

# Digital Integrated Circuits

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Office location : 417 room

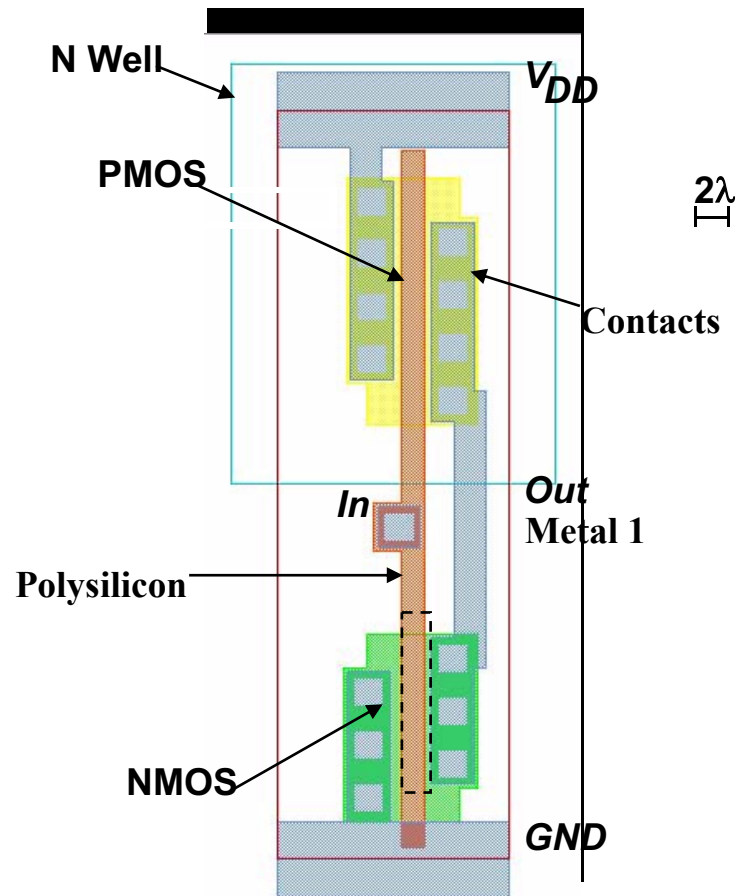
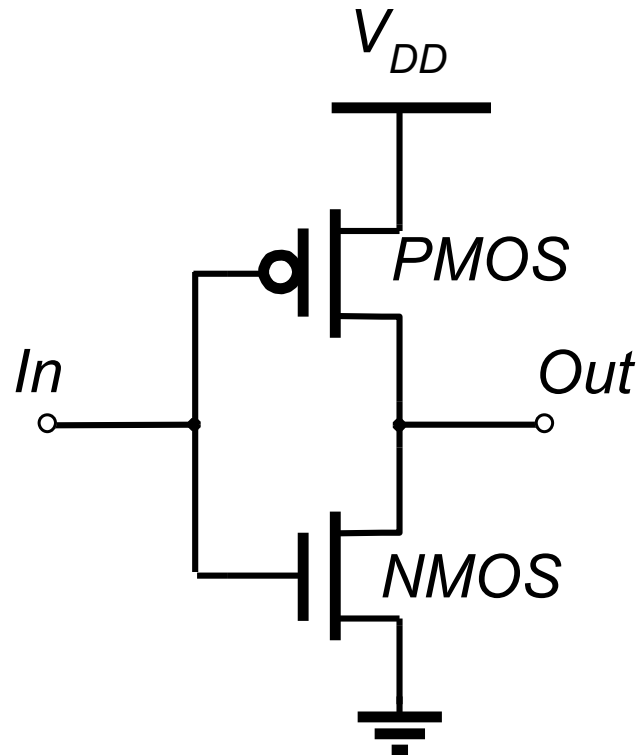
WeiDianZi building, No 800 DongChuan  
road, MinHang Campus

# Chapter 3 Inverter

# CMOS Inverter

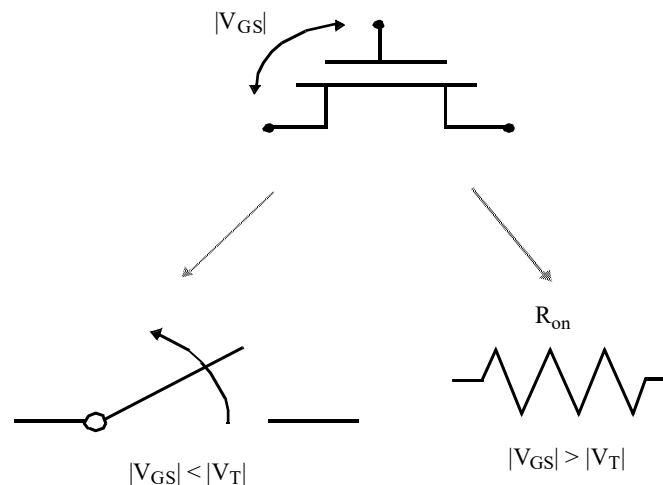
- *At a glance*
- Static behavior
- Dynamic behavior
- Power, Energy, and Energy Delay
- Perspective tech.

# CMOS Inverter

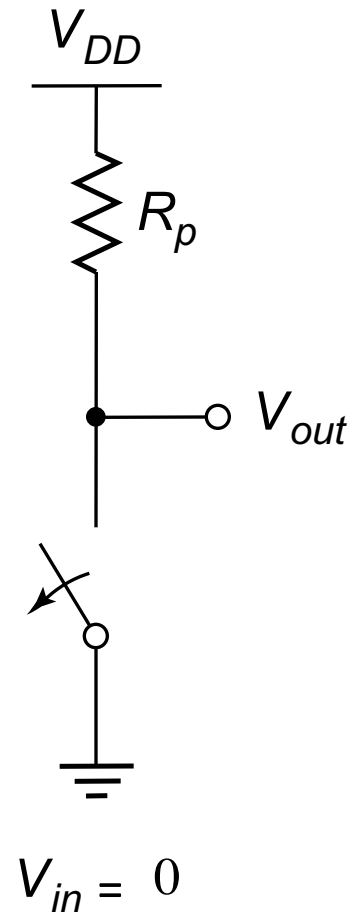
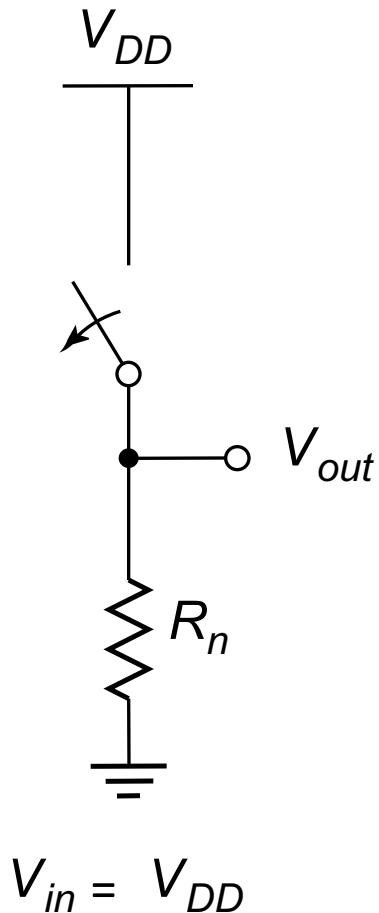


# A First Glance of CMOS

- A switch with infinite off-resistance and a finite on-resistance
  - When  $V_{gs} < V_t$  , MOS is off
  - when  $V_{gs} > V_t$  , MOS is on just like a resistance  $R_{on}$
- when  $V_{in} = V_{dd}$  , nMOS on , pMOS off
- When  $V_{in} = 0$  , pMOS on , nMOS off

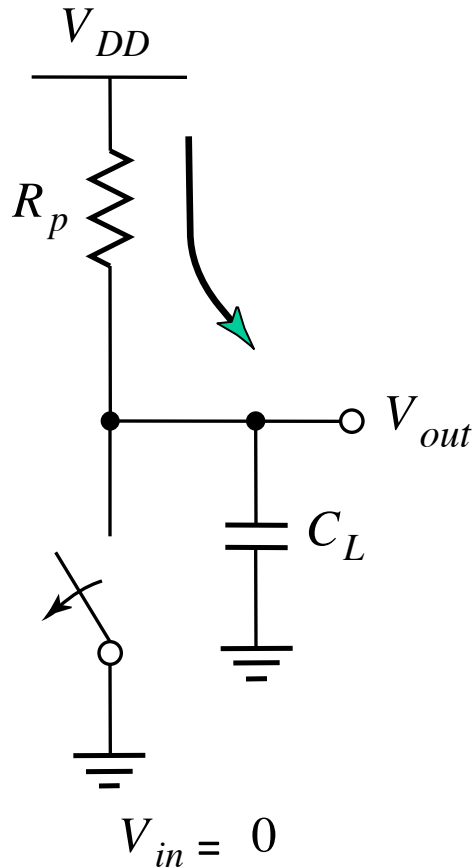


# CMOS Inverter First-Order DC Analysis

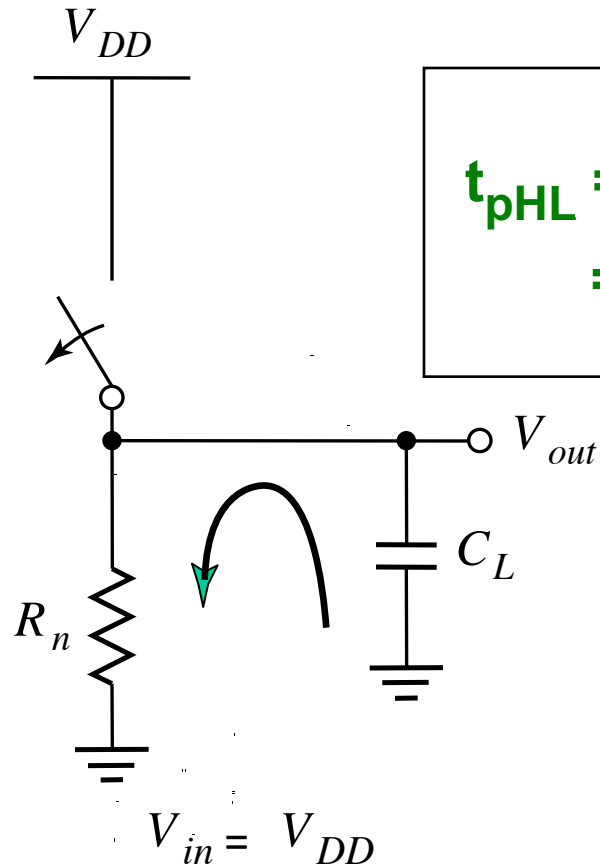


$$\begin{aligned} V_{OL} &= 0 \\ V_{OH} &= V_{DD} \\ V_M &= f(R_n, R_p) \end{aligned}$$

# CMOS Inverter: Transient Response



(a) Low-to-high

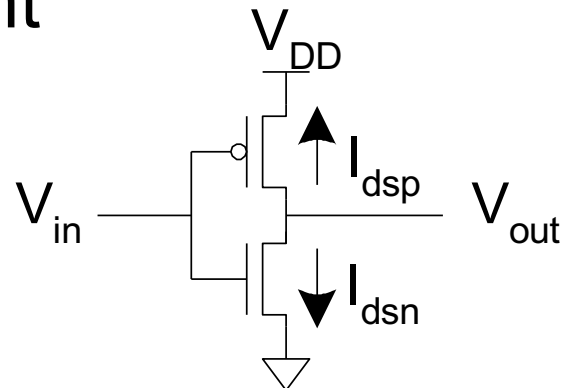


$$t_{pHL} = f(R_{on} \cdot C_L) \\ = 0.69 R_{on} C_L$$

(b) High-to-low

# DC Response

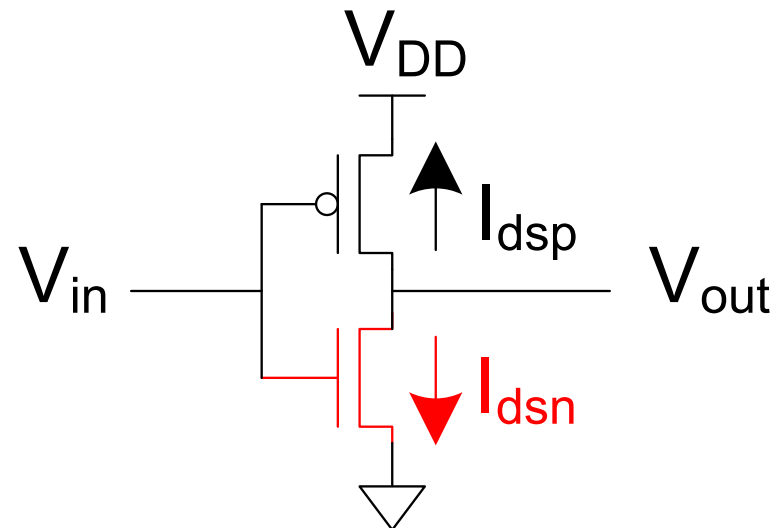
- When  $V_{in} = 0 \Rightarrow V_{out} = V_{DD}$
- When  $V_{in} = V_{DD} \Rightarrow V_{out} = 0$
- In between
  - $V_{out}$  depends on transistor size and current
  - ***By KCL, must settle such that  $I_{dsn} = |I_{dsp}|$***
  - We could solve equations , But graphical solution gives more insight





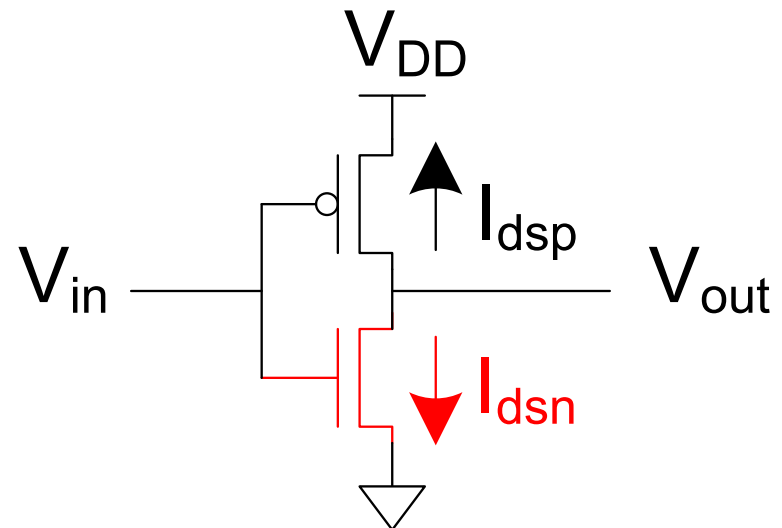
# nMOS Operation

Cutoff	Linear	Saturated/ $V_S$
$V_{gsn} < ?$	$V_{gsn} > ?$  $V_{dsn} < ?$	$V_{gsn} > ?$  $V_{dsn} > ?$



# nMOS Operation

Cutoff	Linear	Saturated/ $V_S$
$V_{gsn} < V_{tn}$	$V_{gsn} > V_{tn}$	$V_{gsn} > V_{tn}$
	$V_{dsn} < V_{gsn} - V_{tn} / V_{dsatn}$	$V_{dsn} > V_{gsn} - V_{tn} / V_{dsatn}$

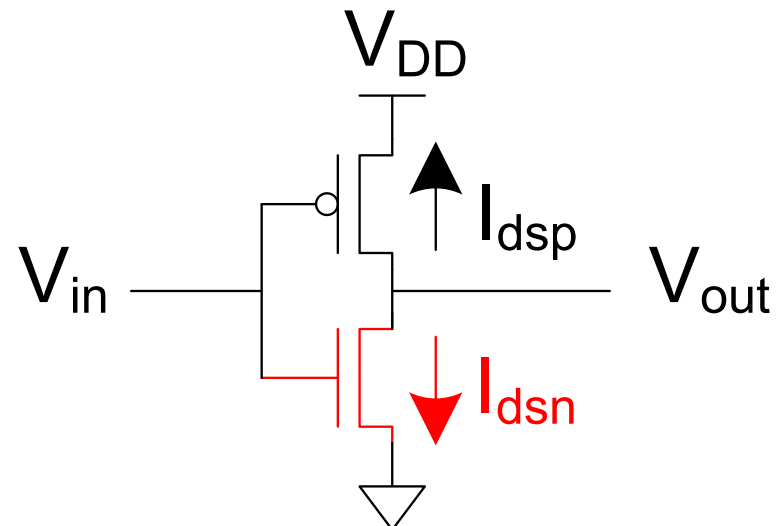


# nMOS Operation

Cutoff	Linear	Saturated/ $V_S$
$V_{gsn} < V_{tn}$	$V_{gsn} > V_{tn}$	$V_{gsn} > V_{tn}$
	$V_{dsn} < V_{gsn} - V_{tn} / V_{dsatn}$	$V_{dsn} > V_{gsn} - V_{tn} / V_{dsatn}$

$$V_{gsn} = V_{in}$$

$$V_{dsn} = V_{out}$$

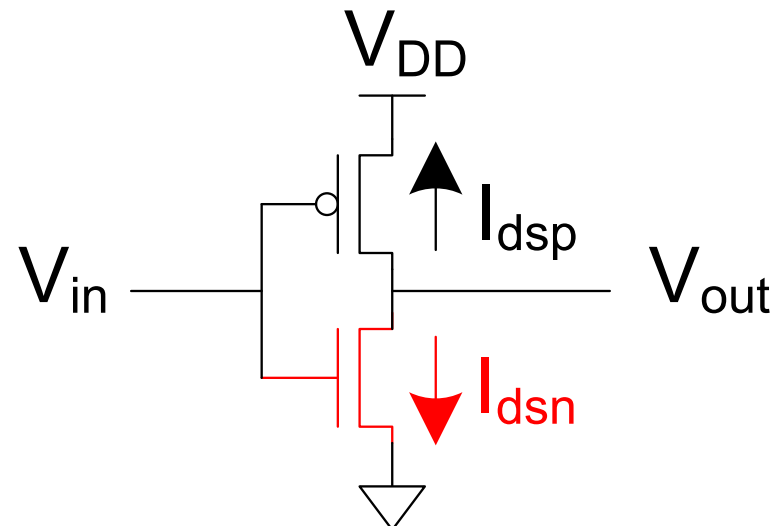


# nMOS Operation

Cutoff	Linear	Saturated/ $V_S$
$V_{gsn} < V_{tn}$ $V_{in} < V_{tn}$	$V_{gsn} > V_{tn}$ $V_{in} > V_{tn}$ $V_{dsn} < V_{gsn} - V_{tn} / V_{dsatn}$ $V_{out} < V_{gsn} - V_{tn} / V_{dsatn}$	$V_{gsn} > V_{tn}$ $V_{in} > V_{tn}$ $V_{dsn} > V_{gsn} - V_{tn} / V_{dsatn}$ $V_{out} > V_{gsn} - V_{tn} / V_{dsatn}$

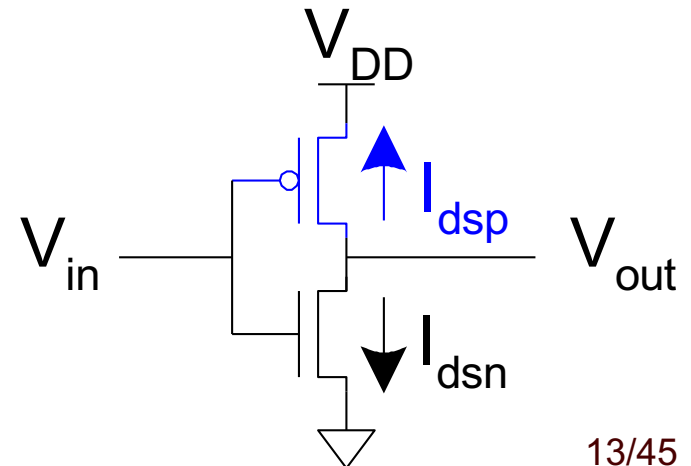
$$V_{gsn} = V_{in}$$

$$V_{dsn} = V_{out}$$



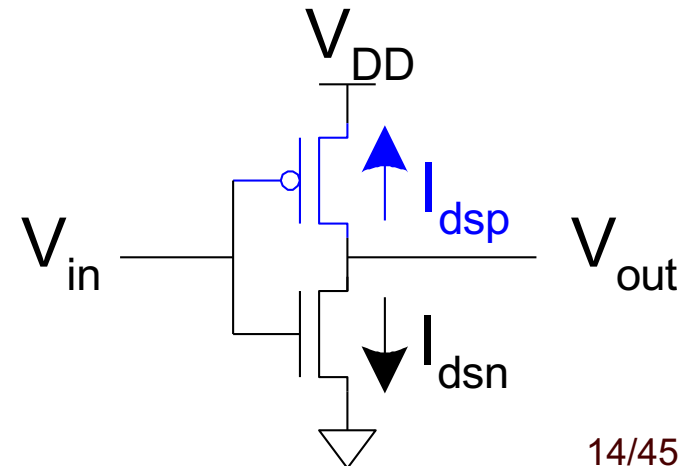
# pMOS Operation

Cutoff	Linear	Saturated/VS
$V_{gsp} > ?$	$V_{gsp} < ?$  $V_{dsp} > ?$	$V_{gsp} < ?$  $V_{dsp} < ?$



# pMOS Operation

Cutoff	Linear	Saturated/VS
$V_{gsp} > V_{tp}$	$V_{gsp} < V_{tp}$	$V_{gsp} < V_{tp}$
	$V_{dsp} > V_{gsp} - V_{tp} / V_{DSATp}$	$V_{dsp} < V_{gsp} - V_{tp} / V_{DSATp}$

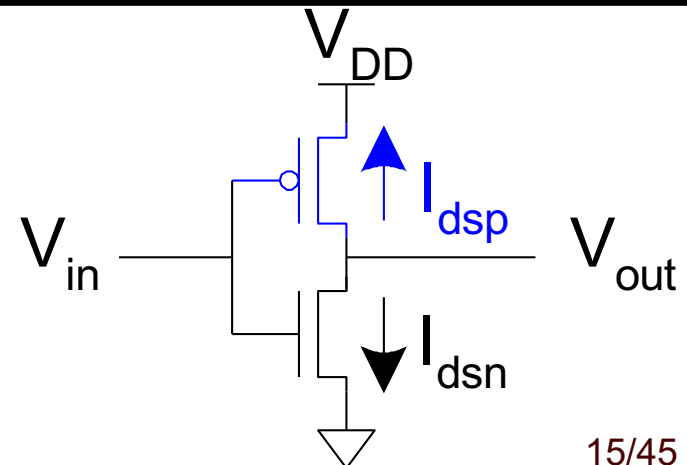


# pMOS Operation

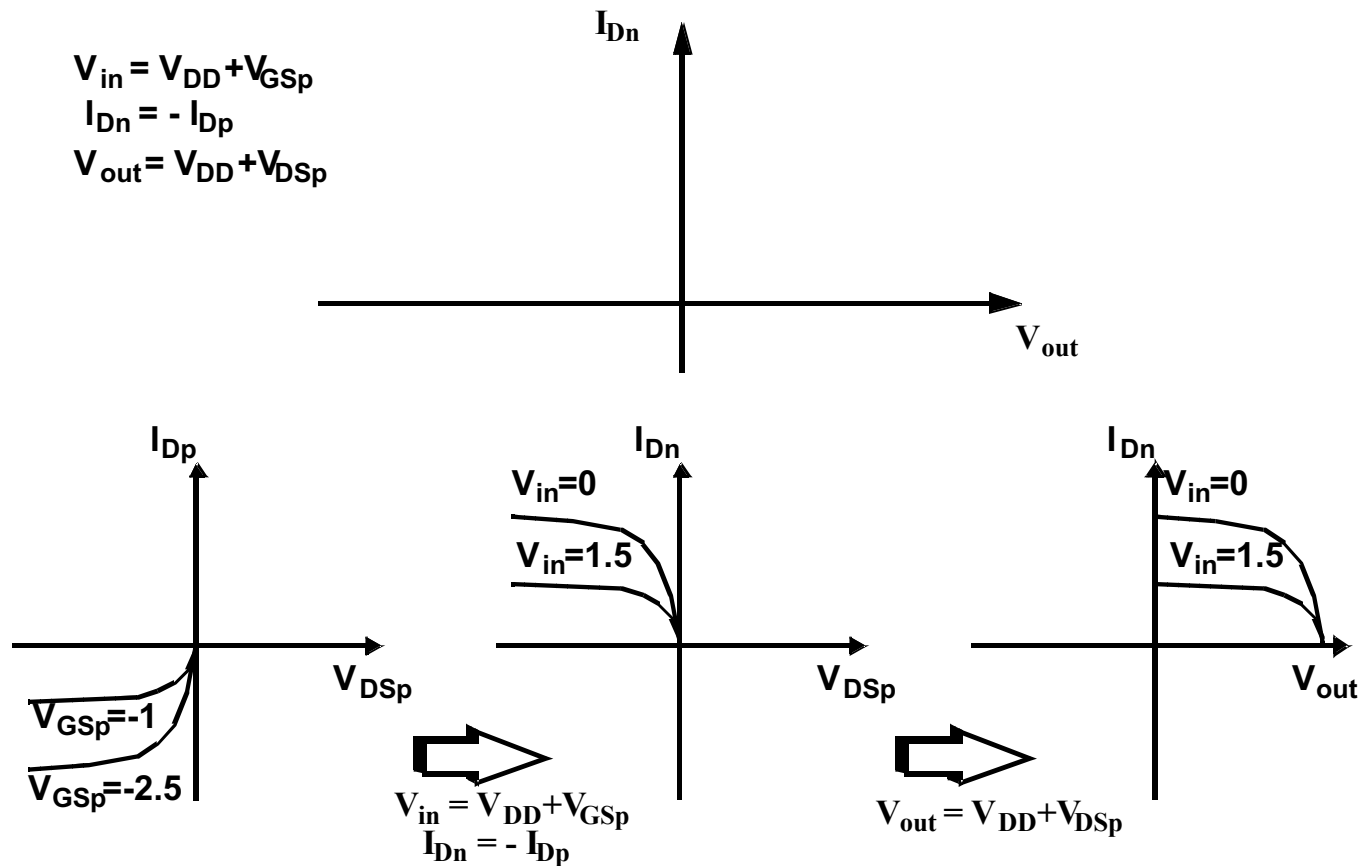
Cutoff	Linear	Saturated/VS
$V_{gsp} > V_{tp}$ $V_{in} > V_{DD} + V_{tp}$	$V_{gsp} < V_{tp}$ $V_{in} < V_{DD} + V_{tp}$ $V_{dsp} > V_{gsp} - V_{tp}/V_{DSATp}$ $V_{out} > V_{in} - V_{tp}/V_{DSATp}$	$V_{gsp} < V_{tp}$ $V_{in} < V_{DD} + V_{tp}$ $V_{dsp} < V_{gsp} - V_{tp}/V_{DSATp}$ $V_{out} < V_{in} - V_{tp}/V_{DSATp}$

$$V_{gsp} = V_{in} - V_{DD}$$

$$V_{dsp} = V_{out} - V_{DD}$$

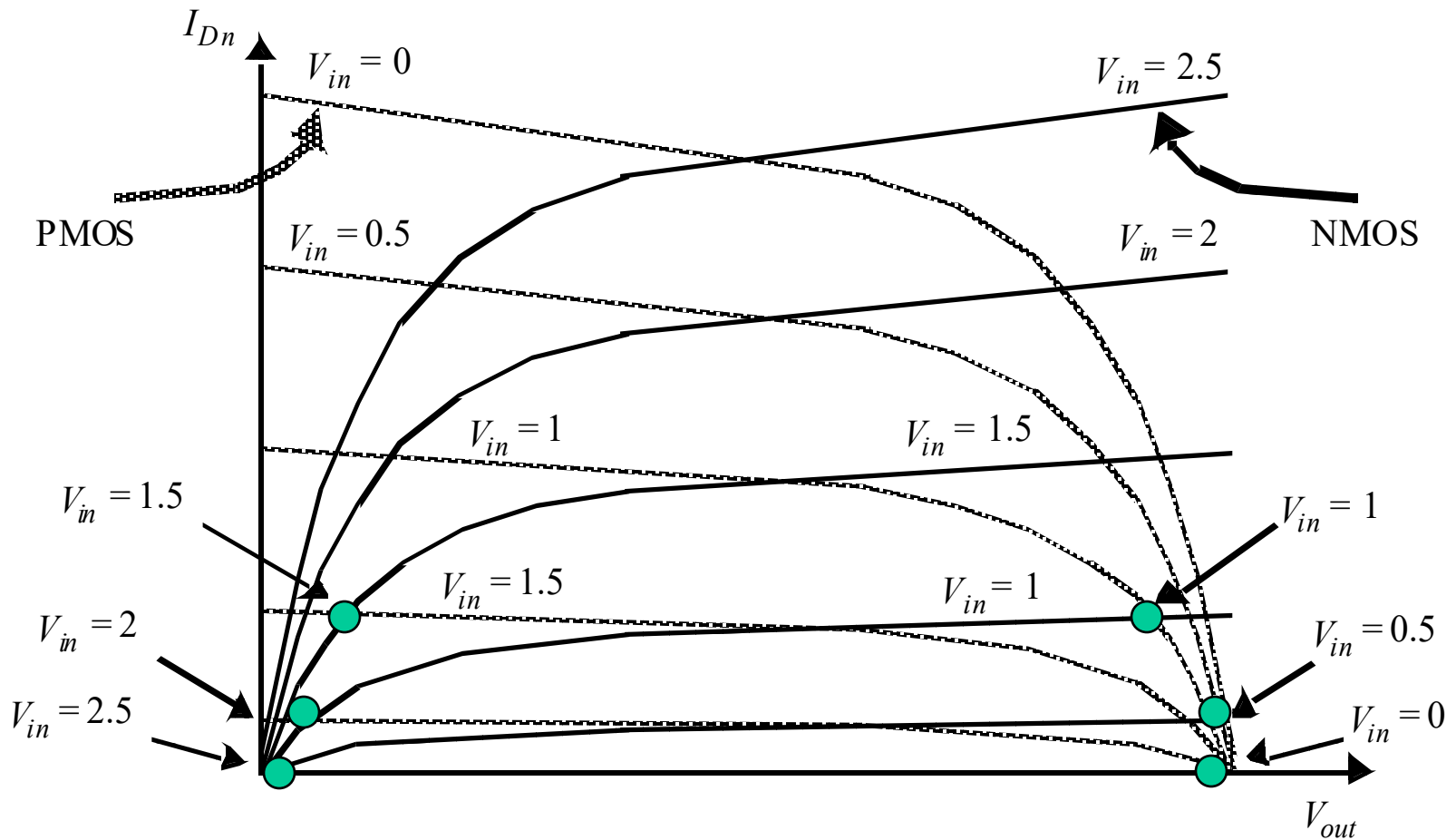


# PMOS Load Lines

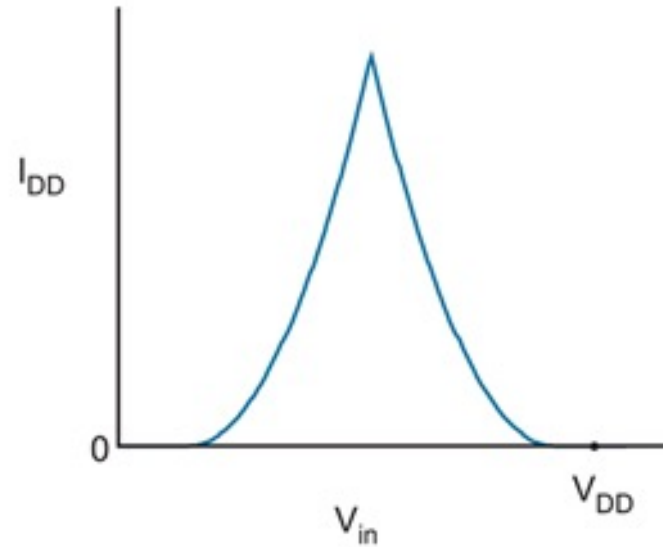
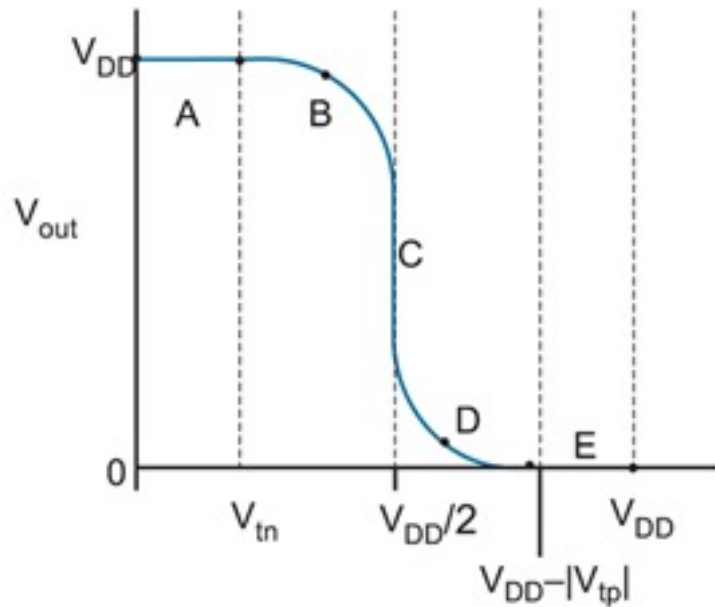




# CMOS Inverter Load Characteristics



# CMOS Inverter VTC



Summary of CMOS inverter operation

Region	Condition	P-device	N-device	output
A	$[0, V_{tn}]$	<b>linear</b>	cutoff	$V_{DD}$
B	$[V_{tn}, V_{DD}/2]$	linear	saturated	$V_{DD}/2$
C	$=V_{DD}/2$	<b>saturated</b>	<b>saturated</b>	X drop
D	$[V_{DD}/2, V_{DD} -  V_{TP} ]$	saturated	linear	$<V_{DD}/2$
E	$[V_{DD} -  V_{TP} , V_{DD}]$	cutoff	<b>linear</b>	0

```
* cmos.sp

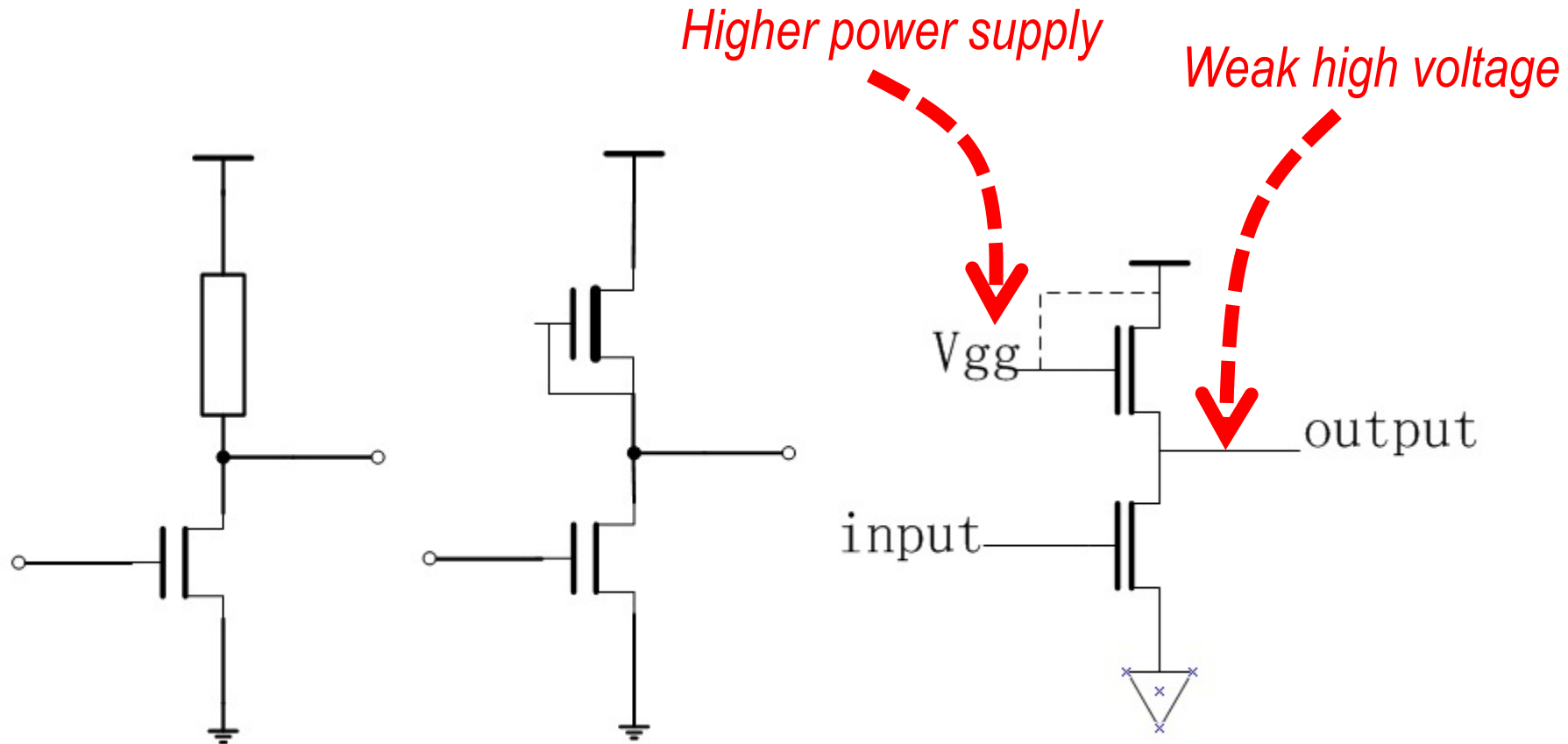
*****
* Parameters and models
*****
.param SUPPLY=1.8
.option scale=90n
.param N1=4
.include '../models/mosistsmc180/mosistsmc180.sp'
.temp 70
.option post

*****
* Simulation netlist
*****
Vdd      vdd      gnd      'SUPPLY'
Vin       a        gnd
M1        y        a        gnd      gnd      NMOS      W=4      L=2

M2        y        a        vdd      vdd      PMOS      W='N1*4' L=2

*****
* Stimulus
*****
.DC Vin 0 SUPPLY 0.01 sweep N1 2 20 2
.end
```

# Other inverters



**Source: Carriers from**

# CMOS properties

- High noise margins, the voltage swing is equal to the supply voltage
- Ratioless circuit structure
- Low output impedance
- High input resistance
- Low power Static power is low

# CMOS Inverter

- At a glance
- *Static behavior*
- Dynamic behavior
- Power, Energy, and Energy Delay
- Perspective tech.

# CMOS inverter static behavior

- *Threshold Voltage*
- Noise Margin
- Gain
- DC robust

$$I_{ds} = \mu_n C_{ox} \frac{W}{L} \left( (V_{gs} - V_T) V_{dsat} - \frac{V_{dsat}^2}{2} \right) \quad k_n = \beta_n = \mu_n C_{ox} \frac{W}{L}$$

$$k_n \left( (V_{gsn} - V_{Tn}) V_{dsatn} - \frac{V_{dsatn}^2}{2} \right) + k_p \left( (V_{gsp} - V_{Tp}) V_{dsatp} - \frac{V_{dsatp}^2}{2} \right) = 0$$

$\xrightarrow{\text{Vin}}$ 
 $\xrightarrow{\text{Vin} - V_{DD}}$

$$k_n \left( (V_M - V_{Tn}) V_{dsatn} - \frac{V_{dsatn}^2}{2} \right) + k_p \left( (V_M - V_D - V_{Tp}) V_{dsatp} - \frac{V_{dsatp}^2}{2} \right) = 0$$

$$V_M = \frac{\left( V_{Tn} + \frac{V_{dsatn}}{2} \right) + \gamma \left( V_{DD} + V_{Tp} + \frac{V_{dsatp}}{2} \right)}{1 + \gamma} \approx \frac{\gamma V_{DD}}{1 + \gamma}$$

$$\gamma = \frac{k_p V_{dsatp}}{k_n V_{dsatn}} = \frac{W_p v_{dsatp}}{W_n v_{dsatn}}$$

$\frac{P}{n}$   $\nearrow$  ,  $V_M$   $\nearrow$

$$V_{sat} \approx V_c = L E_c = L \frac{2 v_{sat}}{\mu_{eff}}$$



$$I_{ds} = \mu_n C_{ox} \frac{W}{L} \left( (V_{gs} - V_T) V_{dsat} - \frac{V_{dsat}^2}{2} \right) \quad k_n = \beta_n = \mu_n C_{ox} \frac{W}{L}$$

$$k_n \left( (V_{gsn} - V_{Tn}) V_{dsat_n} - \frac{V_{dsat_n}^2}{2} \right) + k_p \left( (V_{gsp} - V_{Tp}) V_{dsat_p} - \frac{V_{dsat_p}^2}{2} \right) = 0$$

$$k_n \left( (V_M - V_{Tn}) V_{dsat_n} - \frac{V_{dsat_n}^2}{2} \right) + k_p \left( (V_M - V_D - V_{Tp}) V_{dsat_p} - \frac{V_{dsat_p}^2}{2} \right) = 0$$

$$\frac{W_p/L_p}{W_n/L_n} \approx \frac{k'_n V_{dsat_n} \left( V_M - V_{Tn} - \frac{V_{dsat_n}}{2} \right)}{k'_p V_{dsat_p} \left( V_{DD} - V_M + V_{Tp} + \frac{V_{dsat_p}}{2} \right)}$$

$V_M$   
 $= \frac{V_{DD}}{2}$

# Switching threshold of CMOS inverter

Parameters for manual model of generic 0.25um CMOS process(minimum length device)

	$V_{T0}(V)$	$\gamma(V^{0.5})$	$V_{DSAT}(V)$	$k'(A/V^2)$	$\lambda(V^{-1})$
<i>NMOS</i>	0.43	0.4	0.63	$115 \times 10^{-6}$	0.06
<i>PMOS</i>	-0.4	-0.4	-1	$-30 \times 10^{-6}$	-0.1

Assuming  $W_p/W_n=8$ , calculating  $V_M=?$

$$r = \frac{k'_p \frac{W_p}{L_p} V_{DSAT_p}}{k'_n \frac{W_n}{L_n} V_{DSAT_n}} = \frac{-30 * (-1)}{115 * 0.63} * 8 = 3.3$$

$$L_p = L_n =$$

$$V_M = \frac{(V_{Tn} + \frac{V_{DSAT_n}}{2}) + r(2.5 + V_{Tp} + \frac{V_{DSAT_p}}{2})}{1 + r}$$

$$= \frac{(0.43 + \frac{0.63}{2}) + 3.3(2.5 - 0.4 - \frac{0.4}{2})}{1 + 3.3} = \frac{0.75 + 3.3 * 1.9}{1 + 3.3} = 1.63V$$

# Switching threshold of CMOS inverter

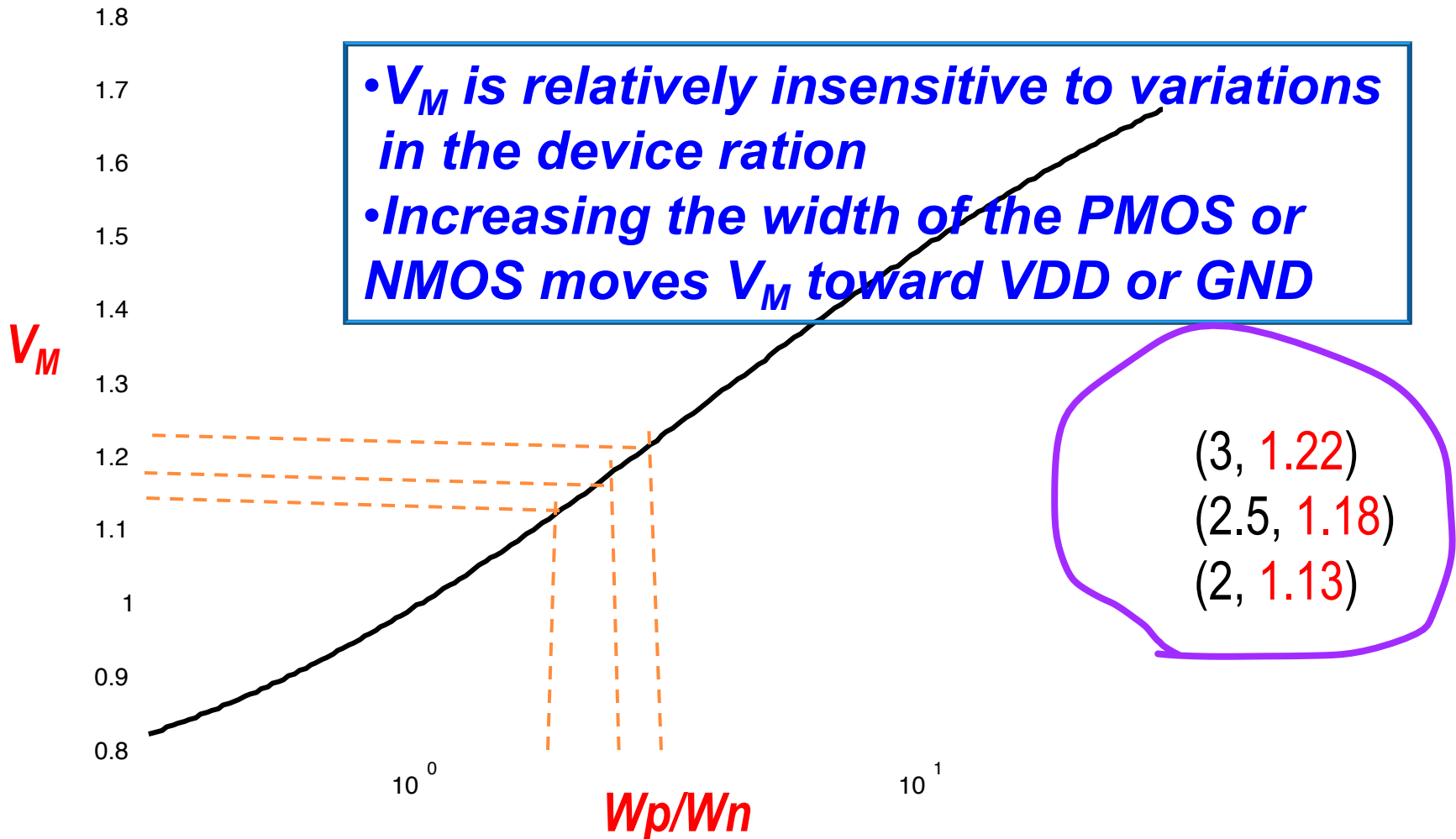
Parameters for manual model of generic 0.25um CMOS process(minimum length device)

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<i>PMOS</i>	-0.4	-0.4	-1	$-30 \times 10^{-6}$	-0.1

$$\frac{(W/L)_p}{(W/L)_n} = \frac{k'_n V_{DSAT_n} (V_M - V_{Tn} - \frac{V_{DSAT_n}}{2})}{-k'_p V_{DSAT_p} (V_{DD} - V_M + V_{Tp} + \frac{V_{DSAT_p}}{2})} = \frac{115 \times 10^{-6} \times 0.63 \times (1.25 - 0.43 - \frac{0.63}{2})}{30 \times 10^{-6} \times 1 \times (1.25 - 0.4 - \frac{1}{2})} = 3.5$$

***This rate let  $V_M = V_{dd}/2!$***

# Switching Threshold as a function of Transistor Ratio



# CMOS inverter static behavior

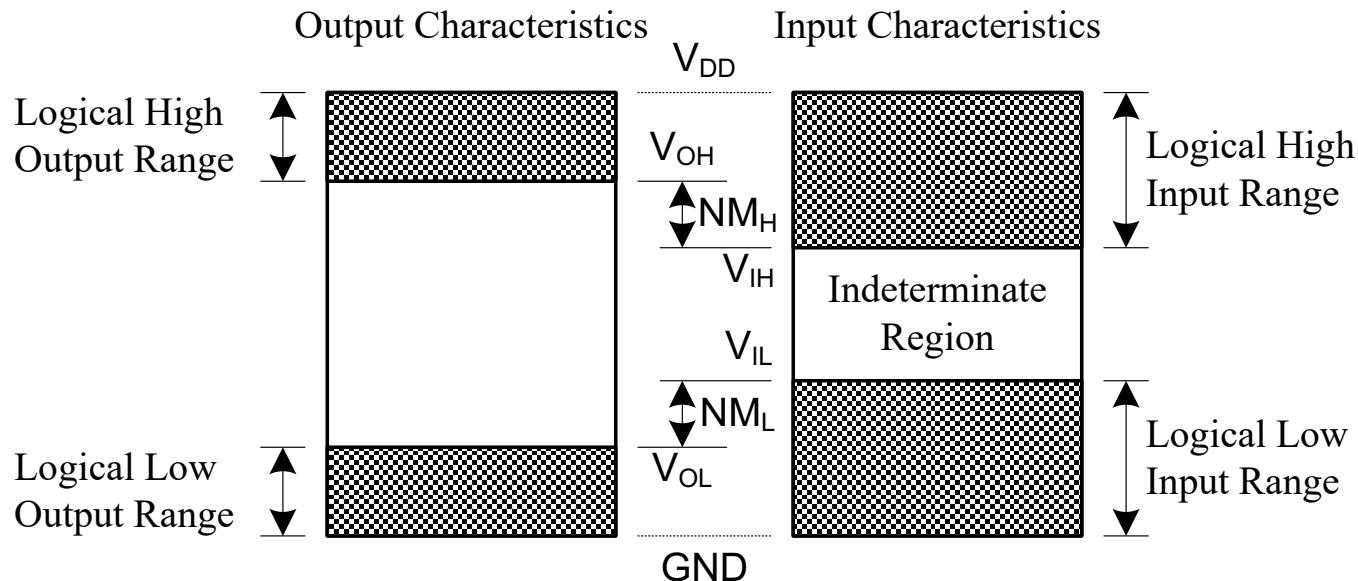
- Threshold Voltage
- ***Noise Margin***
- Gain
- DC robust

# Noise Margin

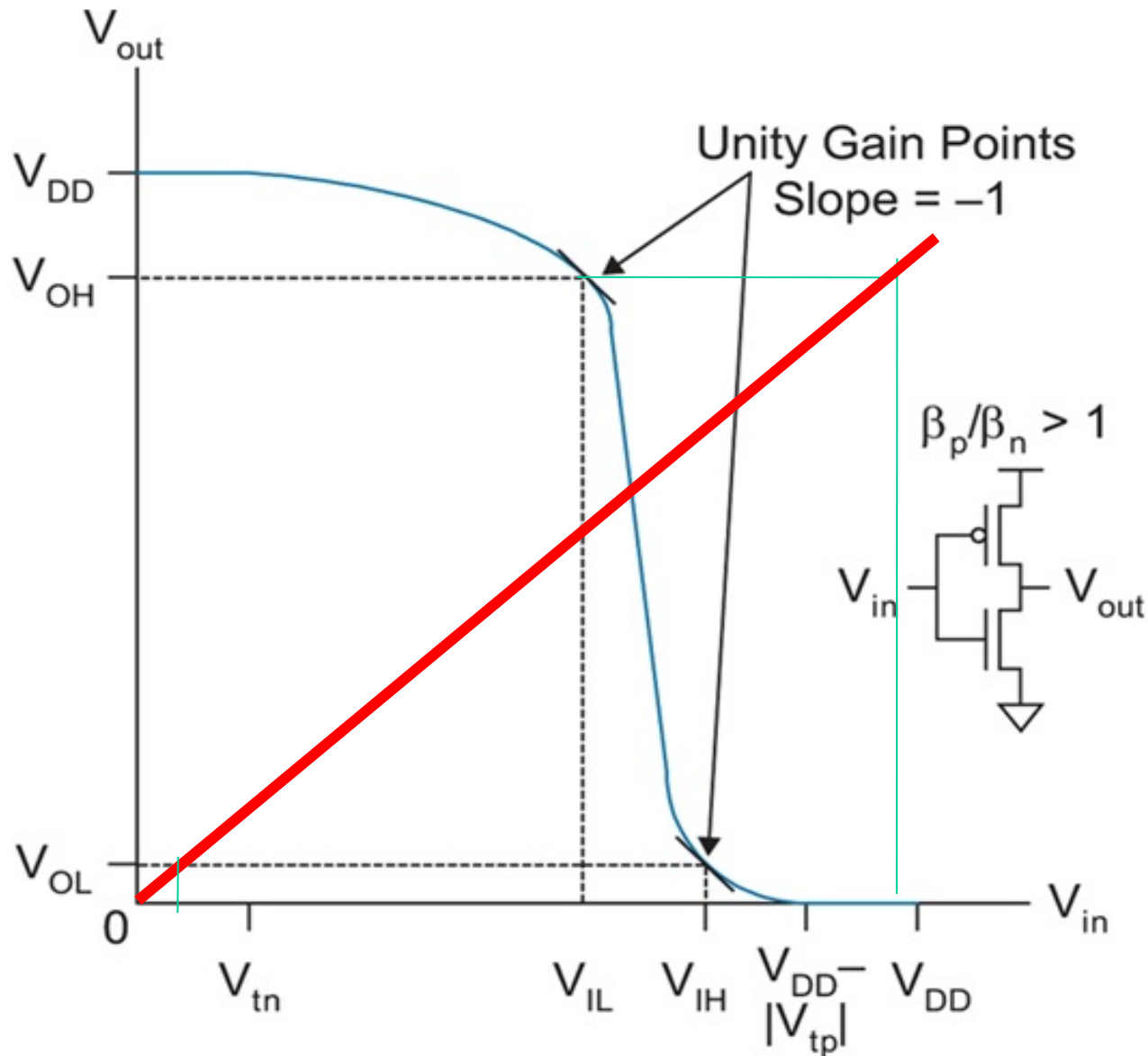
- $NM_L$ 
  - The difference in **maximum LOW** input voltage recognized by the receiving gate and the **maximum LOW** output voltage produced by the driving gate
  - $NM_L = V_{IL} - V_{OL}$
- $NM_H$ 
  - The difference in **minimum HIGH** input voltage recognized by the receiving gate and the **minimum HIGH** output voltage produced by the driving gate
  - $NM_H = V_{OH} - V_{IH}$

# Noise Margin

- $V_{IH}$  = minimum HIGH input voltage
- $V_{IL}$  = maximum LOW input voltage
- $V_{OH}$  = minimum HIGH output voltage
- $V_{OL}$  = maximum LOW output voltage



# Determining $V_{IH}$ and $V_{IL}$

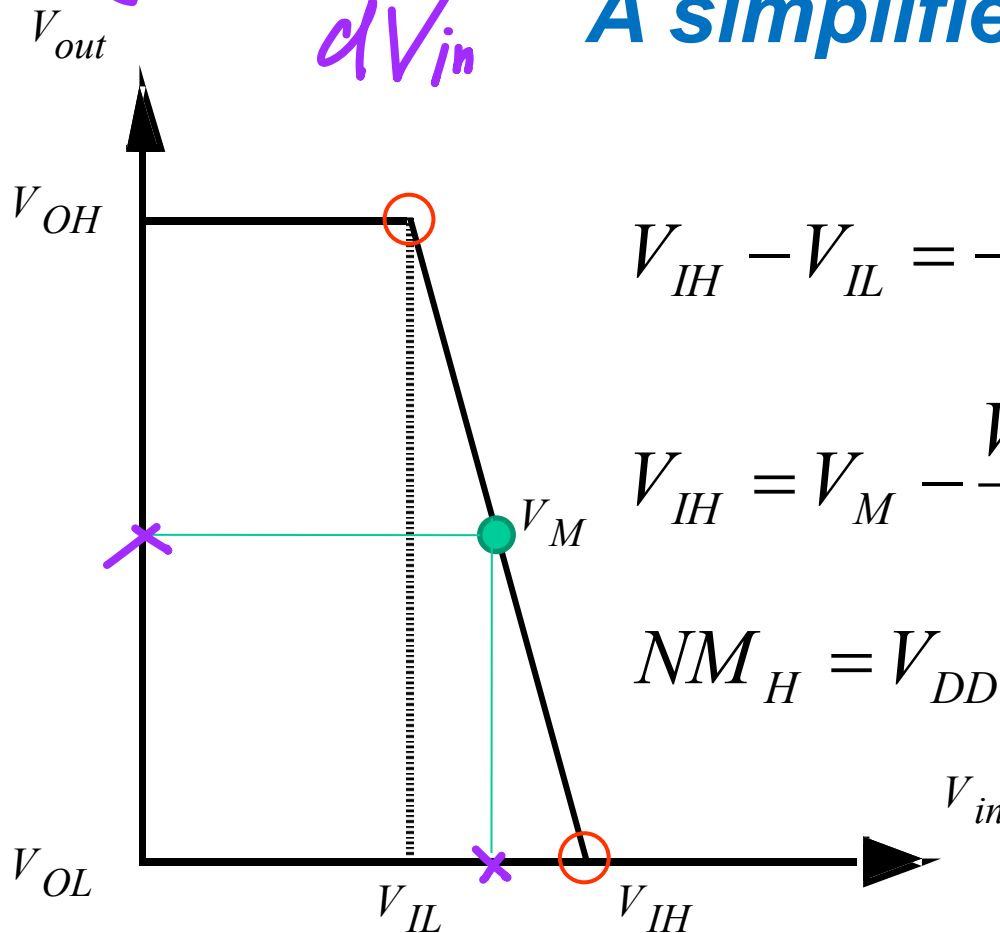




# Determining $V_{IH}$ and $V_{IL}$

$$g = \frac{dV_{out}}{dV_{in}}$$

***A simplified approach!***



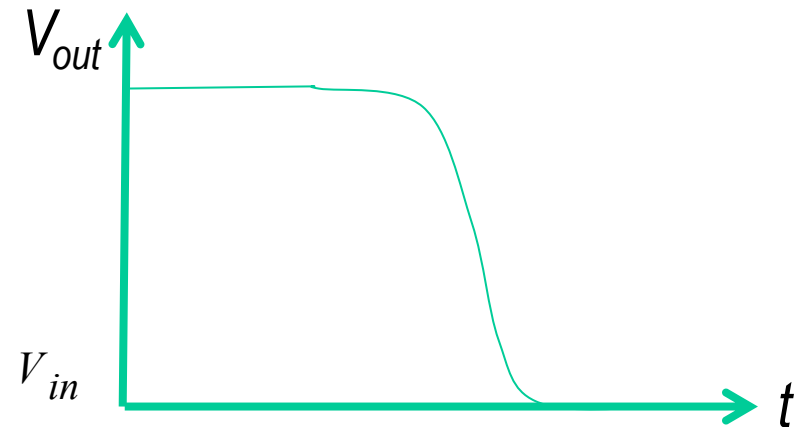
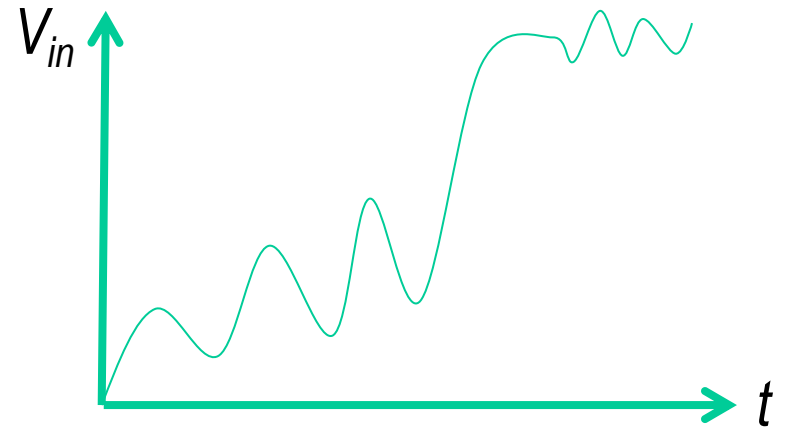
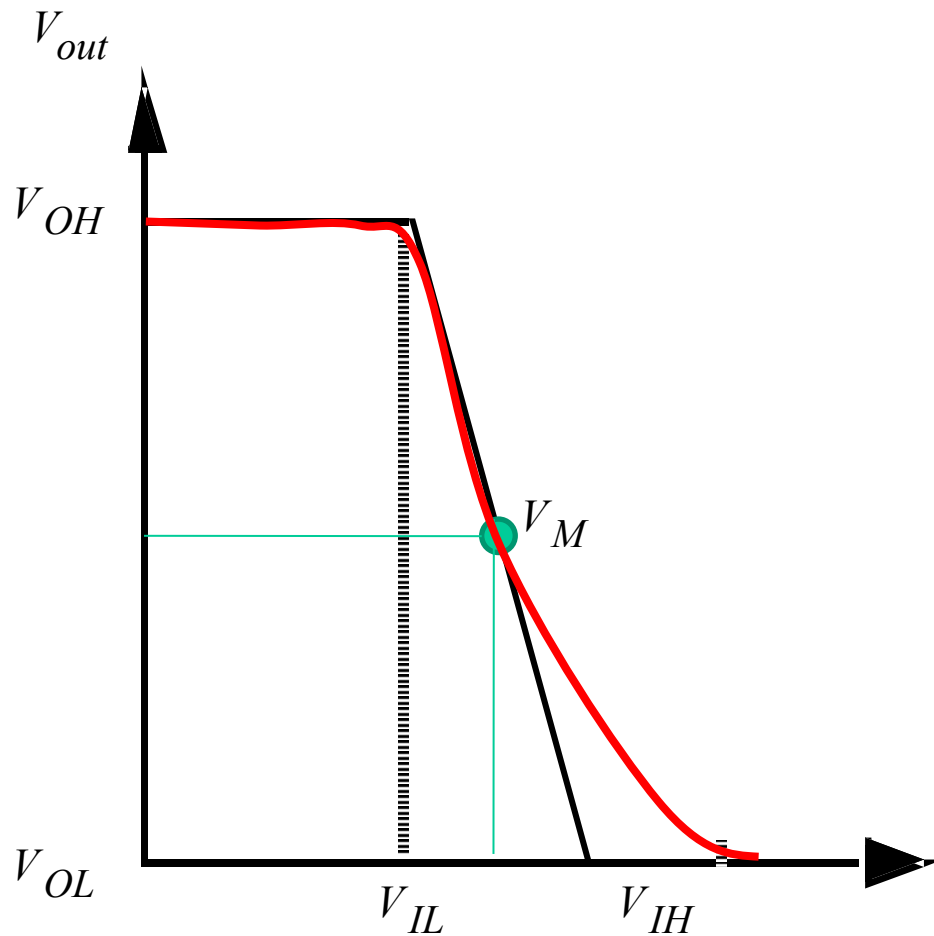
$$V_{IH} - V_{IL} = -\frac{(V_{OH} - V_{OL})}{g} = \frac{-V_{DD}}{g}$$

$$V_{IH} = V_M - \frac{V_M}{g}, V_{IL} = V_M + \frac{V_{DD} - V_M}{g}$$

$$NM_H = V_{DD} - V_{IH}, NM_L = V_{IL}$$

$V_M$  increase,  $NM_H$  decrease,  $NM_L$  increase

# Determining $V_{IH}$ and $V_{IL}$



# Example

**$g=-30$ ,  $V_{dd}=2.5V$ ,  $V_M=1.0V$  Please estimate  $NM_H$  and  $NM_L$**

$$V_{IH} = V_M - V_M/g = 1.0 * (1 + 1/30) = 1.03V$$

$$V_{IL} = (V_{DD} - V_M)/g + V_M = -1.5/30 + 1.0 = 0.95V$$

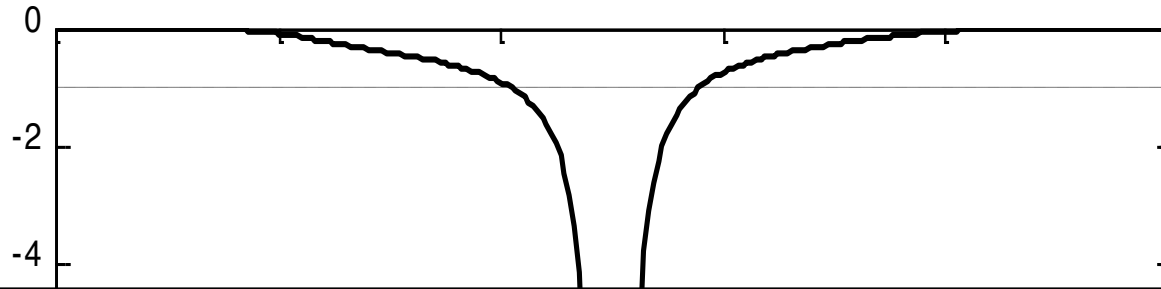
$$NM_H = V_{OH} - V_{IH} = 2.5 - 1.03 = 1.47V$$

$$NM_L = V_{IL} - V_{OL} = 0.95V$$

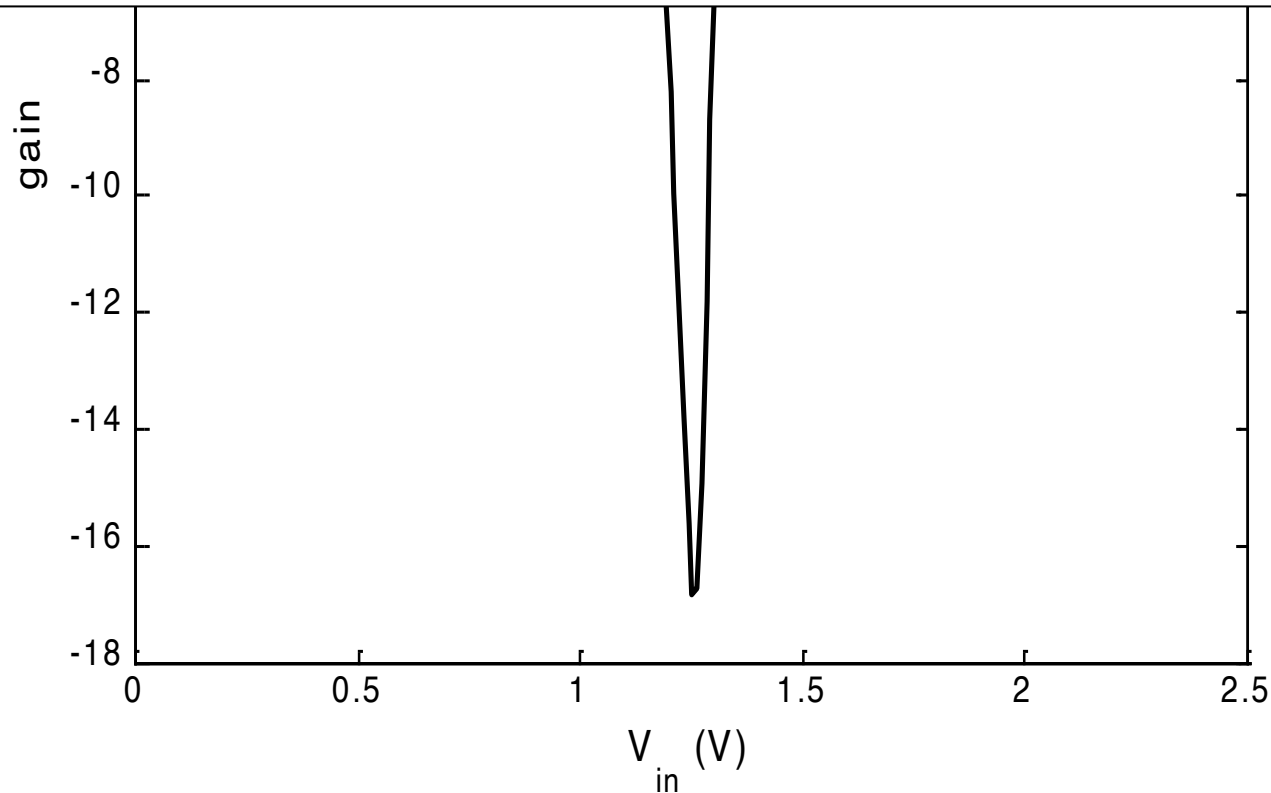
# CMOS inverter static behavior

- Threshold Voltage
- Noise Margin
- *Gain*
- DC robust

# Inverter Gain



$$k_n V_{DSAT_n} (V_{in} - V_{Tn} - \frac{V_{DSAT_n}}{2})(1 + \lambda V_{out}) + k_p V_{DSAT_p} (V_{in} - V_{DD} - V_{Tp} - \frac{V_{DSAT_p}}{2})(1 + \lambda(V_{DD} - V_{out})) = 0$$



# Inverter Gain

$$\begin{aligned}
 g &= \left. \frac{dV_{out}}{dV_{in}} \right|_{V_{in}=V_M} \\
 &= - \left. \frac{k_n V_{DSATn} (1 + \lambda_n V_{out}) + k_p V_{DSATp} (1 + \lambda_p V_{out} - \lambda_p V_{DD})}{\lambda_n k_n V_{DSATn} (V_{in} - V_{Tn} - V_{DSATn}/2) + \lambda_p k_p V_{DSATp} (V_{in} - V_{DD} - V_{Tp} - V_{DSATp}/2)} \right|_{V_{in}=V_M} \\
 &\approx - \frac{k_n V_{DSATn} (1 + \lambda_n V_{out}) + k_p V_{DSATp} (1 + \lambda_p V_{out} - \lambda_p V_{DD})}{k_n V_{DSATn} (V_{in} - V_{Tn} - V_{DSATn}/2) (\lambda_n - \lambda_p)} \\
 &= - \frac{1 + \gamma}{(\underline{V_M} - V_{Tn} - V_{DSATn}/2) (\lambda_n - \lambda_p)}
 \end{aligned}$$

$\gamma$  ↑. not sure  $g$  ↑  
**Ratio increase, Gain increase**

$$\gamma = \frac{k'_p \frac{W_p}{L_p} V_{DSATp}}{k'_n \frac{W_n}{L_n} V_{DSATn}}$$

\*\*\*\*\*  
\* CMOS Inverter 直流特性

```
*****  
.param N1=4  
.param SUPPLY=0.7  
.option scale=125n  
*.lib '../lib/cmos25.lib' tt  
.include '../lib/16nm_HP.pm'  
.temp 70  
.option post  
.probe I(*) V(*)  
*****
```

\* Netlist

```
*****  
Vin a gnd  
Vdd vdd gnd 'SUPPLY'  
M1 y a gnd gnd NMOS W=4 L=2  
M2 y a vdd vdd PMOS W='N1*4' L=2  
*****
```

\* 激励

```
*****
```

```
.DC Vin 0 'SUPPLY' 0.01 sweep N1 2 30 2
```

```
*****
```

\* 测量

```
*****
```

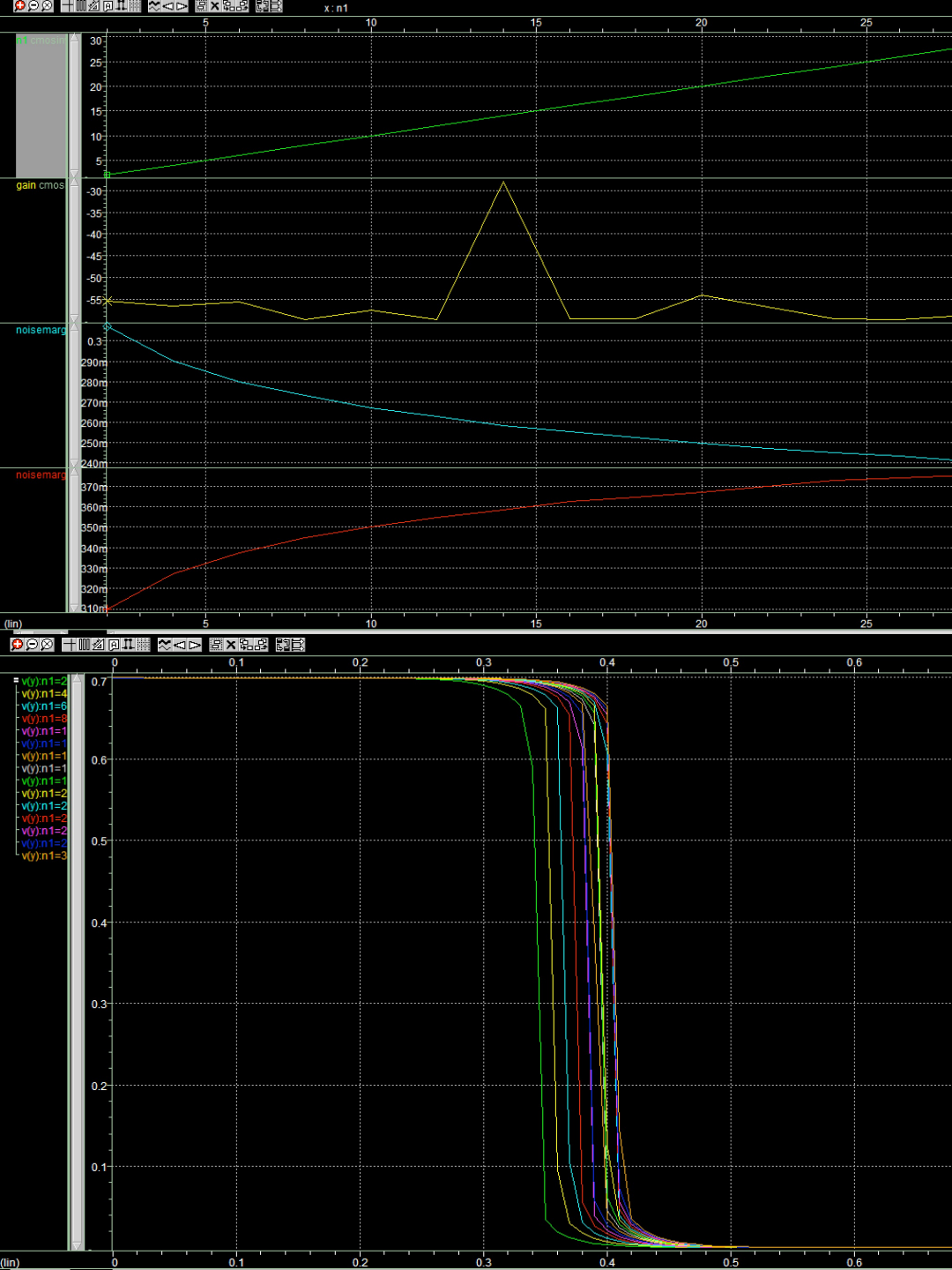
\*噪声容限

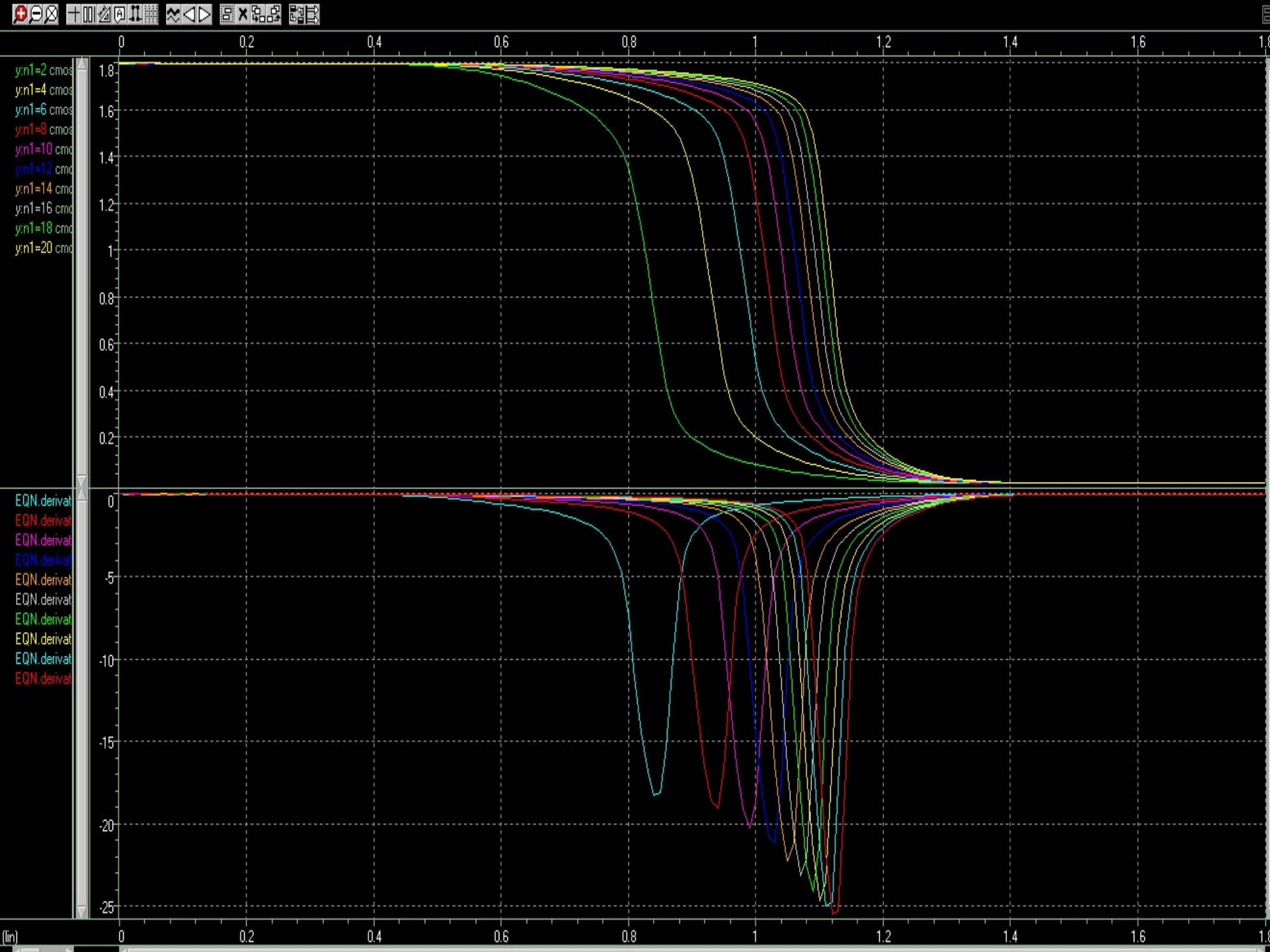
```
.measure DC vm FIND V(a) WHEN V(a)=V(y)  
.measure DC vil FIND V(a) WHEN deriv('V(y)')=-1 cross=1  
.measure DC vih FIND V(a) WHEN deriv('V(y)')=-1 cross=2
```

```
.measure DC voh FIND V(y) WHEN deriv('V(y)')=-1 cross=1  
.measure DC vol FIND V(y) WHEN deriv('V(y)')=-1 cross=2  
.measure noisemargin_H param=('voh-vih')  
.measure noisemargin_L param=('vil-vol')
```

\*增益

```
.measure DC gain deriv V(y) when V(y)=Vm  
.end
```







# An example

An inverter in the generic 0.25um CMOS technology designed with a PMOS-to-NMOS ratio of 3.4 and with the NMOS transistor minimum size ( $W=0.375\mu\text{m}$ ,  $L=0.25\mu\text{m}$ ,  $W/L=1.5$ ),  $V_{dd}=2.5\text{V}$ , Please give the gain of  $V_M$ , and  $V_{IL}$ ,  $V_{IH}$ ,  $NM_L$ ,  $NM_H$ , VTC curve

Parameters for manual model of generic 0.25um CMOS process(minimum length device)

	$V_{TD}(V)$	$\gamma(V^{0.5})$	$V_{DSAT}(V)$	$K'(A/V^2)$	$\lambda(V^{-1})$
NMOS	0.43	0.4	0.63	$115 \times 10^{-6}$	0.06
PMOS	-0.4	-0.4	-1	$-30 \times 10^{-6}$	-0.1

$$\gamma = \frac{k'_p W_p / L_p V_{DSATp}}{k'_n W_n / L_n V_{DSATn}} = \frac{-30 * (-1)}{115 * 0.63} * 3.4 = 1.4$$

$$g = - \frac{1 + \gamma}{\left( V_M - V_{Tn} - \frac{V_{DSATn}}{2} \right) (\lambda_n - \lambda_p)} = - \frac{1 + 1.4}{\left( 1.25 - 0.43 - \frac{0.63}{2} \right) (0.06 + 0.1)}$$

$$= - \frac{2.4}{0.505 * 0.16} = -30$$

# An example: Inverter Gain

Parameters for manual model of generic 0.25um CMOS process(minimum length device)

	$V_{T0}(V)$	$\gamma(V^{0.5})$	$V_{DSAT}(V)$	$k'(A/V^2)$	$\lambda(V^{-1})$
<i>NMOS</i>	0.43	0.4	0.63	$115 \times 10^{-6}$	0.06
<i>PMOS</i>	-0.4	-0.4	-1	$-30 \times 10^{-6}$	-0.1

$$I_D(V_M) = k_n V_{DSAT_n} \left( V_M - V_{Tn} - \frac{V_{DSAT_n}}{2} \right) (1 + \lambda_n V_{out})$$

$$= 115 \times 10^{-6} \times \frac{0.375}{0.25} \times 0.63 \times \left( 1.25 - 0.43 - \frac{0.63}{2} \right) \times (1 + 0.06 \times 1.25) = 59 \times 10^{-6} \text{ A}$$

$$g = \frac{1}{I_D(V_M)} \frac{k_n V_{DSAT_n} + k_p V_{DSAT_p}}{\lambda_n - \lambda_p}$$

$$= \frac{1}{59 \times 10^{-6}} \frac{115 \times 10^{-6} \times \frac{0.375}{0.25} \times 0.63 + 30 \times 10^{-6} \times 3.4 \times \frac{0.375}{0.25} \times 1.0}{0.06 + 0.1} = -27.5$$

$$\gamma = \frac{k'_p W_p / L_p V_{DSATp}}{k'_n W_n / L_n V_{DSATn}} = \frac{-30 * (-1)}{115 * 0.63} * 3.4 = 1$$

$$g = - \frac{1+\gamma}{(V_M - V_{Tn} - V_{DSATn}/2)(\lambda_n - \lambda_p)} = \frac{2}{(1.25 - 0.43 - 0.63/2)(0.06 + 0.1)} = -24.75$$

$$\begin{aligned} g &= \left. \frac{dV_{out}}{dV_{in}} \right|_{V_{in}=V_M} = - \frac{k_n V_{DSATn} (1 + \lambda_n V_{out}) + k_p V_{DSATp} (1 + \lambda_p V_{out} - \lambda_p V_{DD})}{\lambda_n k_n V_{DSATn} (V_{in} - V_{Tn} - V_{DSATn}/2) + \lambda_p k_p V_{DSATp} (V_{in} - V_{DD} - V_{Tp} - V_{DSATp}/2)} = \\ &= - \frac{(1 + \lambda_n V_{out}) + \gamma (1 + \lambda_p V_{out} - \lambda_p V_{DD})}{\lambda_n (V_{in} - V_{Tn} - V_{DSATn}/2) + \lambda_p \gamma (V_{in} - V_{DD} - V_{Tp} - V_{DSATp}/2)} = \\ &= - \frac{(1 + 0.6 * 1.25) + 1 * (1 - 0.1 * 1.25)}{0.06 * (1.25 - 0.43 - 0.315) - 0.1 (-1.25 + 0.4 + 0.5)} = -40 \end{aligned}$$

An inverter in the generic 0.25um CMOS technology designed with a PMOS-to-NMOS ratio of 3.4 and with the NMOS transistor minimum size ( $W=0.375\mu\text{m}$ ,  $L=0.25\mu\text{m}$ ,  $W/L=1.5$ ),  $V_{DD}=2.5\text{V}$ , Please give the gain of  $V_M$ , and  $V_{IL}$ ,  $V_{IH}$ ,  $NM_L$ ,  $NM_H$ , VTC curve

Parameters for manual model of generic 0.25um CMOS process(minimum length device)

	$V_{TD}(V)$	$\gamma(V^{0.5})$	$V_{DSAT}(V)$	$K'(A/V^2)$	$\lambda(V^{-1})$
NMOS	0.43	0.4	0.63	$115 \times 10^{-6}$	0.06
PMOS	-0.4	-0.4	-1	$-30 \times 10^{-6}$	-0.1

$$V_{IH} = V_M - V_M / g = 1.25 * (1 + 1/30) = 1.29V$$

$$V_{IL} = (V_{DD} - V_M) / g + V_M = -1.25/30 + 1.25 = 1.21V$$

$$NM_H = V_{OH} - V_{IH} = 2.5 - 1.29 = 1.21V$$

$$NM_L = V_{IL} - V_{OL} = 1.21V$$

# An example

An inverter in the generic 0.25um CMOS technology designed with a PMOS-to-NMOS ratio of **16** and with the NMOS transistor minimum size ( $W=0.375\mu\text{m}$ ,  $L=0.25\mu\text{m}$ ,  $W/L=1.5$ ) ,  $V_{DD}=2.5\text{V}$  , Please give the gain of  $V_M$ , and  $V_{IL}$ ,  $V_{IH}$ ,  $NM_L$ ,  $NM_H$ ,  $VTC$  curve

Parameters for manual model of generic 0.25um CMOS process(minimum length device)

	$V_{TO}(V)$	$\gamma(V^{0.5})$	$V_{DSAT}(V)$	$K'(A/V^2)$	$\lambda(V^{-1})$
<i>NMOS</i>	0.43	0.4	0.63	$115 \times 10^{-6}$	0.06
<i>PMOS</i>	-0.4	-0.4	-1	$-30 \times 10^{-6}$	-0.1

$$\gamma = \frac{k'_p W_p / L_p V_{DSATp}}{k'_n W_n / L_n V_{DSATn}} = \frac{-30 * (-1)}{115 * 0.63} * 16 = 6.6$$

$$V_M = \frac{(V_{Tn} + V_{DSATn}/2) + \gamma(V_{DD} + V_{Tp} + V_{DSATp}/2)}{1 + \gamma}$$

$$= \frac{(0.43 + 0.63/2) + 6.6 * (2.5 - 0.4 - 1/2)}{1 + 6.6} = \frac{0.745 + 6.6 * 1.6}{1 + 6.6} = 1.49$$

# An example

An inverter in the generic 0.25um CMOS technology designed with a PMOS-to-NMOS ratio of **16** and with the NMOS transistor minimum size ( $W=0.375\mu\text{m}$ ,  $L=0.25\mu\text{m}$ ,  $W/L=1.5$ ) ,  $V_{dd}=2.5\text{V}$  , Please give the gain of  $V_M$ , and  $V_{IL}$ ,  $V_{IH}$ ,  $NM_L$ ,  $NM_H$ ,  $VTC$  curve

Parameters for manual model of generic 0.25um CMOS process(minimum length device)

	$V_{TD}(V)$	$\gamma(V^{0.5})$	$V_{DSAT}(V)$	$K'(A/V^2)$	$\lambda(V^{-1})$
<i>NMOS</i>	0.43	0.4	0.63	$115 \times 10^{-6}$	0.06
<i>PMOS</i>	-0.4	-0.4	-1	$-30 \times 10^{-6}$	-0.1

$$g = - \frac{1 + \gamma}{\left( V_M - V_{Tn} - V_{DSATn}/2 \right) (\lambda_n + \lambda_p)} = - \frac{1 + 6.6}{\left( 1.49 - 0.43 - \frac{0.63}{2} \right) (0.06 + 0.1)}$$

$$= - \frac{7.6}{0.745 * 0.16} = -64$$

# An example

An inverter in the generic 0.25um CMOS technology designed with a PMOS-to-NMOS ratio of **16** and with the NMOS transistor minimum size ( $W=0.375\mu\text{m}$ ,  $L=0.25\mu\text{m}$ ,  $W/L=1.5$ ) ,  $V_{DD}=2.5\text{V}$  , Please give the gain of  $V_M$ , and  $V_{IL}$ ,  $V_{IH}$ ,  $NM_L$ ,  $NM_H$ ,  $VTC$  curve

Parameters for manual model of generic 0.25um CMOS process(minimum length device)

	$V_{TO}(V)$	$\gamma(V^{0.5})$	$V_{DSAT}(V)$	$K'(A/V^2)$	$\lambda(V^{-1})$
<i>NMOS</i>	0.43	0.4	0.63	$115 \times 10^{-6}$	0.06
<i>PMOS</i>	-0.4	-0.4	-1	$-30 \times 10^{-6}$	-0.1

$$V_{IH} = V_M - V_M / g = 1.49 * (1 + 1/64) = 1.51V$$

$$V_{IL} = (V_{DD} - V_M) / g + V_M = -1.25/30 + 1.25 = 1.47V$$

$$NM_H = V_{OH} - V_{IH} = 2.5 - 1.51 = 0.99V$$

$$NM_L = V_{IL} - V_{OL} = 1.47V$$

# A example : noise margin

$$g = -V_M / (V_{IH} - V_M)$$

$$V_{IH} = V_M - V_M / g = 1.25 * (1 + 1/27.5) = 1.3V$$

$$g = -(V_{DD} - V_M) / (V_M - V_{IL})$$

$$V_{IL} = (V_{DD} - V_M) / g + V_M = -1.25 / 27.5 + 1.25 = 1.2V$$

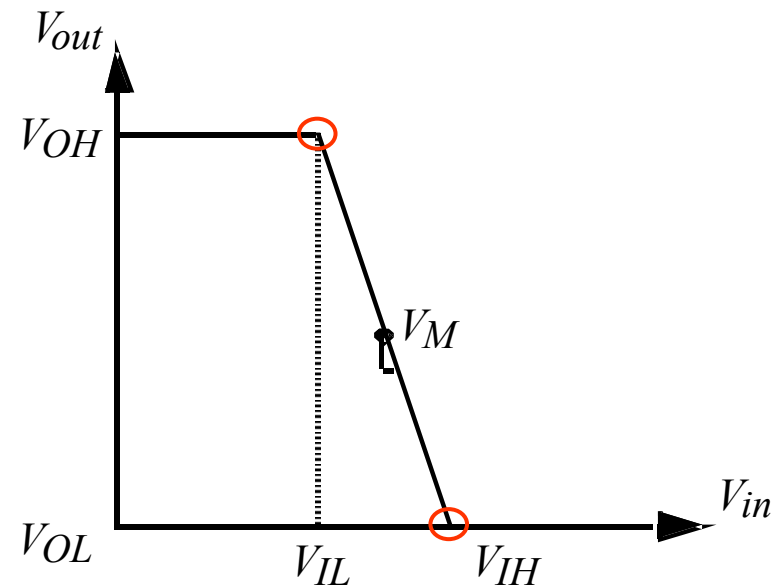
$$NM_H = V_{OH} - V_{IH} = 2.5 - 1.3 = 1.2V$$

$$NM_L = V_{IL} - V_{OL} = 1.2V$$

*Simulated valued*

$$V_{IL} = 1.03V$$

$$V_{IH} = 1.45V$$



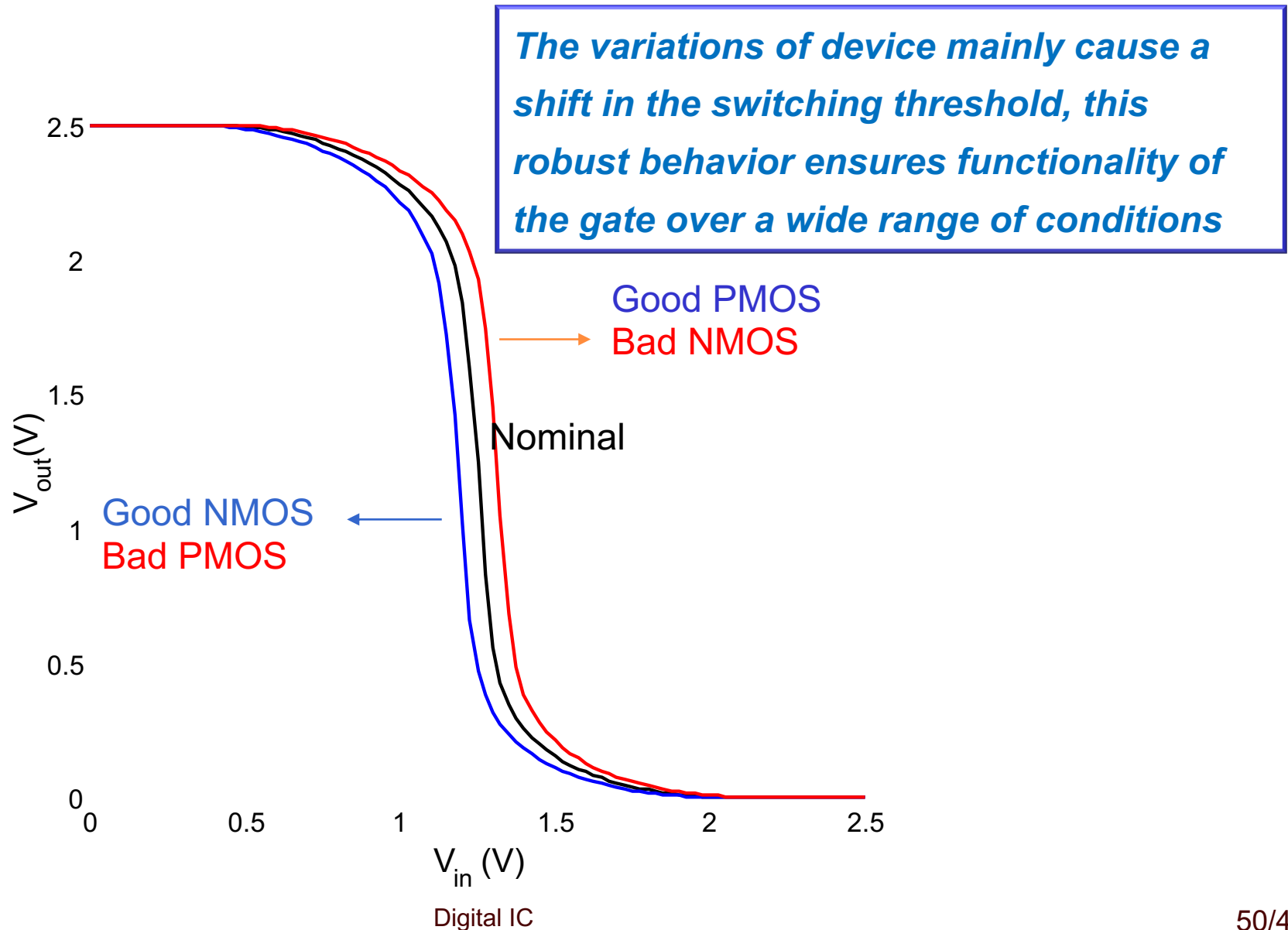
- **Gain is overestimated**
- **Linear approximation of VTC**



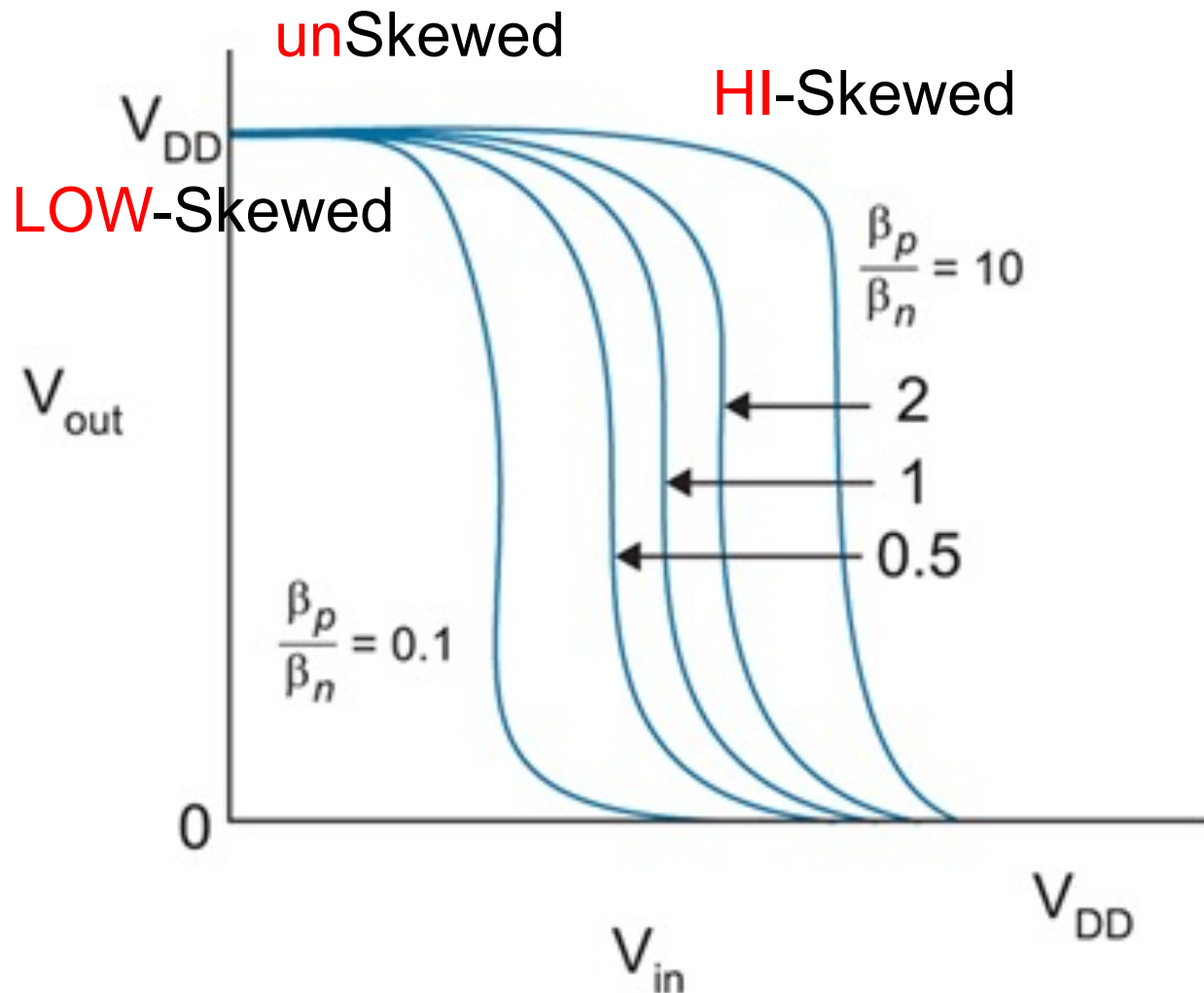
# CMOS static behavior

- CMOS threshold voltage
- CMOS noise margin
- CMOS gain
- *DC robust*

# Impact of Process Variations

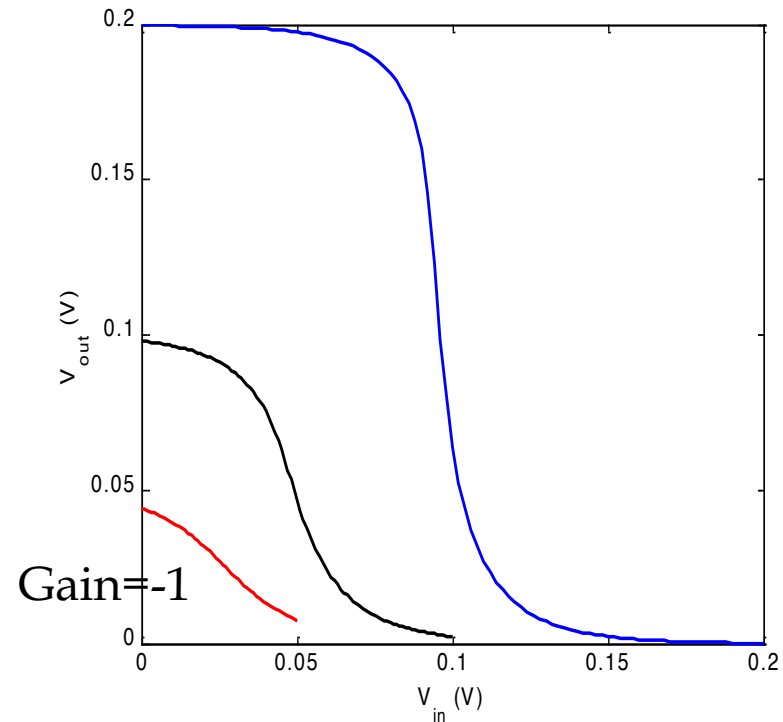
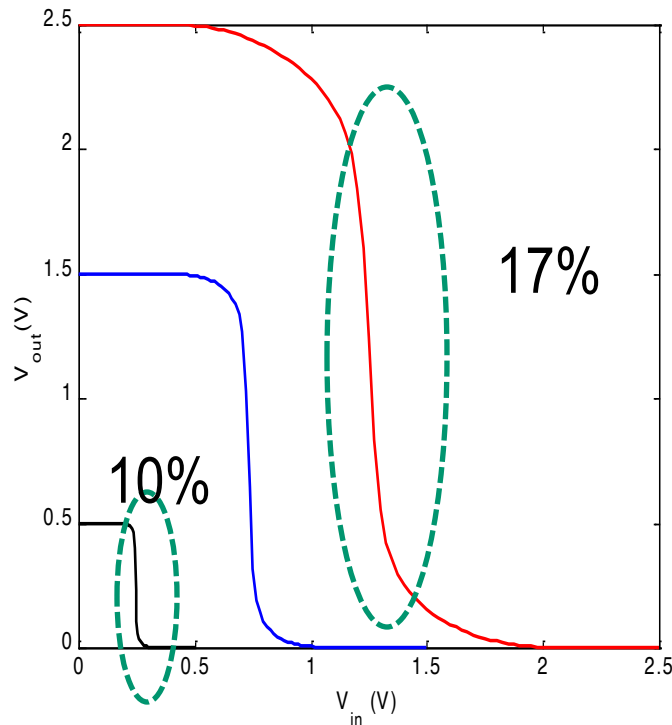


# Transfer characteristics of skewed inverter



# Gain as a function of $V_{DD}$

$$V_{DD\min} > 2 \sim 4 \frac{kT}{q} = 2 \sim 4 \cdot 25mV$$



$$g = - \frac{1 + \gamma}{(V_M - V_{Tn} - V_{DSATn}/2)(\lambda_n + \lambda_p)}$$

**Low power voltage is useful!  
But not too low**

