, on average, do wires switch?*

**Clock frequency:**  
*Increasing...*

# Transistor Sizing for Minimum Energy



- *Goal: Minimize Energy of whole circuit*
  - Design parameters:  $f$  and  $V_{DD}$
  - $t_p \leq t_{pref}$  of circuit with  $f=1$  and  $V_{DD}=V_{ref}$

$$t_p = t_{p0} \left( \left( 1 + \frac{f}{\gamma} \right) + \left( 1 + \frac{F}{f\gamma} \right) \right) \quad I_{DSAT} = k' \frac{W}{L} (V_{GT} V_{DSAT} - \frac{V_{DSAT}^2}{2})$$

$$t_{p0} \propto \frac{V_{DD}}{V_{DD} - V_{TE}}$$

$$t_{pLH} = \ln(2) R_{eqn} C = 0.69 * \frac{3V_{DD}}{4I_{DSATn}} (1 - \lambda V_{DD}) C_L$$

# Transistor Sizing (2)

- **Performance Constraint ( $\gamma=1$ )**

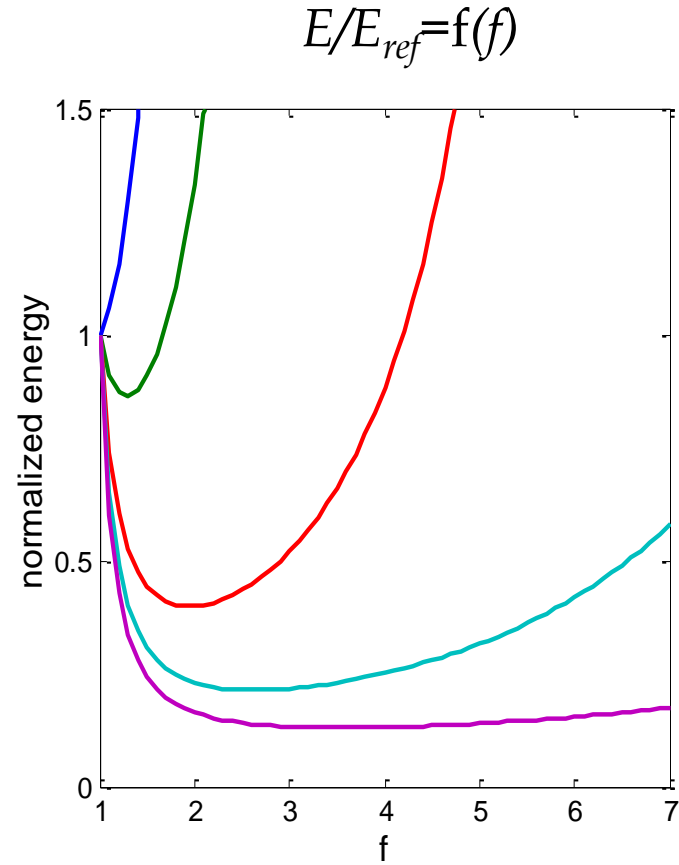
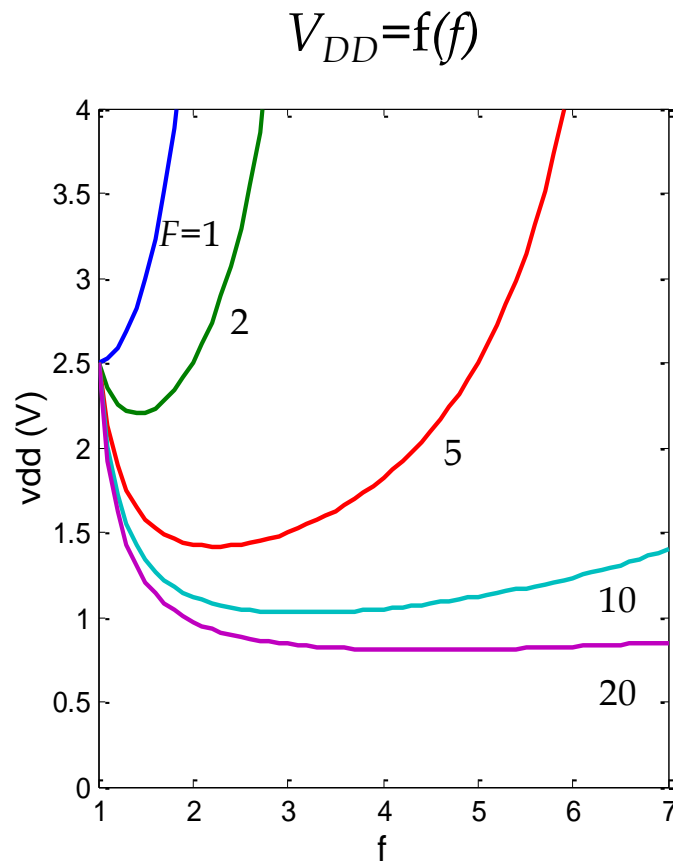
$$\frac{t_p}{t_{pref}} = \frac{t_{p0}}{t_{p0ref}} \frac{\left(2 + f + \frac{F}{f}\right)}{(3 + F)} = \frac{V_{DD}}{V_{ref}} \frac{V_{ref} - V_{TE}}{V_{DD} - V_{TE}} \frac{\left(2 + f + \frac{F}{f}\right)}{(3 + F)} = 1$$

- **Energy for single Transition**

$$E = V_{DD}^2 C_{g1} [(1 + \gamma)(1 + f) + F]$$

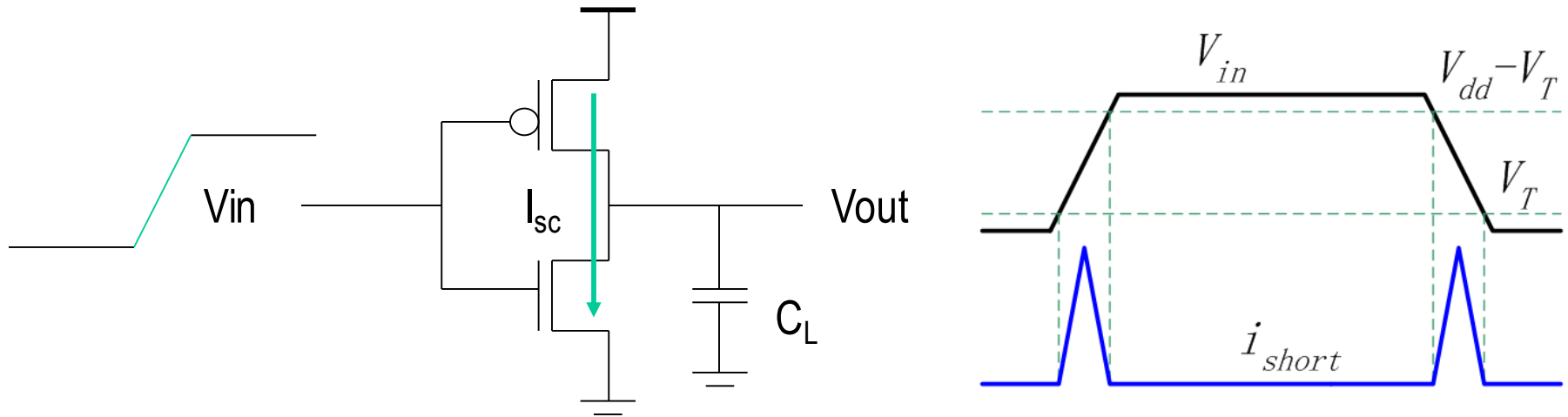
$$\frac{E}{E_{ref}} = \left(\frac{V_{DD}}{V_{ref}}\right)^2 \left(\frac{2 + 2f + F}{4 + F}\right)$$

# Transistor Sizing (3)





# Short Circuit Power Consumption



***Finite slope of the input signal causes a direct current path between  $V_{DD}$  and GND for a short period of time during switching when both the NMOS and PMOS transistors are conducting.***

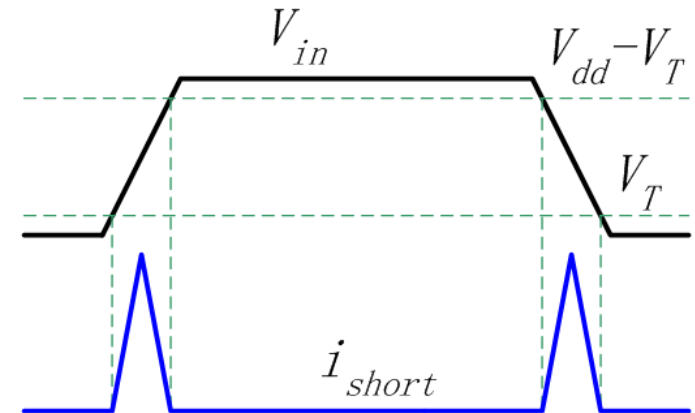
# Short Circuit power consumption

- Energy of ever switch activity

$$E_{dp} = V_{dd} \frac{I_{peak}}{2} t_{rise} + V_{dd} \frac{I_{peak}}{2} t_{fall}$$

- Average power

$$P_{dp} = \frac{t_{rise} + t_{fall}}{2} V_{dd} I_{peak} f$$

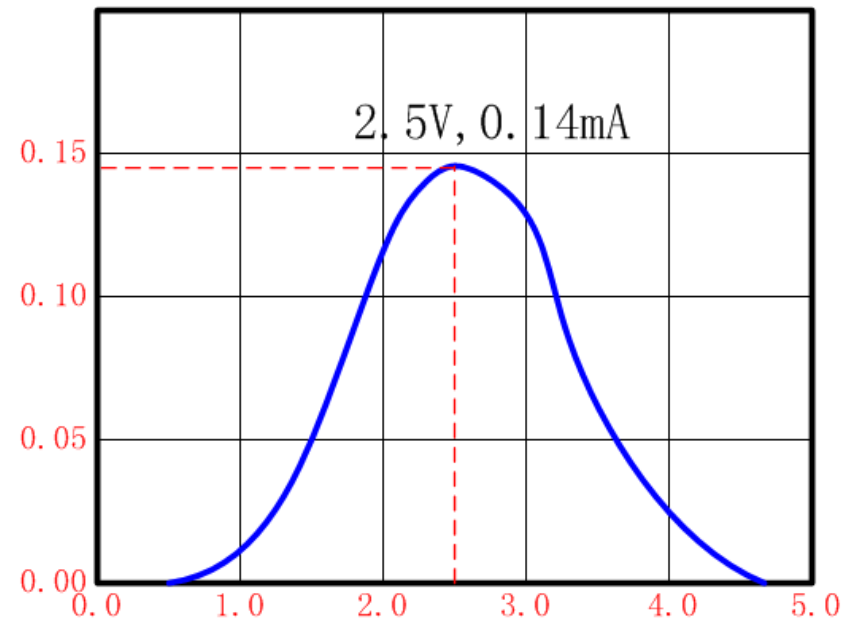


- Short circuit occupy no more than 20% of dynamic power**

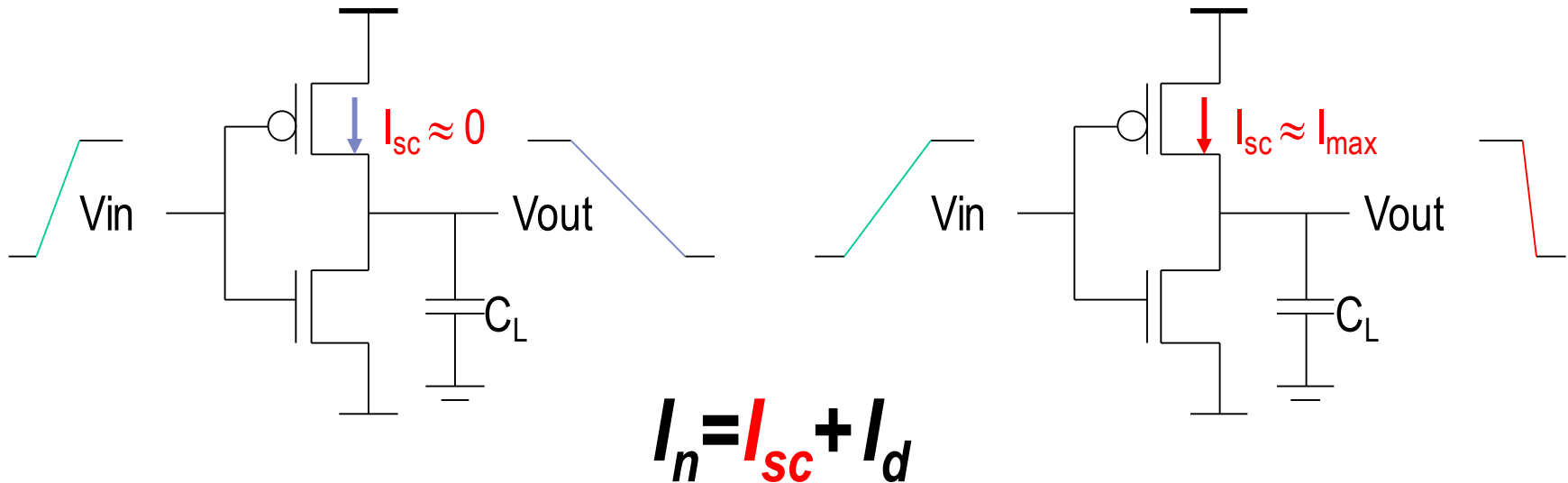
$$t_{sc} = \frac{V_{DD} - 2V_T}{V_{DD}} t_s \approx \frac{V_{DD} - 2V_T}{V_{DD}} \times \frac{t_{r(f)}}{0.8}$$

# An example

- Assume rise/fall time both are 300ps
- Short circuit power:  
 $300\text{ps} \times 5\text{V} \times 0.14\text{mA} = 0.21\text{pJ}$
- Dynamic power  
 $30\text{pF} \times 5\text{V} \times 5\text{V} = 0.75\text{pJ}$



# Impact of $C_L$ on $P_{sc}$



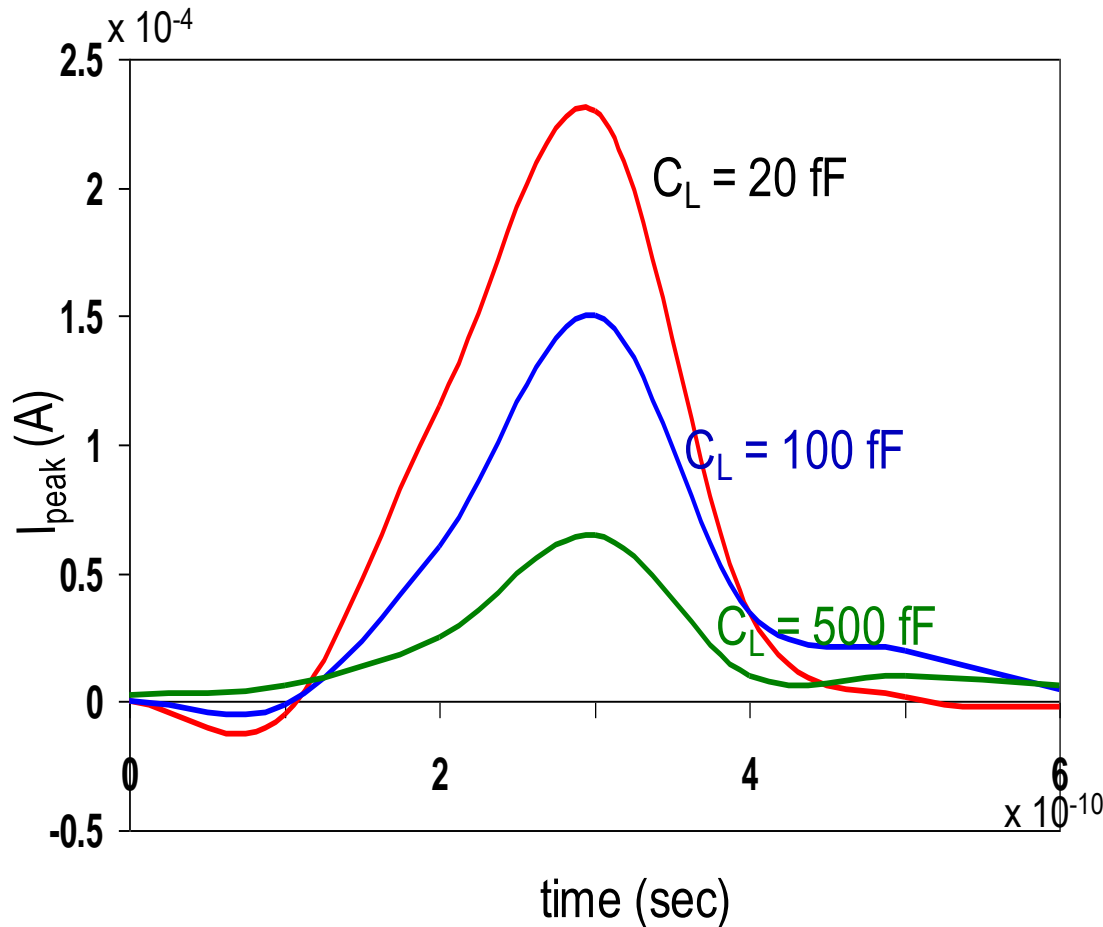
## Large capacitive load

*Output fall time significantly larger than input rise time.*

## Small capacitive load

*Output fall time substantially smaller than the input rise time.*

# $I_{\text{peak}}$ as a Function of $C_L$



When load capacitance is small,  $I_{\text{peak}}$  is large.

Short circuit dissipation is minimized by matching the rise/fall times of the input and output signals - slope engineering.

500 psec input slope

.global gnd

.subckt inv a y vdd1 N=4 P=8

M1 y a gnd gnd NMOS W=N L=2

+ AS=N+5 PS='2\*N+10' AD='N+5' PD='2\*N+10'

M2 y a vdd1 vdd1 PMOS W='2\*N' L=2

+ AS='P+5' PS='2\*P+10' AD='P+5' PD='2\*P+10'

.ends

\*-----

\*simulation netlist

\*-----

vdd1 vdd1 gnd 'SUPPLY'

vdd2 vdd2 gnd 'SUPPLY'

vin a gnd PULSE 0 'SUPPLY' 0ps 100ps 100ps 40000ps 84000ps

\*shape input waveform

X1 a b vdd1 inv N=2

\*reshape input waveform

X2 b c vdd1 inv N=2\*\*2

\*device under test

x3 c d vdd1 inv N=8

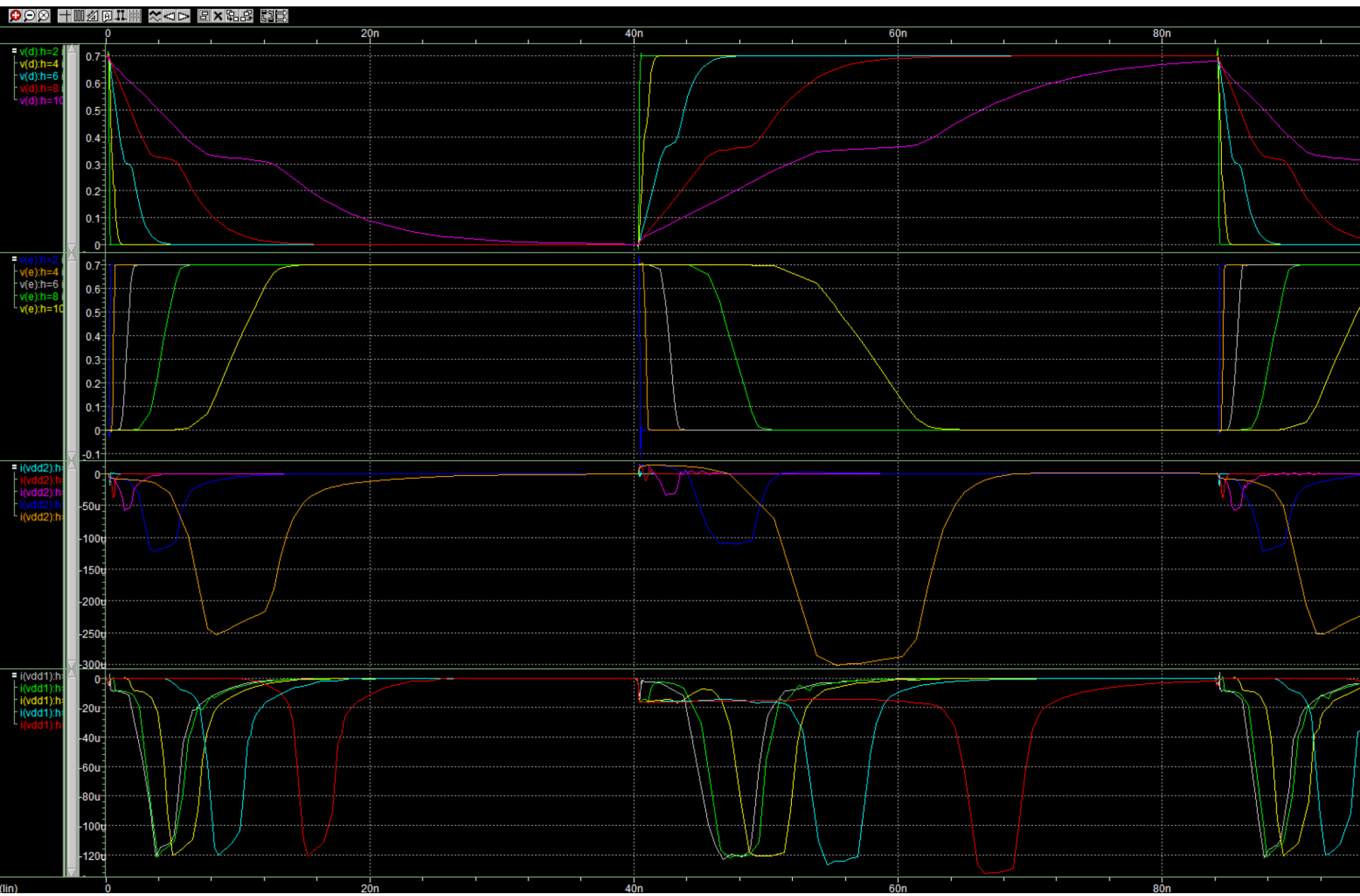
\*load

x4 d e vdd2 inv N=H\*\*4

\*load on load

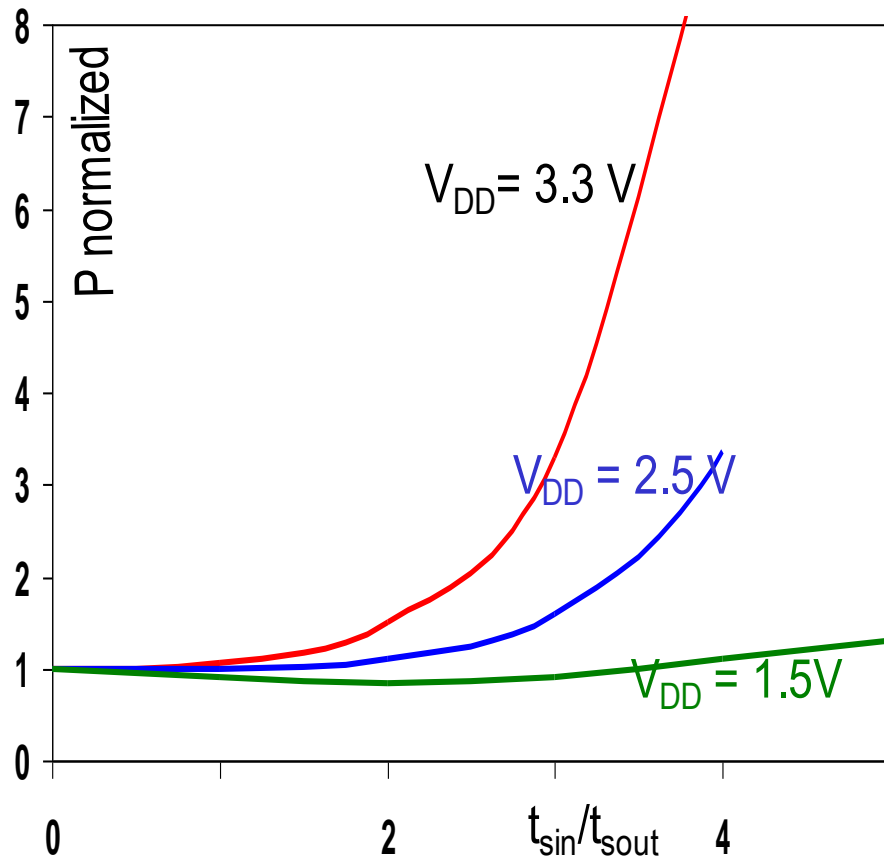
x5 e f vdd1 inv N=8

X6 f h vdd1 inv N=4\*1000



Digital IC

# $P_{sc}$ as a Function of Rise/Fall Times



**When load capacitance is small ( $t_{sin}/t_{sout} > 2$  for  $V_{DD} > 2\text{V}$ ) the power is dominated by  $P_{sc}$**

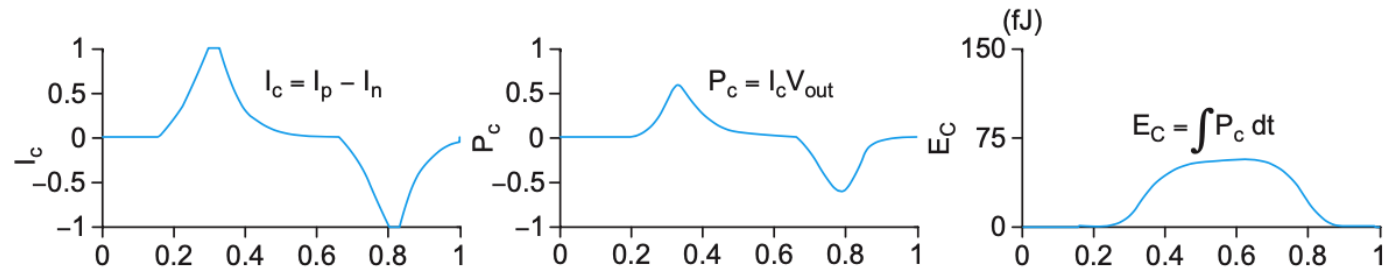
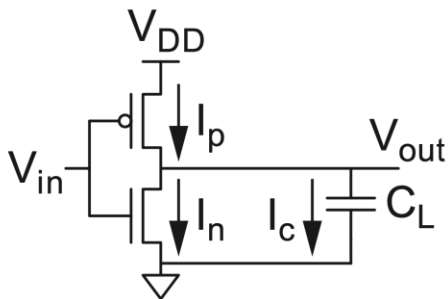
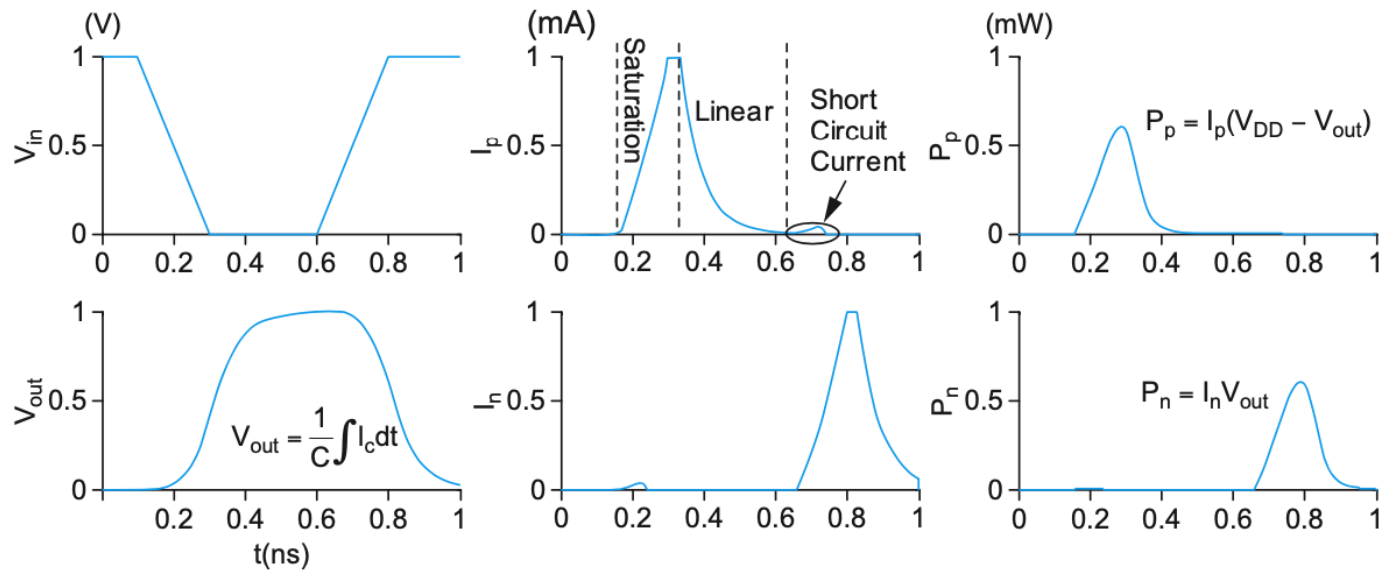
**If  $V_{DD} < V_{Tn} + |V_{Tp}|$  then  $P_{sc}$  is eliminated since both devices are never on at the same time.**

$W/L_p = 1.125\text{ }\mu\text{m}/0.25\text{ }\mu\text{m}$   
 $W/L_n = 0.375\text{ }\mu\text{m}/0.25\text{ }\mu\text{m}$   
 $C_L = 30\text{ fF}$

normalized wrt zero input rise-time dissipation

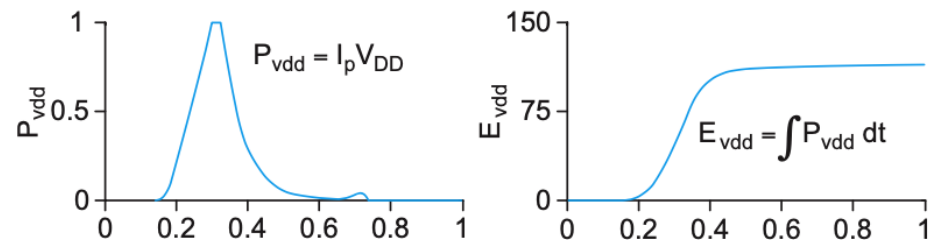


# Switching power



**FIGURE 5.4**  
CMOS inverter

**FIGURE 5.5** Inverter switching voltage, current, power, and energy



# Power Testing

inverter - 记事本

文件(F) 编辑(E) 格式(O) 查看(V) 帮助(H)

```
* cmos.sp
.option node
*.option list
.probe I(*) V(*)
*****
* Parameters and models
*****
.param SUPPLY=0.7
.option scale=16n
.param N1=3
.include '../LIB/16nm_HP.pm'

.temp 70
.option post

*****
* Simulation netlist
*****
Vdd      vdd      gnd      'SUPPLY'
Vin       a       gnd      PULSE(0 'SUPPLY' 0ps 50ps 50ps 800ps 1950ps
M1        y       a       gnd      gnd      NMOS    W=4      L=2
M2        y       a       vdd      vdd      PMOS    W=N1*3    L=2

*C1 y a 1f

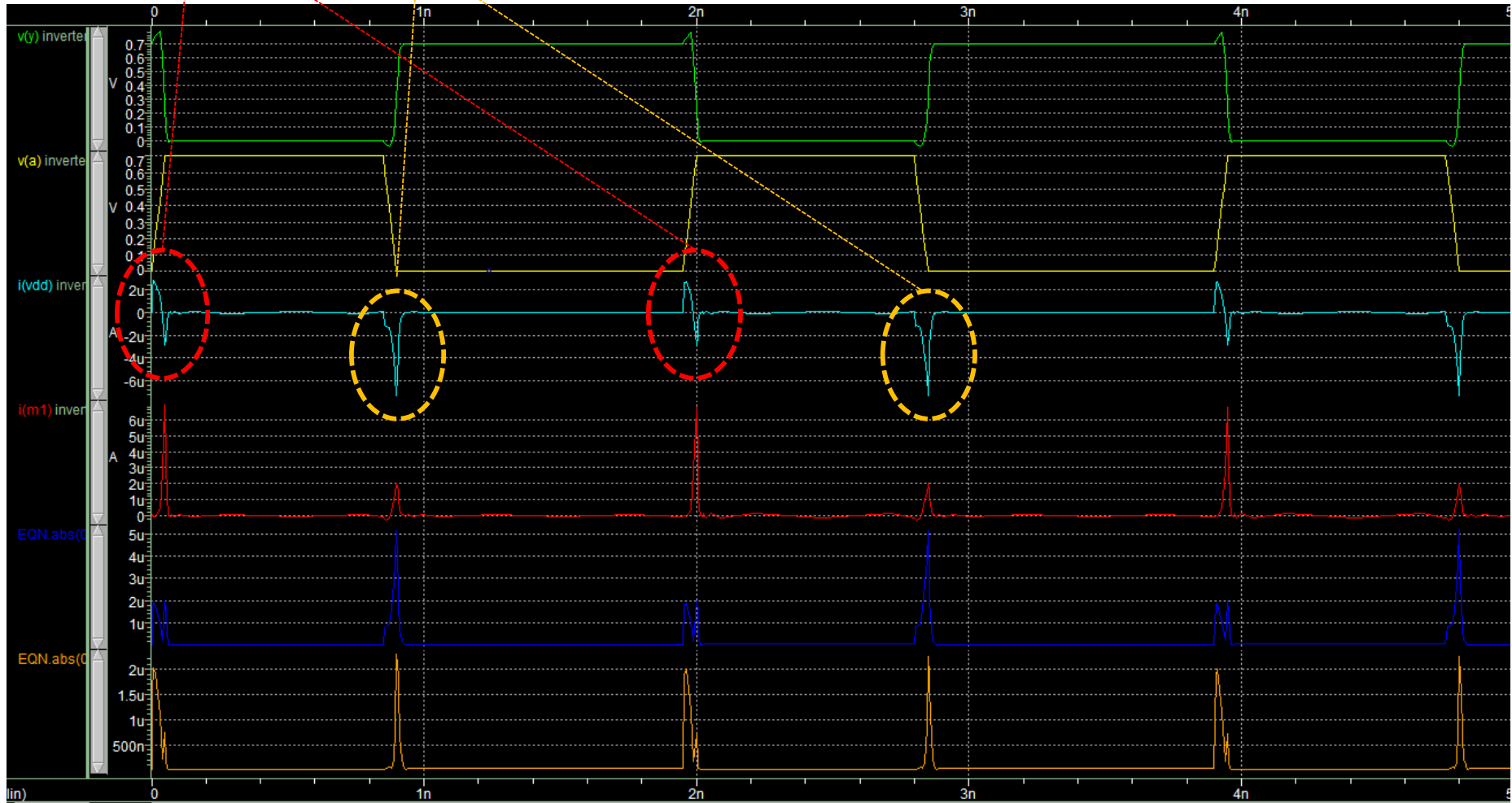
*****
* Stimulus
*****

*.tran 100p 10n
*.plot i(y)
* sweep N1 2 30 2
.tran 10p 5n
*sweep supply 0.2 1.5 0.1
.MEASURE TRAN tdlay TRIG V(a) VAL = 'SUPPLY/2' RISE = 1 TARG V(y) VAL = 'SUPPLY/2' FALL = 1
.end
```

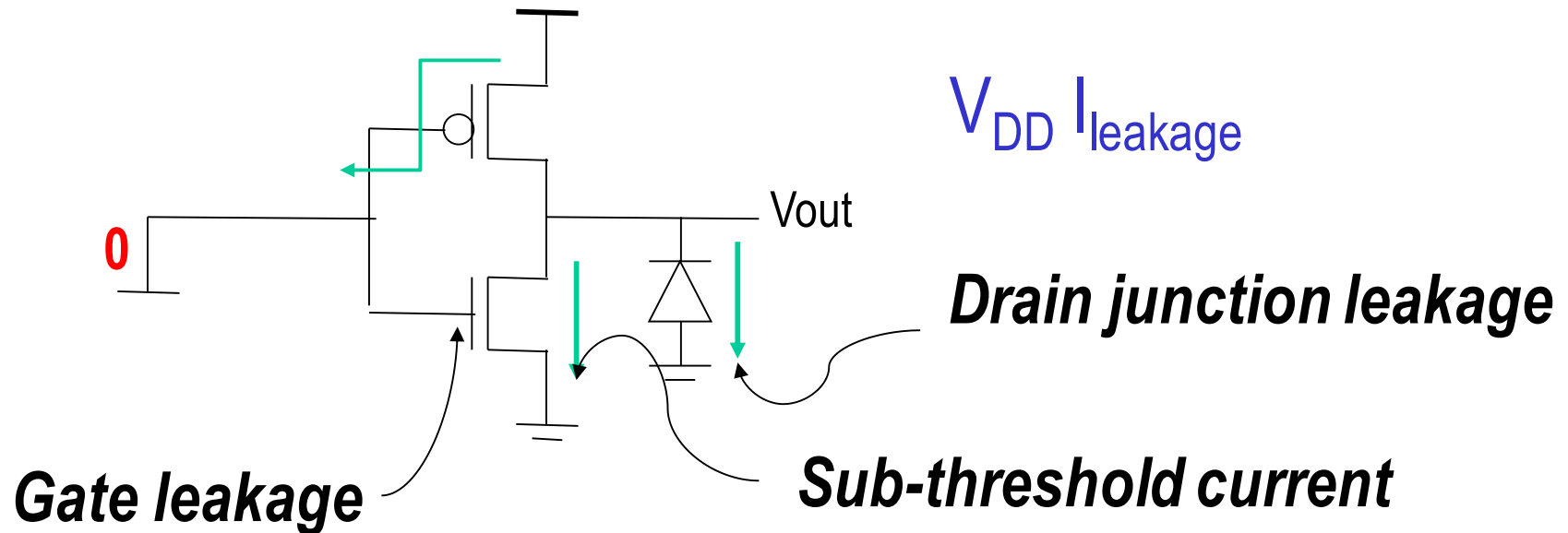
# Power Testing

Short current

charge current



# Leakage (Static) Power Consumption



***Sub-threshold current is the dominant factor.  
All increase exponentially with temperature!***

# TSMC Processes Leakage and $V_T$

	<b>CL018 G</b>	<b>CL018 LP</b>	<b>CL018 ULP</b>	<b>CL018 HS</b>	<b>CL015 HS</b>	<b>CL013 HS</b>
$V_{dd}$	1.8 V	1.8 V	1.8 V	2 V	1.5 V	1.2 V
$T_{ox}$ (effective)	42 Å	42 Å	42 Å	42 Å	29 Å	24 Å
$L_{gate}$	0.16 $\mu m$	0.16 $\mu m$	0.18 $\mu m$	0.13 $\mu m$	0.11 $\mu m$	0.08 $\mu m$
$I_{DSat}$ (n/p) ( $\mu A/\mu m$ )	600/260	500/180	320/130	780/360	860/370	920/400
$I_{off}$ (leakage) ( $\rho A/\mu m$ )	20	1.60	0.15	300	1,800	13,00
$V_{Tn}$	0.42 V	0.63 V	0.73 V	0.40 V	0.29 V	0.25 V
FET Perf. (GHz)	30	22	14	43	52	80

From MPR, 2000

# Principles for Power Reduction

- ***Prime choice: Reduce voltage!***
  - Recent years have seen an acceleration in supply voltage reduction
  - Design at very low voltages still open question
- ***Reduce switching activity***
- ***Reduce physical capacitance***
  - Device Sizing: for  $F=20$ 
    - $f_{opt}(\text{energy})=3.53$ ,  
 $f_{opt}(\text{performance})=4.47$

# Review: Energy & Power Equations

$$E = C_L V_{DD}^2 P_{0 \rightarrow 1} + t_{sc} V_{DD} I_{peak} + V_{DD} I_{leakage} 1/f_{clock}$$

$$f_{0 \rightarrow 1} = P_{0 \rightarrow 1} * f_{clock}$$

$$P = C_L V_{DD}^2 f_{0 \rightarrow 1} + t_{sc} V_{DD} I_{peak} f + V_{DD} I_{leakage}$$

Dynamic power  
(~90% today and  
decreasing relatively)

Short-circuit power  
(~8% today and  
decreasing absolutely)

Leakage power  
(~2% today and  
*increasing*)

# CMOS Inverter

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- Static behavior
- Dynamic behavior
- Power, Energy, and Energy Delay
- ***Perspective tech.***



# Technology Evolution

## *International Technology Roadmap for Semiconductors*

Year of Introduction	1999	2000	2001	2004	2008	2011	2014
Technology node [nm]	180		130	90	60	40	30
Supply [V]	1.5-1.8	1.5-1.8	1.2-1.5	0.9-1.2	0.6-0.9	0.5-0.6	0.3-0.6
Wiring levels	6-7	6-7	7	8	9	9-10	10
Max frequency [GHz], Local-Global	1.2	1.6-1.4	2.1-1.6	3.5-2	7.1-2.5	11-3	14.9-3.6
Max mP power [W]	90	106	130	160	171	177	186
Bat. power [W]	1.4	1.7	2.0	2.4	2.1	2.3	2.5

Node years: 2007/65nm, 2010/45nm, 2013/33nm, 2016/23nm

# Technology Scaling Models

- ***Full scaling(constant electrical)***
  - Ideal model –dimensions and voltage scale together by the same factor  $S$
- ***Fixed voltage scaling***
  - Most common model until recently- only dimensions scale, voltages remain constant
- ***General scaling***
  - Most realistic for today's situation-voltages and dimensions scale with different factors

# Scaling Relationships for Long Channel Devices

Parameter	Relation	Full Scaling	General Scaling	Fixed Voltage Scaling
$W, L, t_{ox}$		$1/S$	$1/S$	$1/S$
$V_{DD}, V_T$		$1/S$	$1/U$	$1$
$N_{SUB}$	$V/W_{depl}^2$	$S$	$S^2/U$	$S^2$
Area/Device	$WL$	$1/S^2$	$1/S^2$	$1/S^2$
$C_{ox}$	$1/t_{ox}$	$S$	$S$	$S$
$C_L$	$C_{ox}WL$	$1/S$	$1/S$	$1/S$
$k_n, k_p$	$C_{ox}W/L$	$S$	$S$	$S$
$I_{av}$	$k_{n,p} V^2$	$1/S$	$S/U^2$	$S$
$t_p$ (intrinsic)	$C_L V / I_{av}$	$1/S$	$U/S^2$	$1/S^2$
$P_{av}$	$C_L V^2 / t_p$	$1/S^2$	$S/U^3$	$S$
PDP	$C_L V^2$	$1/S^3$	$1/SU^2$	$1/S$

# Difference between long and short channels

$$I_{Dsat} = \frac{k'_n}{2} \frac{W}{L} (V_{GS} - V_T)^2$$

$$I_D \Big|_{V_{DS}=V_{DSAT}} = \mu_n C_{ox} \frac{W}{L} \left[ (V_{GS} - V_T) V_{DSAT} - \frac{V_{DSAT}^2}{2} \right]$$
$$= v_{sat} C_{ox} W (V_{GT} - \frac{V_{DSAT}}{2})$$

# Transistor Scaling (velocity-saturated devices)

Parameter	Relation	Full Scaling	General Scaling	Fixed-Voltage Scaling
$W, L, t_{ox}$		$1/S$	$1/S$	$1/S$
$V_{DD}, V_T$		$1/S$	$1/U$	1
$N_{SUB}$	$V/W_{depl}^2$	$S$	$S^2/U$	$S^2$
✓ $Area/Device$	$WL$	$1/S^2$	$1/S^2$	$1/S^2$
$C_{ox}$	$1/t_{ox}$	$S$	$S$	$S$
$C_{gate}$	$C_{ox}WL$	$1/S$	$1/S$	$1/S$
$k_n, k_p$	$C_{ox}W/L$	$S$	$S$	$S$
$I_{sat}$	$C_{ox}WV$	$1/S$	$1/U$	1
$Current\ Density$	$I_{sat}/Area$	$S$	$S^2/U$	$S^2$
$R_{on}$	$V/I_{sat}$	1	1	1
✓ $Intrinsic\ Delay$	$R_{on}C_{gate}$	$1/S$	$1/S$	$1/S$
✓ $P$	$I_{sat}V$	$1/S^2$	$1/U^2$	1
$Power\ Density$	$P/Area$	1	$S^2/U^2$	$S^2$

# summary

- CMOS at a glance
- CMOS static behavior
  - VTC, noise margin, threshold voltage
- CMOS dynamic behavior
  - Capacitance mosaic, delay
  - Ratio pMOS/nMOS:3.5,2.4,1.6
  - Optimizing inverter sizing
- Power
  - Power mosaic
  - Optimizing dynamic power
  - Short power consideration