$$Homework1$$
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$1.pf:$

$$firstly, we\ prove: f(n) = O(n^2)$$
$$\exists c > 0, c = \frac{4}{3}\ and\ n_0 = 5$$
$$When\ n \geq 5, we\ have\ : f(n) = \frac{1}{3}n^2 + 5n \geq 0$$
$$and\ f(n) = \frac{1}{3}n^2 + 5n \leq \frac{4}{3}n^2,$$
$$which\ is\ equivalent\ to\ n^2 - 5n \geq 0,\ apparently\ it\ is\ true.$$
$$So, f(n) = O(n^2).$$
$$secondly, we\ prove: f(n) = \Omega(n^2)$$
$$\exists c > 0, c = \frac{1}{6}\ and\ n_0 = 0$$
$$When\ n \geq 0, we\ have\ : cn^2 = \frac{1}{6}n^2 \geq 0$$
$$and\ f(n) = \frac{1}{3}n^2 + 5n \geq \frac{1}{6}n^2,$$
$$which\ is\ equivalent\ to\ \frac{1}{6}n^2 + 5n \geq 0,\ apparently\ it\ is\ true.$$
$$So, f(n) = \Omega(n^2).$$
$$from\ above\ all, we\ can\ derive f(n) = \Theta(n^2)$$

$2.Solution:$

$$set\ m = log_2 n,$$
$$\Rightarrow T(2^m) = 4T(2^{\frac{m}{2}}) + m$$
$$set\ S(m) = T(2^m),$$
$$\Rightarrow S(m) = 4S(\frac{m}{2}) + m$$
$$then\ we\ will\ prove\ S(m) \leq c_1 m^2 - c_2 m,\ \ c_1, c_2 > 0$$
$$subprove:$$
$$S(m) = 4S(\frac{m}{2}) + m$$
$$\leq 4(c_1(\frac{m}{2})^2) - c_2(\frac{m}{2})) + m$$
$$= c_1 m^2 - 2c_2 m + m$$
$$= c_1 m^2 - c_2 m - (c_2 m - m) \leq c_1 m^2 - c_2 m$$
$$so,\ S(m) \leq cm^2,\ \ c > 0,$$

$$so, S(m) = O(m^2)$$
$$then\ we\ will\ prove : S(m) \geq c_3 m^2,\ c_3 > 0,$$
$$subprove :$$
$$S(m) = 4S(\frac{m}{2}) + m$$
$$\geq 4(c_3(\frac{m}{2})^2) + m$$
$$= c_3 m^2 + m$$
$$\geq c_3 m^2$$
$$so, S(m) \geq cm^2,\ c > 0,$$
$$so, S(m) = \Omega(m^2)$$
$$\Rightarrow S(m) = \Theta(m^2)$$
$$\Rightarrow T(n) = \Theta(log^2 n)$$

$3.Solution :$

$$a = 2 > 1, b = 2 > 1,$$
$$n^{log_b a} = n$$
$$f(n) = nlog(n+10) = \Omega(n^{\epsilon}), \epsilon > 0$$
$$2nlog(\frac{n}{2} + 10) \leq cnlog(n+10), it\ fails\ when\ n\ is\ pretty\ large,$$
$$so\ \text{"主定理"} fails.$$
$$.$$
$$T(n) = 2T(\frac{n}{2}) + nlog(n+10)$$
$$= 2^i T(\frac{n}{2^i}) + n(log(n+10) + ... + log(\frac{n}{2^{i-1}} + 10)) \quad (*)$$
$$and\ \frac{n}{2^i} = 1 \Rightarrow i = log_2 n$$
$$\geq nT(1) + nlog(\frac{n^i}{2^{\frac{(i-1)i}{2}}})$$
$$= nT(1) + n(logn)^2 - n \times \frac{(i-1)i}{2}$$
$$= \Theta(nlognlogn)$$
$$so,\ T(n) = \Omega(nlognlogn)$$
$$and,\ if\ n\ is\ quite\ large\ (like\ n > 10)$$
$$(*) \leq nT(1) + nlog(2n \times n \times \frac{n}{2} \times ... \times \frac{n}{2^{i-2}})$$
$$= nT(1) + nlog(\frac{n^i}{2^{\frac{(i-3)i}{2}}})$$
$$= nT(1) + n(logn)^2 - n \times \frac{(i-3)i}{2}$$

$$= \Theta(nlognlogn)$$
$$so, T(n) = O(nlognlogn)$$
$$from\ above\ all, T(n) = \Theta(nlognlogn)$$

$4.Solutions:$

$$when\ n \leq 14,\ T(n) = d,$$
$$when\ n > 14,\ set\ n = m + 14,$$
$$then\ we\ have\ T(m + 14) = 2T(\frac{m}{2} + 14) + m + 14,$$
$$then\ set\ S(m) = T(m + 14),$$
$$then\ S(m) = 2S(\frac{m}{2}) + m + 14,$$
$$from\ the\ conclusion\ before:$$
$$we\ have\ S(m) = \Theta(mlogm)$$
$$T(n) = S(n - 14) = \Theta((n - 14)log(n - 14)) = \Theta(nlogn)$$
$$So\ T(n) = \Theta(nlogn)$$
$$in\ all,\ T(n) = \Theta(nlogn),\ n > 14$$
$$T(n) = d,\ n \leq 14$$