

Digital Integrated Circuits

***Designing Combinational
Logic Circuits***

Fuyuzhuo

Static CMOS logic

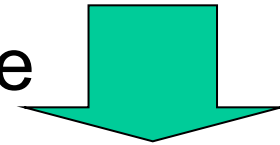
- CMOS static characteristic
- CMOS propagate delay
- Large fan-in technology
- **CMOS power analysis**

Power consumption of combinational circuit

Modify inverter Transistor size

- Rise-fall time
- Switching activity
- Device thresholds and temperature

$$P_{dyn} = C_L V_{DD}^2 f$$

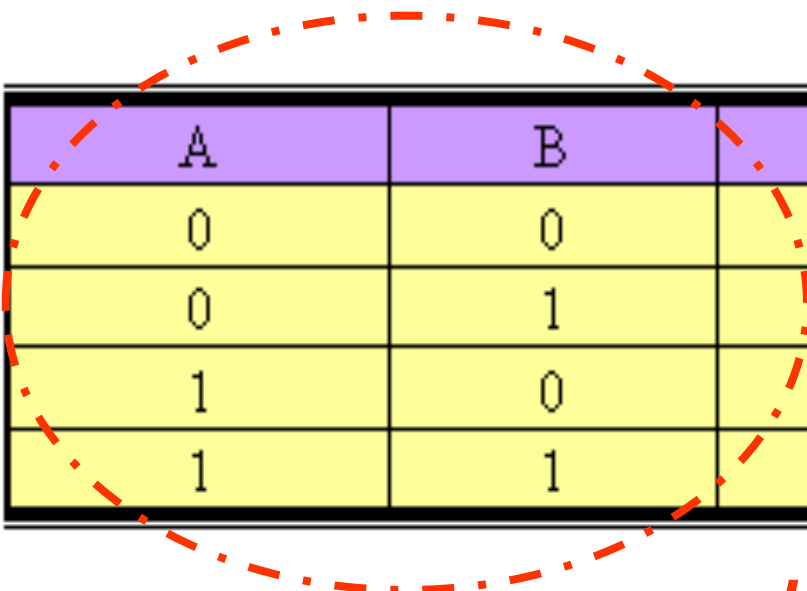


$$P_{dyn} = C_L V_{DD}^2 f_{0 \rightarrow 1} = C_L V_{DD}^2 \alpha_{0 \rightarrow 1} f$$

$$\alpha_{0 \rightarrow 1} = p_0 p_1 = p_0 (1 - p_0)$$

$$\alpha_{0 \rightarrow 1} = \frac{N_0}{2^N} \left(1 - \frac{N_0}{2^N}\right)$$

An example of NAND



A	B	OUT
0	0	1
0	1	1
1	0	1
1	1	0

Uniform input distribution


$$P_1 = 1 - P_A P_B$$

$$P_{0 \rightarrow 1} = (1 - P_1) P_1$$

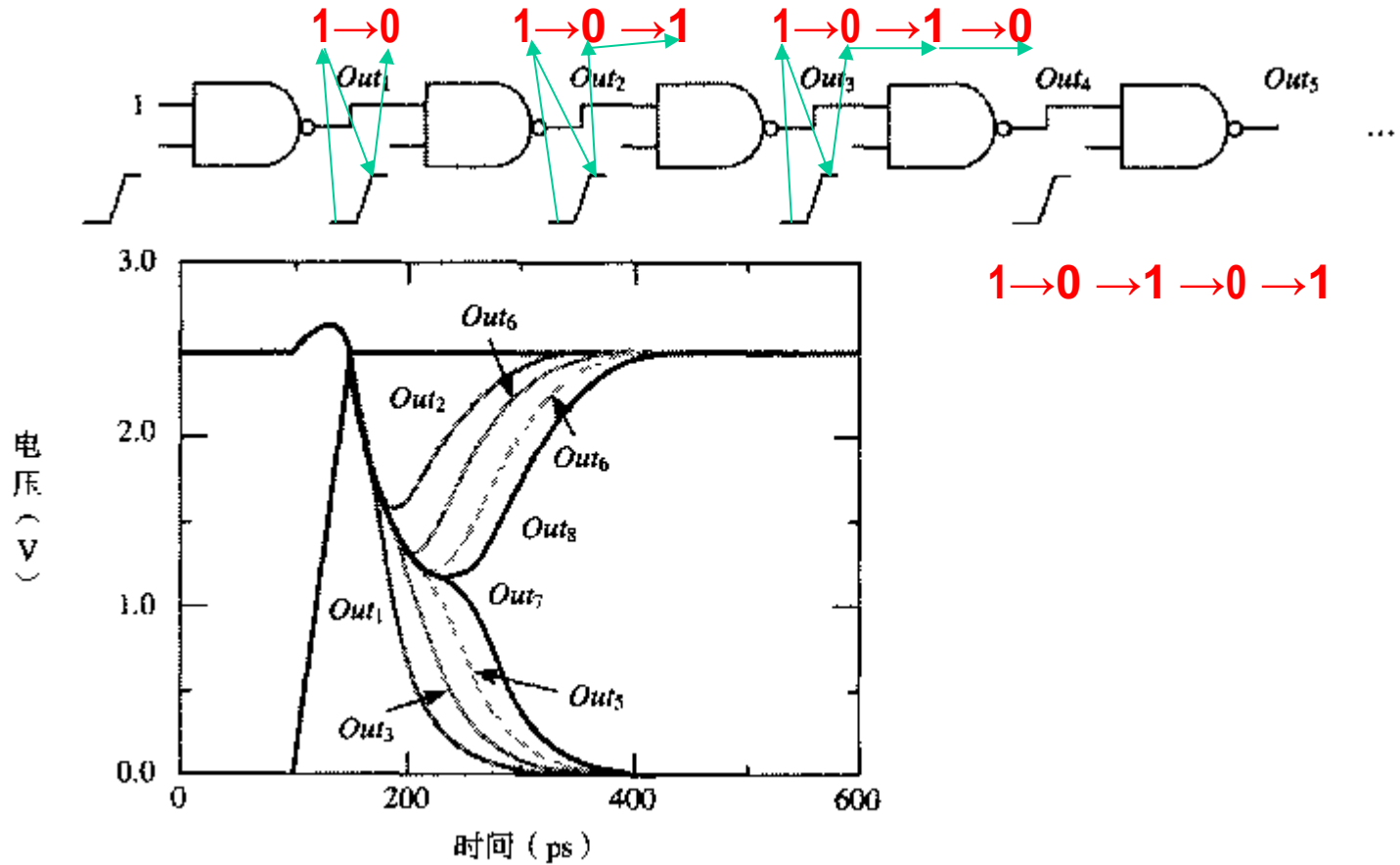
$$P_{0 \rightarrow 1} = P_0 P_1 = (1 - P_1) P_1 = \frac{1}{4} \frac{3}{4} = \frac{3}{16}$$

In general...

Output transition probabilities for static logic gates

	P_1 	$\alpha_{0 \rightarrow 1}$
AND2	$p_A p_B$	$(1-p_1)p_1$
AND3	$p_A p_B p_C$	
OR2	$1 - \bar{p}_A \bar{p}_B$	
NAND2	$1 - p_A p_B$	
NOR2	$\bar{p}_A \bar{p}_B$	
XOR	$p_A \bar{p}_B + \bar{p}_A p_B$	

Glitch because of delay

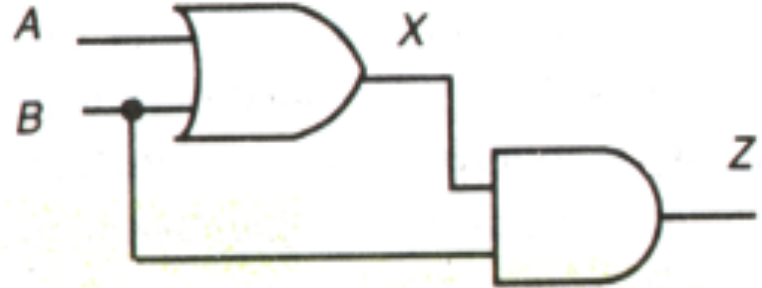
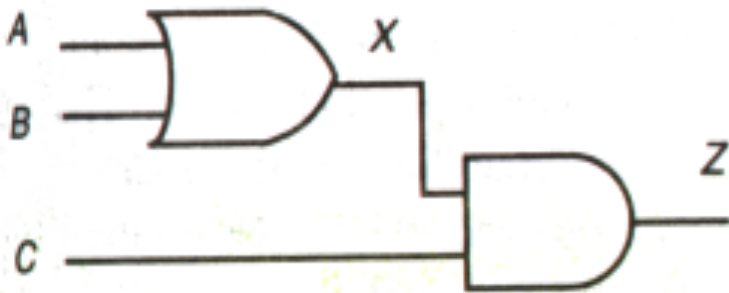


A funny example

OR 0→1 transition probability is 3/16

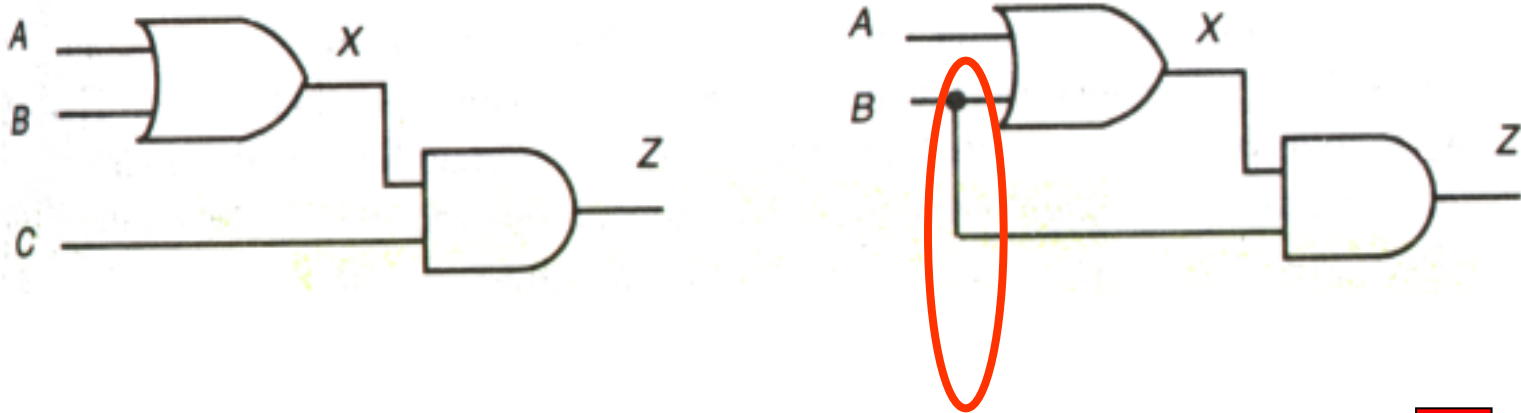
OR output 1 probability is 3/4

Z point 0→1 probability is $(1 - 3/4 * 1/2)(3/4 * 1/2) = 15/64$



It is still right?

The question comes from assuming of
uncorrelated input

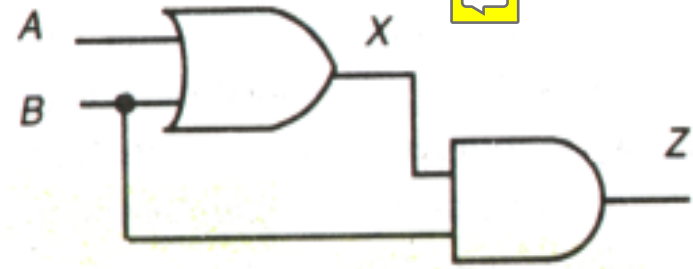
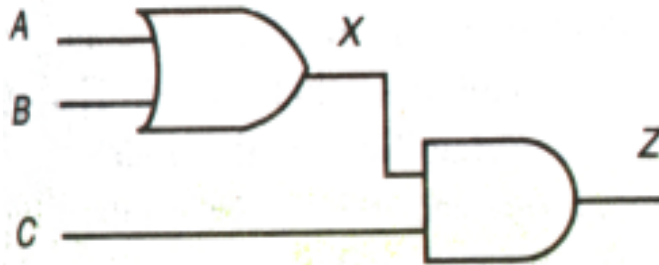


Reconvergent fanout

B

B

Conditional probabilities analysis approach



$$P(Z = 0) = 1 - P(Z = 1)$$

$$P(Z = 1) = P(B = 1, X = 1) = P(B = 1)P(X = 1)$$

**Power evaluation of large network
mainly come from probabilities model**



$$P(Z = 0) = 1 - P(B = 1, X = 1)$$

$$P(Z = 1) = P(B = 1, X = 1) = P(X = 1|B = 1)P(B = 1) = P(B = 1)$$

Design for Low Power

Outline

Power and Energy

Dynamic Power

Static Power

Low Power Design

Power and Energy

- Power is drawn from a voltage source attached to the V_{DD} pin(s) of a chip.

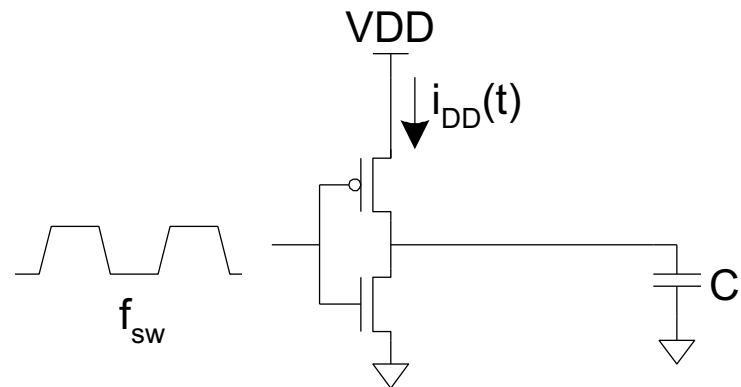
- Instantaneous Power: $P(t) = i_{DD}(t)V_{DD}$

- Energy:
$$E = \int_0^T P(t)dt = \int_0^T i_{DD}(t)V_{DD}dt$$

- Average Power:
$$P_{\text{avg}} = \frac{E}{T} = \frac{1}{T} \int_0^T i_{DD}(t)V_{DD}dt$$

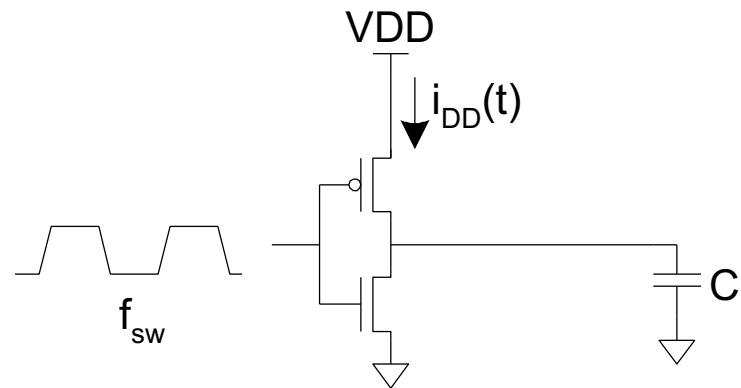
Dynamic Power

- Dynamic power is required to charge and discharge load capacitances when transistors switch.
- One cycle involves a rising and falling output.
- On rising output, charge $Q = CV_{DD}$ is required
- On falling output, charge is dumped to GND
- This repeats Tf_{sw} times over an interval of T



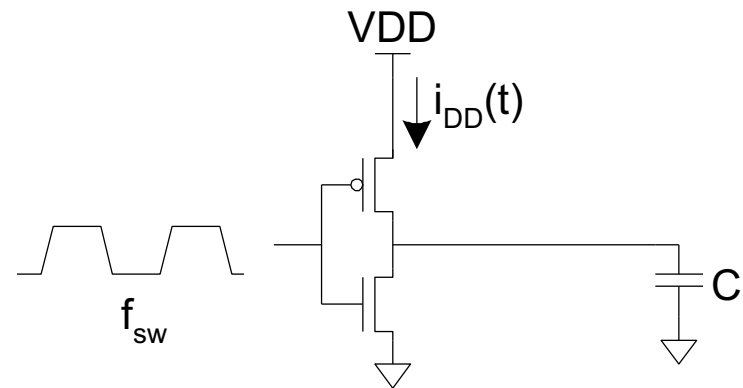
Dynamic Power Cont.

$$P_{\text{dynamic}} =$$



Dynamic Power Cont.

$$\begin{aligned}P_{\text{dynamic}} &= \frac{1}{T} \int_0^T i_{DD}(t) V_{DD} dt \\&= \frac{V_{DD}}{T} \int_0^T i_{DD}(t) dt \\&= \frac{V_{DD}}{T} [T f_{\text{sw}} C V_{DD}] \\&= C V_{DD}^2 f_{\text{sw}}\end{aligned}$$



Activity Factor

- Suppose the system clock frequency = f
- Let $f_{sw} = \alpha f$, where α = activity factor
 - If the signal is a clock, $\alpha = 1$
 - If the signal switches once per cycle, $\alpha = 1/2$
 - Dynamic gates:
 - Switch either 0 or 2 times per cycle, $\alpha = 1/2$
 - Static gates:
 - Depends on design, but typically $\alpha = 0.1$
- Dynamic power:
$$P_{dynamic} = \alpha C V_{DD}^2 f$$

Short Circuit Current

- When transistors switch, both nMOS and pMOS networks may be momentarily ON at once
- Leads to a blip of “short circuit” current.
- $< 10\%$ of dynamic power if rise/fall times are comparable for input and output

Example

- 200 Mtransistor chip
 - 20M logic transistors
 - Average width: 12λ
 - 180M memory transistors
 - Average width: 4λ
 - 1.2 V 100 nm process
 - $C_g = 2 \text{ fF}/\mu\text{m}$

Dynamic Example

- Static CMOS logic gates: activity factor = 0.1
- Memory arrays: activity factor = 0.05 (many banks!)
- Estimate dynamic power consumption per MHz.
Neglect wire capacitance and short-circuit current.

Dynamic Example

- Static CMOS logic gates: activity factor = 0.1
- Memory arrays: activity factor = 0.05 (many banks!)
- Estimate dynamic power consumption per MHz.
Neglect wire capacitance.

$$C_{\text{logic}} = (20 \times 10^6)(12\lambda)(0.05\mu m / \lambda)(2 fF / \mu m) = 24 nF$$

$$C_{\text{mem}} = (180 \times 10^6)(4\lambda)(0.05\mu m / \lambda)(2 fF / \mu m) = 72 nF$$

$$P_{\text{dynamic}} = [0.1C_{\text{logic}} + 0.05C_{\text{mem}}](1.2)^2 f = 8.6 \text{ mW/MHz}$$

Static Power

- Static power is consumed even when chip is quiescent.
 - Ratioed circuits burn power in fight between ON transistors
 - Leakage draws power from nominally OFF devices

$$I_{ds} = I_{ds0} e^{\frac{V_{gs} - V_t}{n v_T}} \left[1 - e^{\frac{-V_{ds}}{v_T}} \right]$$

$$V_t = V_{t0} - \eta V_{ds} + \gamma \left(\sqrt{\phi_s + V_{sb}} - \sqrt{\phi_s} \right)$$

Ratio Example

- The chip contains a 32 word x 48 bit ROM
 - Uses pseudo-nMOS decoder and bitline pullups
 - On average, one wordline and 24 bitlines are high
- Find static power drawn by the ROM
 - $\beta = 75 \mu\text{A}/\text{V}^2$
 - $V_{tp} = -0.4\text{V}$

Ratio Example

- The chip contains a 32 word x 48 bit ROM
 - Uses pseudo-nMOS decoder and bitline pullups
 - On average, one wordline and 24 bitlines are high
- Find static power drawn by the ROM
 - $\beta = 75 \mu\text{A}/\text{V}^2$
 - $V_{tp} = -0.4\text{V}$

- Solution:

$$I_{\text{pull-up}} = \beta \frac{(V_{DD} - |V_{tp}|)^2}{2} = 24\mu\text{A}$$

$$P_{\text{pull-up}} = V_{DD} I_{\text{pull-up}} = 29\mu\text{W}$$

$$P_{\text{static}} = (31 + 24)P_{\text{pull-up}} = 1.6 \text{ mW}$$

Leakage Example

- The process has two threshold voltages and two oxide thicknesses.
- Subthreshold leakage:
 - 20 nA/ μm for low V_t
 - 0.02 nA/ μm for high V_t
- Gate leakage:
 - 3 nA/ μm for thin oxide
 - 0.002 nA/ μm for thick oxide
- Memories use low-leakage transistors everywhere
- Gates use low-leakage transistors on 80% of logic

Leakage Example Cont.

- Estimate static power:

Leakage Example Cont.

- Estimate static power:

- High leakage: $(20 \times 10^6)(0.2)(12\lambda)(0.05\mu m / \lambda) = 2.4 \times 10^6 \mu m$
- Low leakage: $(20 \times 10^6)(0.8)(12\lambda)(0.05\mu m / \lambda) +$
 $(180 \times 10^6)(4\lambda)(0.05\mu m / \lambda) = 45.6 \times 10^6 \mu m$

$$I_{static} = (2.4 \times 10^6 \mu m) \left[(20nA / \mu m) / 2 + (3nA / \mu m) \right] +$$
$$(45.6 \times 10^6 \mu m) \left[(0.02nA / \mu m) / 2 + (0.002nA / \mu m) \right]$$
$$= 32mA$$

$$P_{static} = I_{static} V_{DD} = 38mW$$

Leakage Example Cont.

- Estimate static power:

- High leakage: $(20 \times 10^6)(0.2)(12\lambda)(0.05\mu m / \lambda) = 2.4 \times 10^6 \mu m$

- Low leakage: $(20 \times 10^6)(0.8)(12\lambda)(0.05\mu m / \lambda) +$
 $(180 \times 10^6)(4\lambda)(0.05\mu m / \lambda) = 45.6 \times 10^6 \mu m$

$$I_{static} = (2.4 \times 10^6 \mu m) \left[(20nA / \mu m) / 2 + (3nA / \mu m) \right] +$$
$$(45.6 \times 10^6 \mu m) \left[(0.02nA / \mu m) / 2 + (0.002nA / \mu m) \right]$$
$$= 32mA$$

$$P_{static} = I_{static} V_{DD} = 38mW$$

- If no low leakage devices, $P_{static} = 749 mW$ (!)

Low Power Design

- Reduce dynamic power
 - α :
 - C:
 - V_{DD} :
 - f:
- Reduce static power

Low Power Design

- Reduce dynamic power
 - α : clock gating, sleep mode
 - C :
 - V_{DD} :
 - f :
- Reduce static power

Low Power Design

- Reduce dynamic power
 - α : clock gating, sleep mode
 - C: small transistors (esp. on clock), short wires
 - V_{DD} :
 - f:
- Reduce static power

Low Power Design

- Reduce dynamic power
 - α : clock gating, sleep mode
 - C : small transistors (esp. on clock), short wires
 - V_{DD} : lowest suitable voltage
 - f :
- Reduce static power

Low Power Design

- Reduce dynamic power
 - α : clock gating, sleep mode
 - C : small transistors (esp. on clock), short wires
 - V_{DD} : lowest suitable voltage
 - f : lowest suitable frequency
- Reduce static power

Low Power Design

- Reduce dynamic power
 - α : clock gating, sleep mode
 - C: small transistors (esp. on clock), short wires
 - V_{DD} : lowest suitable voltage
 - f: lowest suitable frequency
- Reduce static power
 - Selectively use ratioed circuits
 - Selectively use low V_t devices
 - Leakage reduction:
stacked devices, body bias, low temperature