John Hodge
ECE 5105: Final Exam Document
12/14/15

Problem 1

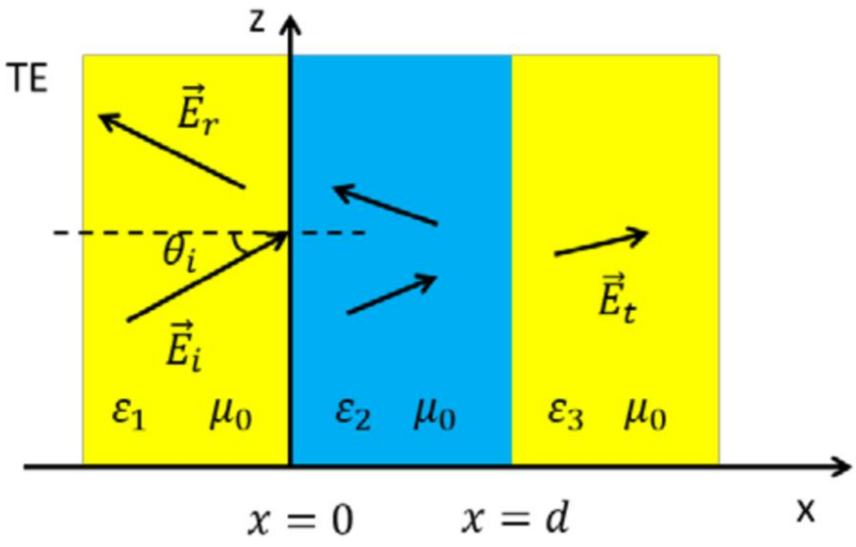

Figure 1: Diagram of Structure for Problem 1

Angular frequency of the incident wave: ω = 3*10^11 rad/s

Free-space wavelength: ω = (2* π)*(c/λ) = (2* π)*((3*10^8 m/s)/(λ)) = 3*10^11 rad/s → λ = π/500 = 6.28mm

More generally: λ = (2* π)/k

$$\varepsilon_1 = \varepsilon_3 = 4\varepsilon_0$$

Incident Angle of $\vec{E}_i$: $\theta_i = 60^o$

The electric field of the incident wave contains the $\vec{e}_y$component only -> TE polarization case.

General TE wave: $\vec{E}_n = \{A_n e^{-jk_{nx}x} + B_n e^{+jk_{nx}x}\}e^{j(\omega t - \beta z)}\vec{e}_y$

Two Cases:

Case A: $\varepsilon_2 = \varepsilon_0$

Case B: $\varepsilon_2 = 8\varepsilon_0$

Range of thickness for the 2nd layer: d = 0.01mm to 10.0mm

General form of k-vector: $\vec{k} = k_x\vec{e}_x + \beta\vec{e}_z$

$$(k_{nx})^2 + \beta^2 = \omega^2\varepsilon_1\mu_0 = k_n^2$$

Given the geometry:

$$\tan\theta_i = \frac{\beta}{k_x} \rightarrow \tan 60^o = \frac{\beta}{k_x}$$

$$\beta = k_x \tan 60^o = \sqrt{3}\, k_x$$

$$(k_x)^2 + (\sqrt{3}\, k_x)^2 = \omega^2\varepsilon_1\mu_0 = k_i^2 \rightarrow (k_x)^2 + 3k_x^2 = \omega^2\varepsilon_1\mu_0$$

$4k_x^2$ = (3*10^11 rad/s)^2*(4)*(8.854*10^-12 F/m)*(4*pi*10^-7 H/m)

k_x = 1000.68 (1/m)

$$(k_x)^2 + \beta^2 = \omega^2\varepsilon_1\mu_0 \rightarrow (1000.68\tfrac{1}{m})^2 + \beta^2 = \omega^2\varepsilon_1\mu_0$$

$(1000.68\frac{1}{m})^2 + \beta^2$ = (3*10^11 rad/s)^2*(4)*(8.854*10^-12 F/m)*(4*pi*10^-7 H/m)

β = 1733.23 (1/m) = $\sqrt{3}\, k_x = \sqrt{3}\,(1000.68\frac{1}{m})$

Relevant Equations From Class Notes (09/29 and 10/08):

$$\begin{bmatrix} A_t \\ B_t \end{bmatrix} = T_{TE}\begin{bmatrix} A_i \\ B_i \end{bmatrix}$$

Boundary Condition:
Outgoing uniform wave in the transmission layer

$$k_{tx} = \sqrt{\omega^2\varepsilon_t\mu_t - \beta^2} \qquad \beta < \omega\sqrt{\varepsilon_t\mu_t}$$

$$\begin{bmatrix} A_i \\ B_i \end{bmatrix} = \begin{bmatrix} (T_{TE}^{-1})_{11} & (T_{TE}^{-1})_{12} \\ (T_{TE}^{-1})_{21} & (T_{TE}^{-1})_{22} \end{bmatrix}\begin{bmatrix} A_t \\ B_t \end{bmatrix} \quad \begin{bmatrix} A_t \\ B_t \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad \begin{bmatrix} A_i \\ B_i \end{bmatrix} = \begin{bmatrix} (T_{TE}^{-1})_{11} \\ (T_{TE}^{-1})_{21} \end{bmatrix}$$

$$T_{TE} = [(D_t^{TE})^{-1} D_1^{TE} P_1^{TE}][(D_1^{TE})^{-1} D_i^{TE}]$$

$$D_n^{TE} = \begin{bmatrix} 1 & 1 \\ \frac{k_{nx}}{\mu_n} & -\frac{k_{nx}}{\mu_n} \end{bmatrix} \quad P_n^{TE} = \begin{bmatrix} e^{-jk_{nx}L_n} & 0 \\ 0 & e^{jk_{nx}L_n} \end{bmatrix}$$

We will use these equations to analyze the reflection and transmission of the EM structure depicted in this problem.

Layer 1:

$k_i = \omega\sqrt{\varepsilon_1 \mu_0}$ = (3*10^11 rad/s)*sqrt((4)*(8.854*10^-12 F/m)*(4*pi*10^-7 H/m))

$$k_i = \omega\sqrt{\varepsilon_1 \mu_0} = 2000.36 \frac{1}{m}$$

$k_i = \omega\sqrt{\varepsilon_1 \mu_0} > \beta \rightarrow$ No Total Internal Reflection (TIR)

k_{ix} = 1000.68 (1/m)

$$D_i^{TE} = \begin{bmatrix} 1 & 1 \\ \frac{k_{ix}}{\mu_0} & -\frac{k_{ix}}{\mu_0} \end{bmatrix}$$

Layer 2 (Case A):

$k_{2A} = \omega\sqrt{\varepsilon_{2A} \mu_0}$ = (3*10^11 rad/s)*sqrt((1)*(8.854*10^-12 F/m)*(4*pi*10^-7 H/m))

$$k_{2A} = \omega\sqrt{\varepsilon_{2A} \mu_0} = 1000.68 \frac{1}{m}$$

$k_{2A} = \omega\sqrt{\varepsilon_{2A} \mu_0} < \beta \rightarrow TIR \rightarrow k_{2A\_x} = j\kappa_{2A\_x} \rightarrow$ No propagation in the x-direction. Completely imaginary k-vector signifies attenuation in this layer without a propagation component.

$k_{2A\_x} = j\kappa_{2A\_x}$ = j*1415.18 (1/m)

$$D_{2A}^{TE} = \begin{bmatrix} 1 & 1 \\ \frac{k_{2Ax}}{\mu_0} & -\frac{k_{2Ax}}{\mu_0} \end{bmatrix}$$

$$P_{2A}^{TE} = \begin{bmatrix} e^{-jk_{2Ax}d} & 0 \\ 0 & e^{+jk_{2Ax}d} \end{bmatrix}$$

Layer 2 (Case B):

$k_{2B} = \omega\sqrt{\varepsilon_{2B}\mu_0}$ = (3*10^11 rad/s)*sqrt((8)*(8.854*10^-12 F/m)*(4*pi*10^-7 H/m))

$$k_{2B} = \omega\sqrt{\varepsilon_{2B}\mu_0} = 2830.36\frac{1}{m}$$

$$k_{2B} = \omega\sqrt{\varepsilon_{2B}\mu_0} > \beta \rightarrow \text{No TIR}$$

$$k_{2B\_x} = 2237.60\ (1/\text{m})$$

$$D_{2B}^{TE} = \begin{bmatrix} 1 & 1 \\ \frac{k_{2Bx}}{\mu_0} & -\frac{k_{2Bx}}{\mu_0} \end{bmatrix}$$

$$P_{2A}^{TE} = \begin{bmatrix} e^{-jk_{2Bx}d} & 0 \\ 0 & e^{+jk_{2Bx}d} \end{bmatrix}$$

Layer 3:

$k_t = \omega\sqrt{\varepsilon_3\mu_0}$ = (3*10^11 rad/s)*sqrt((4)*(8.854*10^-12 F/m)*(4*pi*10^-7 H/m))

$$k_t = \omega\sqrt{\varepsilon_3\mu_0} = 2000.36\frac{1}{m}$$

$$k_t = \omega\sqrt{\varepsilon_3\mu_0} > \beta \rightarrow \text{No Total Internal Reflection (TIR)}$$

$$k_{tx} = 1000.68\ (1/\text{m})$$

$$D_t^{TE} = \begin{bmatrix} 1 & 1 \\ \frac{k_{tx}}{\mu_0} & -\frac{k_{tx}}{\mu_0} \end{bmatrix}$$

Total TE Transfer Matrix:

Case A: $T_{TE} = [(D_t^{TE})^{-1}(D_{2A}^{TE})(P_{2A}^{TE})][(D_{2A}^{TE})^{-1}(D_i^{TE})]$

Case B: $T_{TE} = [(D_t^{TE})^{-1}(D_{2B}^{TE})(P_{2B}^{TE})][(D_{2B}^{TE})^{-1}(D_i^{TE})]$

Summary:

Layer #	$\boldsymbol{\varepsilon_r}$	$\boldsymbol{\lvert\vec{k}\rvert}$	**β**	$\boldsymbol{k_x}$	**TIR?**
1	4	$2000.36\,\frac{1}{m}$	1733.23 (1/m)	1000.68 (1/m)	No
Case A: 2	1	$\mathbf{1000.68\,\frac{1}{m}}$	1733.23 (1/m)	**j*1415.18 (1/m)**	**Yes**
Case B: 2	8	$2830.36\,\frac{1}{m}$	1733.23 (1/m)	2237.60 (1/m)	No
3	4	$2000.36\,\frac{1}{m}$	1733.23 (1/m)	1000.68 (1/m)	No

Table 1: Summary of k-Vectors in Each Layer of the Dielectric Slab Structure

P1 Plot:

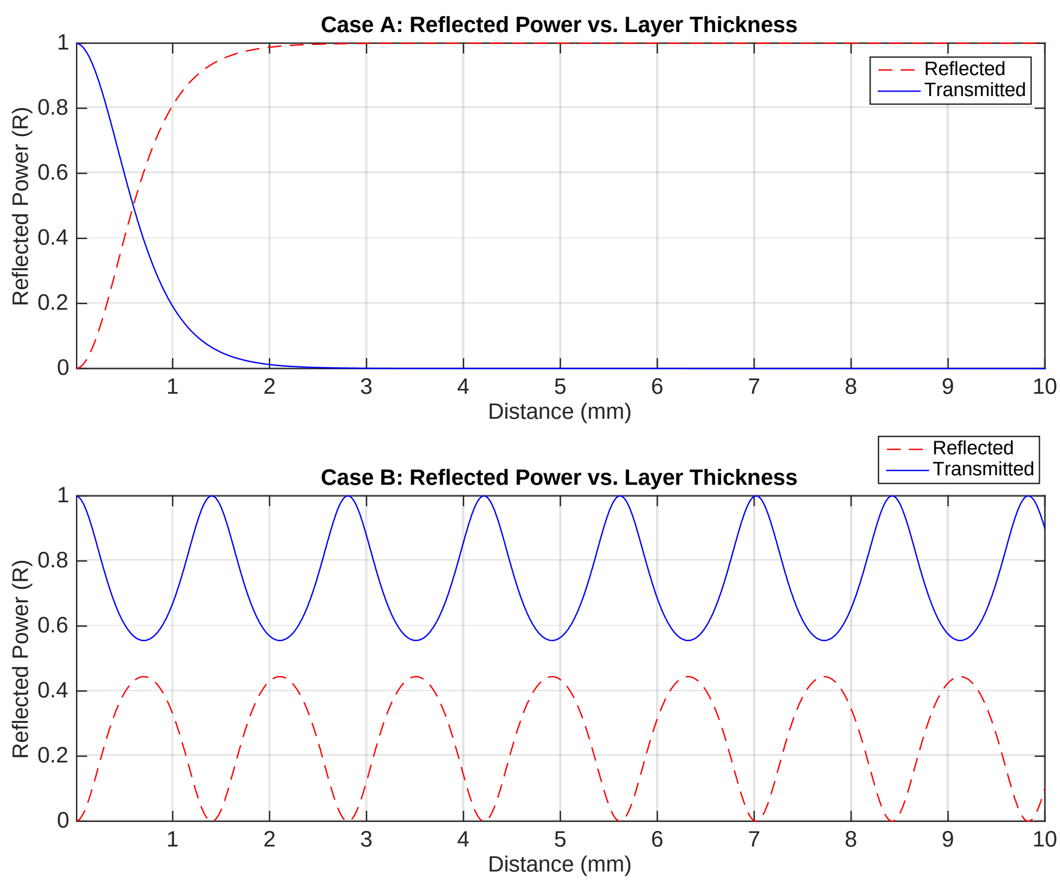

Figure 2: Fraction of Power Reflected and Transmitted vs. Middle Layer Thickness for Case A and Case B in Problem 1

Problem 1 Discussion:

In problem 1, the reflection and transmission of TE polarized EM waves through a three-layer dielectric slab structure is analyzed. This problem analyzes the fraction of EM power reflected (R) and transmitted (T) as a function of the middle layer's thickness for two distinct cases. Case A has a middle layer, which is equivalent to free space ($\varepsilon_r = 1, \mu_r = 1$), and case B has a middle layer ($\varepsilon_r = 8, \mu_r = 1$) with a dielectric constant twice that of the outer layers. The material properties of the outer layers ($\varepsilon_r = 4, \mu_r = 1$), layers 1 and 3, in case A and case B are identical. Despite the dielectric structure being identical in all other ways than the dielectric constant value of the middle layer, this analysis shows that the EM performance of the structure as a function of middle layer thickness for a TE polarized wave is drastically different for the two cases. The purpose of this discussion is to analyze the physical meaning of these results to gain a better understanding of the difference observed between Case A and Case B. All of the materials in these structures are lossless, meaning that there is no imaginary loss tangent component to their material permittivity and permeability values, resulting in the relationship of $R + T = 1$. This means that all power in the system must be transmitted or reflected.

In Case A, nearly all of the power is transmitted and none is reflected as the middle layer thickness approaches zero ($d \rightarrow 0$). This makes intuitive sense because in the absence of the middle layer, there would be no reflection between the two outer layers since they have identical material properties resulting in a perfect impedance match at the boundary. However, as the thickness of the middle layer increases, transmitted power quickly approaches zero and almost all of the power is reflected. Transmitted power is <1.3% when d = 2mm and is <0.1% when d = 3mm. These results seem to indicate that there is no propagating component to the k-vector in the x-direction ($\mathrm{Re}[k_x] = 0$) and only an imaginary attenuation component to the k_x vector. An analysis summarized in Table 1 confirms these results and shows that in layer 2 for Case A that $k_2 = \omega\sqrt{\varepsilon_0\mu_0} < \beta$ resulting in TIR.

After reviewing this analysis, it is interesting in Case A that while there is not a propagating component to the k-vector in the x-direction that power is still transmitted through the system at small values of d (middle layer thickness). This phenomenon is sometimes referred to as frustrated TIR. This is because there is an evanescent wave exponentially decaying with distance in the middle layer, which is a free-space gap. The propagating wave essentially tunnels through the free-space gap when the thickness of the gap is small. Despite the impedance mismatch that causes TIR, surface waves propagate along the material boundary between layers 1 and 2 and attenuate into layer 2. Despite attenuation and no propagating k-vector, enough power is still present at the boundary between layer 2 and layer 3 that the resulting power can continue to propagate into layer 3. This power essentially leaks across the second boundary and continues to propagate when the thickness of the middle layer is small. As the middle layer thickness increases, nearly all of the power is attenuated and TIR occurs resulting in $R \rightarrow 1$. Analysis of Snell's law of refraction also supports the conclusion of TIR. The incident angle of the incoming wave $\theta_i = 60^o > \theta_c = 30^o$.

In Case B, the electric permittivity of the middle layer is eight times that of free-space (and Case A) resulting in a larger refractive index, which is also greater than the outer layers. Due to the greater index of refraction, TIR does not occur because $k_2 = \omega\sqrt{8\varepsilon_0\mu_0} > \beta$. The results from Figure 2 support this claim, showing an appreciable fraction of power transmitted at all middle layer thickness values analyzed (d = 0.01mm to 10mm). Interestingly, the fraction of transmitted and reflected power in Case B is periodic with the middle layer thickness d with a sinusoidal variation. The period on these sinusoidal variations in R and T is approximately d ≈ 1.4mm. The peaks of this sinusoidal variation with middle layer thickness results in a maximum fraction of transmitted power of T=1 and R=0 and a minimum of T ≈0.56 and R ≈ 0.44. Total transmission (T=1) occurs at layer thickness where the reflections are perfectly in-phase, such that there is not destructive interference.

In Case B, the k_x-vector in all three layers is purely with no imaginary component so there is no attenuation component and the wave propagates through all three layers. The variations in the fraction of transmitted and reflected power with middle layer thickness are due to in-phase and partially out-of-phase reflections between the layers.

Problem 2:

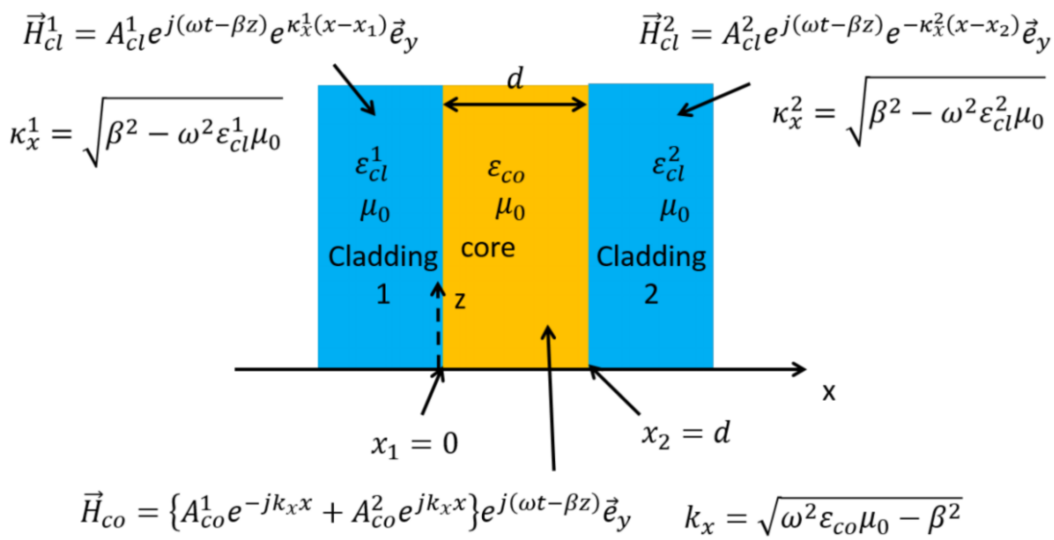

Figure 3: Diagram of EM Structure in Problem 2

$$\varepsilon_{cl}^1 = 2\varepsilon_0;\ \varepsilon_{cl}^2 = 4\varepsilon_0;\ \varepsilon_{co} = 9\varepsilon_0$$

Problem 2a.

Calculate angular frequency using the free-space wavelength of 1μm:

ω = (2* π)*(c/λ) = (2* π)*((3*10^8 m/s)/(1.0 * 10^-6 m)) = 1.885*10^15 rad/s

Calculate propagation constant of fundamental TM mode graphically using TM characteristic equation:

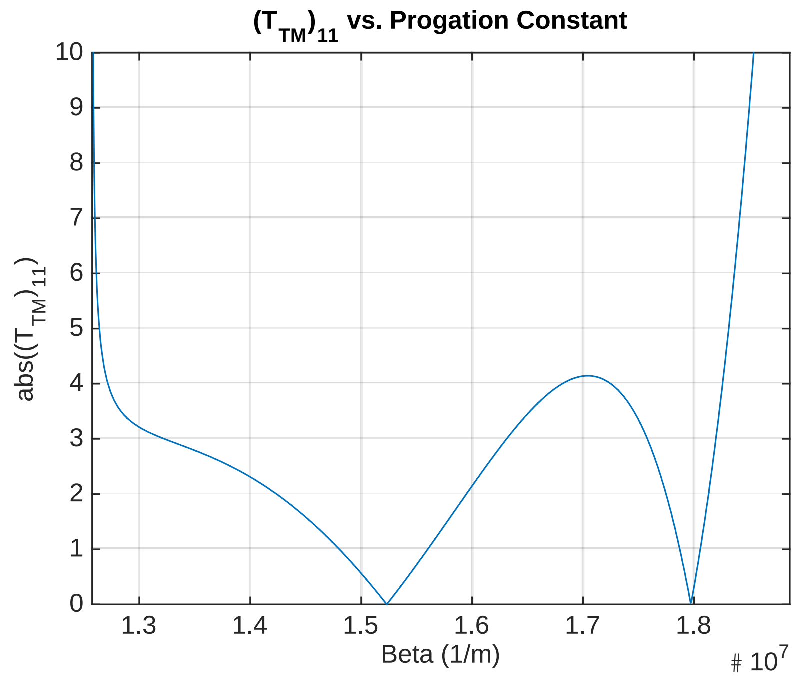

Figure 4: TM Wave Transfer Matrix 1,1 Component vs. Propagation Constant. Used to Calculate Fundamental TM Mode

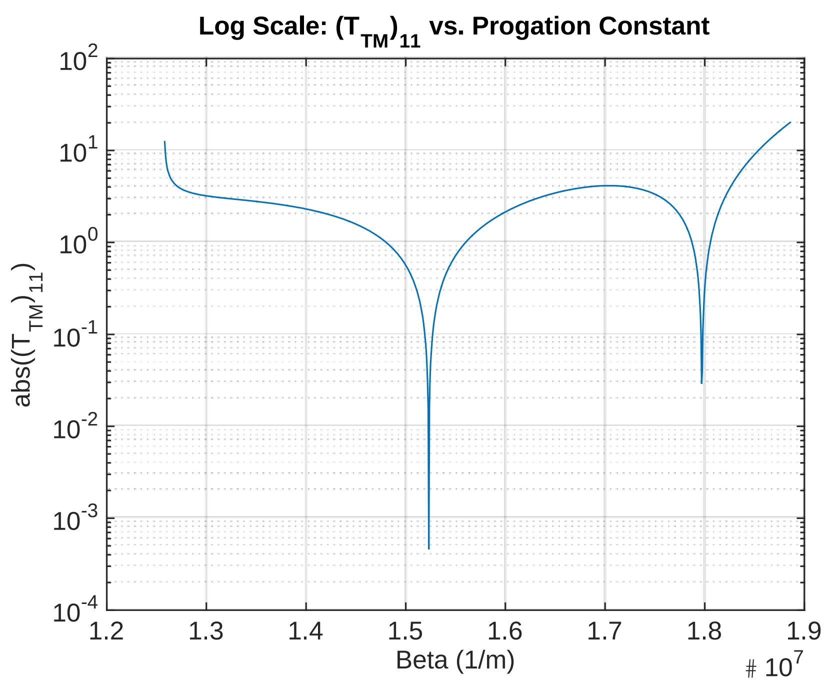

Figure 5: Log Scale Version of TM Wave Transfer Matrix 1,1 Component vs. Propagation Constant. Used to Calculate Fundamental TM Mode

β was found graphically using Matlab to find fundamental TM mode with high frequency where $(T_{TM})_{11} = 0$ (Figures 4 and 5; Matlab code is attached in the Appendix).

Characteristic Equation to Determine Waveguide Modes: $(T_{TM})_{11} = 0$

β = 1.7971*10^7 (1/m) -> ≈ 1.80 * 10^7 (1/m)

A_{cl}^{1} = 1 A/m

Cladding 1 Region: $\vec{H}_{cl}^{1} = A_{cl}^{1} e^{j(\omega t - \beta z)} e^{\kappa_x^1 (x - x_1)} \vec{e}_y$

Core Region: $\vec{H}_{co} = \{A_{co}^{1} e^{-jk_x x} + A_{co}^{2} e^{+jk_x x}\} e^{j(\omega t - \beta z)} \vec{e}_y$

Cladding 2 Region: $\vec{H}_{cl}^{2} = A_{cl}^{2} e^{j(\omega t - \beta z)} e^{-\kappa_x^2 (x - x_2)} \vec{e}_y$

$\kappa_x^1 = \sqrt{\beta^2 - \omega^2 \varepsilon_{cl}^1 \mu_0}$ = 1.5617e+07 (1/m)

$k_x = \sqrt{\omega^2 \varepsilon_{co} \mu_0 - \beta^2}$ = 5.7301e+06 (1/m)

$\kappa_x^2 = \sqrt{\beta^2 - \omega^2 \varepsilon_{cl}^2 \mu_0}$ = 1.2839e+07 (1/m)

*Note: Numerical calculations done in Matlab using code in the Appendix section

Must use boundary conditions to calculate the values for A_{co}^{1}, A_{co}^{2}, and A_{cl}^{2}:

At $x = x_1 = 0$:

Applying the boundary condition that $\vec{H}_{t1} = \vec{H}_{t2}$ such that:

$$\vec{H}_{cl}^{1} \vec{e}_y = \vec{H}_{co} \vec{e}_y$$

$$A_{cl}^{1} e^{j(\omega t - \beta z)} e^{\kappa_x^1 (x - x_1)} \vec{e}_y = \{A_{co}^{1} e^{-jk_x x} + A_{co}^{2} e^{+jk_x x}\} e^{j(\omega t - \beta z)} \vec{e}_y$$

$$A_{cl}^{1} e^{j(\omega t - \beta z)} \vec{e}_y = \{A_{co}^{1} + A_{co}^{2}\} e^{j(\omega t - \beta z)} \vec{e}_y$$

$$A_{cl}^{1} = A_{co}^{1} + A_{co}^{2} = 1 \text{ A/m}$$

Next we want to find the boundary condition for the tangential electric fields at x = 0 where $\vec{E}_{t1} = \vec{E}_{t2}$:

$$\nabla \times \vec{H} = \varepsilon \frac{d\vec{E}}{dt} = j\omega\varepsilon\vec{E} \text{ -> } \vec{E} = \frac{1}{j\omega\varepsilon} \nabla \times \vec{H} \text{ -> } \vec{E} = \frac{1}{j\omega\varepsilon}\left[-\frac{d}{dz} H_y \vec{e}_x + \frac{d}{dx} H_y \vec{e}_z\right]$$

$$E_x = \frac{-1}{j\omega\varepsilon} \frac{d}{dz} H_y$$

$$E_z = \frac{+1}{j\omega\varepsilon} \frac{d}{dx} H_y$$

$$E_y = 0$$

We want to find E_z because that is the tangential component at the boundary (at $x = x_1 = 0$):

$$\vec{E}^1_{cl\_z} = (\frac{+1}{j\omega\varepsilon^1_{cl}})(\kappa^1_x)A^1_{cl}e^{j(\omega t-\beta z)}e^{\kappa^1_x(x-x_1)}\vec{e}_z \text{ -> } \vec{E}^1_{cl\_z} = (\frac{+1}{j\omega\varepsilon^1_{cl}})(\kappa^1_x)A^1_{cl}e^{j(\omega t-\beta z)}\vec{e}_z$$

$$\vec{E}_{co\_z} = (\frac{+1}{j\omega\varepsilon_{co}})(-jk_xA^1_{co}e^{-jk_xx} + jk_xA^2_{co}e^{+jk_xx})\ e^{j(\omega t-\beta z)}\vec{e}_z \text{ ->}$$

$$\vec{E}_{co\_z} = (\frac{+1}{j\omega\varepsilon_{co}})(-jk_xA^1_{co}+ jk_xA^2_{co})\ e^{j(\omega t-\beta z)}\vec{e}_z \text{ -> } \vec{E}_{co\_z} = (\frac{k_x}{\omega\varepsilon_{co}})(A^2_{co}- A^1_{co})\ e^{j(\omega t-\beta z)}\vec{e}_z$$

$$\vec{E}^1_{cl\_z} = \vec{E}_{co\_z} \text{ -> } \left(\frac{+1}{j\omega\varepsilon^1_{cl}}\right)(\kappa^1_x)A^1_{cl}e^{j(\omega t-\beta z)}\vec{e}_z = (\frac{k_x}{\omega\varepsilon_{co}})(A^2_{co}- A^1_{co})\ e^{j(\omega t-\beta z)}\vec{e}_z \text{ ->}$$

$$\left(\frac{\kappa^1_x}{j\omega\varepsilon^1_{cl}}\right)A^1_{cl} = (\frac{k_x}{\omega\varepsilon_{co}})(A^2_{co}- A^1_{co}) \text{ -> } \left(\frac{\kappa^1_x\varepsilon_{co}}{jk_x\varepsilon^1_{cl}}\right)A^1_{cl} = A^2_{co}- A^1_{co} \text{ -> } -j\left(\frac{\kappa^1_x\varepsilon_{co}}{k_x\varepsilon^1_{cl}}\right)(1\,\frac{A}{m}) = A^2_{co}- A^1_{co} \text{ ->}$$

$$-j\left(\frac{\kappa^1_x\varepsilon_{co}}{k_x\varepsilon^1_{cl}}\right) = A^2_{co}- A^1_{co}$$

$$\frac{\varepsilon_{co}}{\varepsilon^1_{cl}} = \frac{9\varepsilon_0}{2\varepsilon_0} = \frac{9}{2}$$

$$-j\ (\frac{9}{2})\left(\frac{\kappa^1_x}{k_x}\right) = A^2_{co}- A^1_{co}$$

Along with $A^1_{cl} = A^1_{co} + A^2_{co}$ = 1 A/m from before. Now we have two equations and two unknowns to solve for A^1_{co} and A^2_{co}. Through algebra it can be shown that (*shown in handwritten appendix section in class dropbox and also calculated using Matlab):

$$A^1_{co} = \frac{1}{2} + j\ (\frac{9}{4})\left(\frac{\kappa^1_x}{k_x}\right)$$

$$A^2_{co} = \frac{1}{2} - j\ (\frac{9}{4})\left(\frac{\kappa^1_x}{k_x}\right)$$

A^1_{co} = 0.5000 + 6.1323i

A^2_{co} = 0.5000 - 6.1323i

At $x = x_2 = d = 0.5$μm:

Applying the boundary condition that $\vec{H}_{t2} = \vec{H}_{t3}$ such that:

$$\vec{H}_{co}\vec{e}_y = \vec{H}_{cl}^2\vec{e}_y$$

$$\{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t-\beta z)}\vec{e}_y = A_{cl}^2 e^{j(\omega t-\beta z)} e^{-\kappa_x^2(x-x_2)}\vec{e}_y$$

$$\{A_{co}^1 e^{-jk_x d} + A_{co}^2 e^{+jk_x d}\} = A_{cl}^2 e^{-\kappa_x^2(x_2-x_2)}$$

$$A_{co}^1 e^{-jk_x d} + A_{co}^2 e^{+jk_x d} = A_{cl}^2$$

A_{cl}^2 = (0.5 + 6.1323i)*e^(-i*(5.7301e+06)*(5.0000e-07)) + (0.5 - 6.1323i)*e^(+i*(5.7301e+06)*(5.0000e-07)) = 2.3866 A/m

A_{cl}^2 = 2.3866 A/m

Summary:

Quantity	Value
ω	1.885*10^15 rad/s
β	1.80 * 10^7 (1/m)
A_{co}^1	0.5000 + 6.1323i
A_{co}^2	0.5000 - 6.1323i
A_{cl}^2	2.3866 A/m
κ_x^1	1.5617e+07 (1/m)
k_x	5.7301e+06 (1/m)
κ_x^2	1.2839e+07 (1/m)

Table 2: Summary of Values Calculated for Problem 2A

P2b. Calculate and plot the z-component of the time-averaged Poynting vector as a function of x.

Time-Averaged Poynting Vector: $< \vec{S} > = \frac{1}{2} Re[\vec{E} \times \vec{H}^*]$

$$< \overrightarrow{S_z} > = \frac{1}{2} Re\left[E_x \times H_y^*\right] + \frac{1}{2} Re\left[E_y \times H_x^*\right]$$

Since this is a TM-wave: $H_x = 0$ -> $\frac{1}{2} Re\left[E_y \times H_x^*\right] = 0$

$$< \overrightarrow{S_z} > = \frac{1}{2} Re\left[E_x \times H_y^*\right]$$

Need to calculate the E-field component in the x-direction in each region using:

$$E_x = \frac{-1}{j\omega\varepsilon}\frac{d}{dz} H_y$$

Need to calculate the Poynting vector in the Cladding 1, Core, and Cladding 2 regions:

* Note: Additional algebra can be found in the handwritten appendix section in class dropbox

Cladding 1 Region:

$$\vec{H}_{cl}^1 = A_{cl}^1 e^{j(\omega t - \beta z)} e^{\kappa_x^1 (x - x_1)} \vec{e}_y$$

$$E_{cl\_x}^1 = \frac{-1}{j\omega\varepsilon_{cl}^1}\frac{d}{dz}\vec{H}_{cl}^1 = \frac{-1}{j\omega\varepsilon_{cl}^1} (-j\beta)\, A_{cl}^1 e^{j(\omega t - \beta z)} e^{\kappa_x^1 (x - x_1)} \vec{e}_x$$

$$E_{cl\_x}^1 = \frac{\beta}{\omega\varepsilon_{cl}^1} A_{cl}^1 e^{j(\omega t - \beta z)} e^{\kappa_x^1 (x - x_1)} \vec{e}_x$$

$$E_x \times H_y^* = (\vec{e}_x \times \vec{e}_y)\frac{\beta}{\omega\varepsilon_{cl}^1}[A_{cl}^1]^2\, e^{+2\kappa_x^1 (x - x_1)}$$

$$< \overrightarrow{S_z} > = \frac{1}{2} Re\left[E_x \times H_y^*\right] = (\vec{e}_z)\frac{\beta}{2\omega\varepsilon_{cl}^1} Re([A_{cl}^1]^2)\, e^{+2\kappa_x^1 (x - x_1)}$$

For region -0.5μm < x < 0μm

Core Region:

$$\vec{H}_{co} = \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t - \beta z)} \vec{e}_y$$

$$E_{co\_x} = \frac{-1}{j\omega\varepsilon_{co}}\frac{d}{dz}\vec{H}_{co} = \frac{-1}{j\omega\varepsilon_{co}} (-j\beta)\, \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t - \beta z)} \vec{e}_x$$

$$E_{co\_x} = \frac{\beta}{\omega\varepsilon_{co}} \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t - \beta z)} \vec{e}_x$$

$$E_x \times H_y^* = (\vec{e}_x \times \vec{e}_y)\frac{\beta}{\omega\varepsilon_{co}}[A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}]\,[A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}]^*$$

Note: Must remember that A_{co}^1 and A_{co}^2 have both real and imaginary components and must be properly conjugated.

$$E_x \times H_y^* = (\vec{e}_z)\frac{\beta}{\omega\varepsilon_{co}}[A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}]\,[A_{co}^{1*} e^{+jk_x x} + A_{co}^{2*} e^{-jk_x x}]$$

$$E_x \times H_y^* = (\vec{e}_z)\frac{\beta}{\omega\varepsilon_{co}}([A_{co}^1 A_{co}^{1*}] + \left[A_{co}^1 A_{co}^{2*} e^{-j2k_x x}\right] + \left[A_{co}^{1*} A_{co}^2 e^{+j2k_x x}\right] + [A_{co}^2 A_{co}^{2*}])$$

$$< \overrightarrow{S_z} > = \frac{1}{2} Re\left[E_x \times H_y^*\right]$$

$$< \overrightarrow{S_z} > = (\vec{e}_z)\frac{\beta}{2\omega\varepsilon_{co}}Re([A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}]\,[A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}]^*)$$

For region 0μm < x < 0.5 μm

Note: Did algebra for more complete algebraic expression, however, plotted in Matlab from this point for simplicity.

$$< \overrightarrow{S_z} > = (\vec{e}_z)\ \frac{\beta}{2\omega\varepsilon_{co}}Re([A_{co}^1 A_{co}^{1*}] + \left[A_{co}^1 A_{co}^{2*} e^{-j2k_x x}\right] + \left[A_{co}^{1*} A_{co}^2 e^{+j2k_x x}\right] + [A_{co}^2 A_{co}^{2*}])$$

Cladding 2 Region:

$$\vec{H}_{cl}^2 = A_{cl}^2 e^{j(\omega t-\beta z)} e^{-\kappa_x^2(x-x_2)}\vec{e}_y$$

$$E_{cl\_x}^2 = \frac{-1}{j\omega\varepsilon_{cl}^2}\frac{d}{dz}\vec{H}_{cl}^2 = \frac{-1}{j\omega\varepsilon_{cl}^2}\ (-j\beta)\ A_{cl}^2 e^{j(\omega t-\beta z)} e^{-\kappa_x^2(x-x_2)}\vec{e}_x$$

$$E_{cl\_x}^2 = \frac{\beta}{\omega\varepsilon_{cl}^2} A_{cl}^2 e^{j(\omega t-\beta z)} e^{-\kappa_x^2(x-x_2)}\vec{e}_x$$

$$E_x \times H_y^* = (\vec{e}_x \times \vec{e}_y)\frac{\beta}{\omega\varepsilon_{cl}^2}[A_{cl}^2]^2\ e^{-2\kappa_x^2(x-x_2)}$$

$$< \overrightarrow{S_z} > = \frac{1}{2}Re\left[E_x \times H_y^*\right] = (\vec{e}_z)\frac{\beta}{2\omega\varepsilon_{cl}^2}Re([A_{cl}^2]^2)\ e^{-2\kappa_x^2(x-x_2)}$$

For region 0.5μm < x < 1.0 μm

Summary: Time-Averaged Poynting Vector (z-component) as a function of x

$$< \overrightarrow{S_z} > = (\vec{e}_z)\begin{cases} \dfrac{\beta}{2\omega\varepsilon_{cl}^1}Re([A_{cl}^1]^2)\ e^{+2\kappa_x^1(x-x_1)}\ ;\text{Cladding 1 Region} \\ \dfrac{\beta}{2\omega\varepsilon_{co}}Re\left(\left[A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\right]\left[A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\right]^*\right);\text{Core Region} \\ \dfrac{\beta}{2\omega\varepsilon_{cl}^2}Re([A_{cl}^2]^2)\ e^{-2\kappa_x^2(x-x_2)}\ ;\text{Cladding 2 Region} \end{cases}$$

Cladding 1 Region: -0.5μm < x < 0μm

Core Region: 0μm < x < 0.5 μm

Cladding 2 Region: 0.5μm < x < 1.0 μm

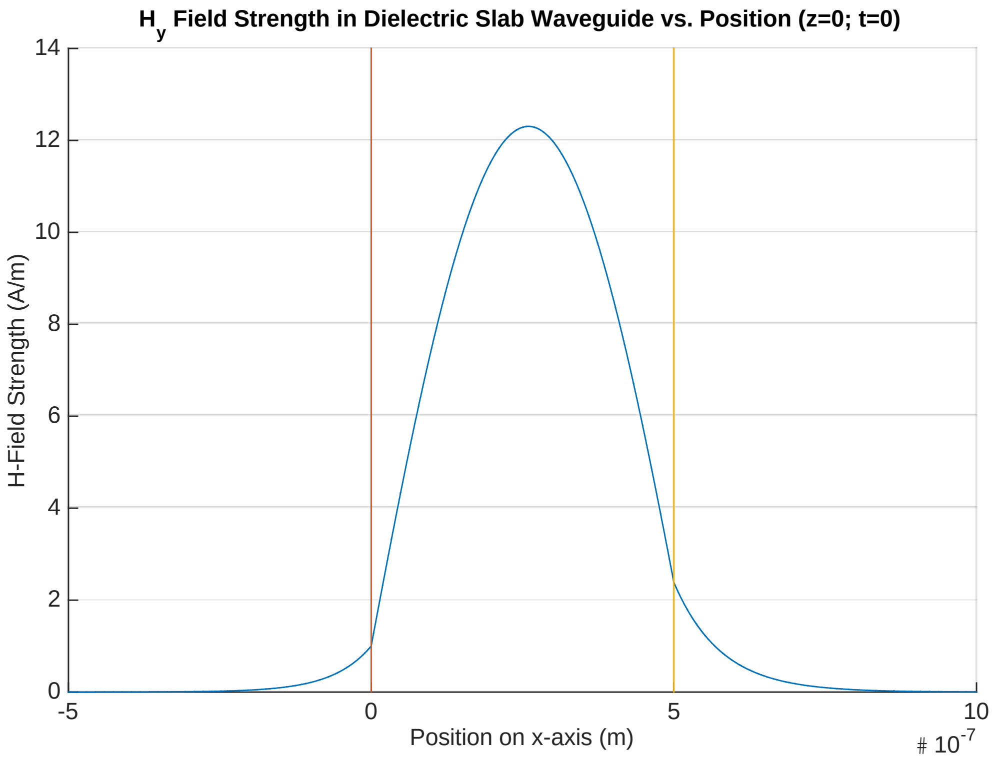

Figure 6: H-Field Strength vs. x-Coordinate Position and z=0 and t=0 for Structure in Problem 2

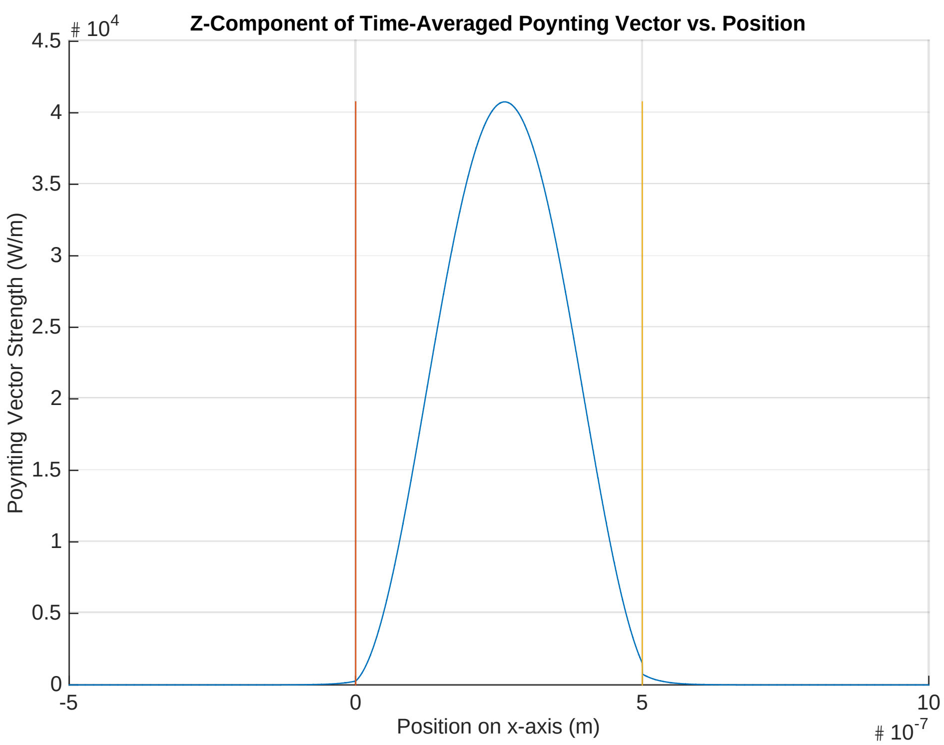

Figure 7: z-Component of the Time-Averaged Poynting Vector as a Function of x in Problem 2b

P2c.

As shown in the class notes, equivalent electric and magnetic currents can be created to represent the following situation:

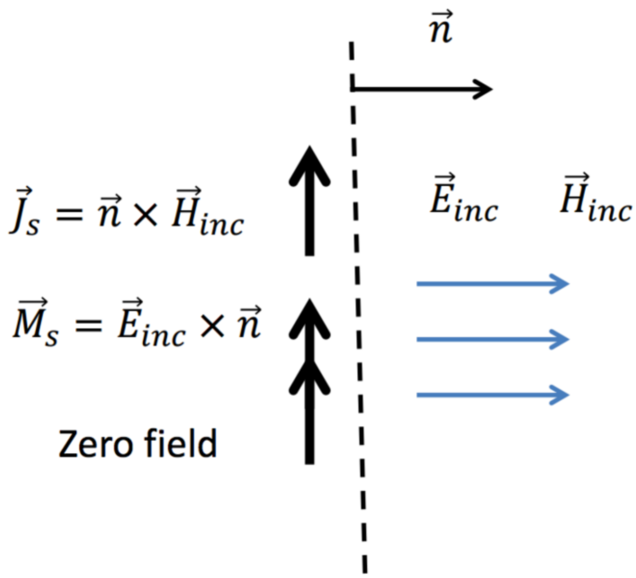

Figure 8: Depiction from class notes of electric and magnetic equivalent sources, which produce a propagating wave in the +normal direction and no EM field in the opposite direction

Equivalent electric and magnetic source currents for the fields represented in Problem 2a will be created using:

$$\vec{J}_s = \vec{n} \times \vec{H}_{inc} \text{ and } \vec{M}_s = \vec{E}_{inc} \times \vec{n}$$

$$\vec{n} = \vec{e}_z$$

These equivalent currents will produce the fundamental TM mode that propagates in the +z direction and the equivalent sources will be constrained to being on the x-y plane. The currents for each region will be solved for individually and then pieced together to create a piecewise function of the equivalent source current distribution. All EM field components must be zero for z < 0;

Cladding 1 Region: For $x < x_1 = 0$.

$$\vec{H}_{cl}^1 = A_{cl}^1 e^{j(\omega t - \beta z)} e^{\kappa_x^1 (x - x_1)} \vec{e}_y$$

$$\vec{J}_{s1} = \vec{n} \times \vec{H}_{inc} = \vec{n} \times \vec{H}_{cl}^1 = \vec{e}_z \times A_{cl}^1 e^{j(\omega t - \beta z)} e^{\kappa_x^1 (x - x_1)} \vec{e}_y$$

$$\boxed{\vec{J}_{s1} = (-\vec{e}_x)\, A_{cl}^1 e^{j(\omega t-\beta z)} e^{\kappa_x^1 (x-x_1)}}$$

In calculating the equivalent magnetic current, the z-component of the $\vec{E}_{inc}$ can be ignored because:

$$\vec{M}_s = \vec{E}_z \times \vec{e}_z = (E_z\, \vec{e}_z) \times \vec{e}_z = 0 \text{ since } (\vec{e}_z \times \vec{e}_z) = 0$$

Since it is a TM wave source only → E_y = 0

The equivalent magnetic source current must be a result of E_x component which will be calculated:

$$E_{cl\_x}^1 = \frac{-1}{j\omega\varepsilon_{cl}^1} \frac{d}{dz} \vec{H}_{cl}^1 = \frac{-1}{j\omega\varepsilon_{cl}^1}\, (-j\beta)\, A_{cl}^1 e^{j(\omega t-\beta z)} e^{\kappa_x^1 (x-x_1)} \vec{e}_x$$

$$E_{cl\_x}^1 = \frac{\beta}{\omega\varepsilon_{cl}^1} A_{cl}^1 e^{j(\omega t-\beta z)} e^{\kappa_x^1 (x-x_1)} \vec{e}_x$$

$$\vec{M}_{s1} = E_{cl\_x}^1\, \vec{e}_x \times \vec{e}_z$$

$$\vec{e}_x \times \vec{e}_z = -\vec{e}_y$$

$$\vec{M}_{s1} = \left(-\vec{e}_y\right) \frac{\beta}{\omega\varepsilon_{cl}^1} A_{cl}^1 e^{j(\omega t-\beta z)} e^{\kappa_x^1 (x-x_1)}$$

Core Region: From x = 0 to x = d = 0.5μm

$$\vec{H}_{co} = \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t-\beta z)} \vec{e}_y$$

$$\vec{J}_{s\_co} = \vec{n} \times \vec{H}_{inc} = \vec{e}_z \times \vec{H}_{co} = \vec{e}_z \times \vec{e}_y \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t-\beta z)}$$

$$\vec{e}_z \times \vec{e}_y = -\vec{e}_x$$

$$\vec{J}_{s\_co} = (-\vec{e}_x)\, \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t-\beta z)}$$

$$E_{co\_x} = \frac{-1}{j\omega\varepsilon_{co}} \frac{d}{dz} \vec{H}_{co} = \frac{-1}{j\omega\varepsilon_{co}}\, (-j\beta)\, \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t-\beta z)} \vec{e}_x$$

$$E_{co\_x} = \frac{\beta}{\omega\varepsilon_{co}} \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t-\beta z)} \vec{e}_x$$

$$\vec{M}_{s\_co} = \vec{E}_{inc} \times \vec{n} = E_{co\_x}\, \vec{e}_x \times \vec{e}_z$$

$$\vec{e}_x \times \vec{e}_z = -\vec{e}_y$$

$$\vec{M}_{s\_co} = (-\vec{e}_y)\, \frac{\beta}{\omega \varepsilon_{co}} \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t - \beta z)}$$

Cladding 2 Region: For x > x_2 = d = 0.5μm. (Very similar to Cladding 1 Region)

$$\vec{H}_{cl}^2 = A_{cl}^2 e^{j(\omega t - \beta z)} e^{-\kappa_x^2 (x - x_2)} \vec{e}_y$$

$$\vec{J}_{s2} = \vec{n} \times \vec{H}_{inc} = \vec{n} \times \vec{H}_{cl}^2 = \vec{e}_z \times A_{cl}^2 e^{j(\omega t - \beta z)} e^{-\kappa_x^2 (x - x_2)} \vec{e}_y$$

$$\boxed{\vec{J}_{s2} = (-\vec{e}_x)\, A_{cl}^2 e^{j(\omega t - \beta z)} e^{-\kappa_x^2 (x - x_2)}}$$

In calculating the equivalent magnetic current, the z-component of the $\vec{E}_{inc}$ can be ignored because:

$$\vec{M}_s = \vec{E}_z \times \vec{e}_z = (E_z\, \vec{e}_z) \times \vec{e}_z = 0 \ \text{ since } \ (\vec{e}_z \times \vec{e}_z) = 0$$

Since it is a TM wave source only → $E_y = 0$

The equivalent magnetic source current must be a result of E_x which will be calculated:

$$E_{cl\_x}^2 = \frac{-1}{j\omega \varepsilon_{cl}^2} \frac{d}{dz} \vec{H}_{cl}^1 = \frac{-1}{j\omega \varepsilon_{cl}^2} (-j\beta)\, A_{cl}^2 e^{j(\omega t - \beta z)} e^{-\kappa_x^2 (x - x_1)} \vec{e}_x$$

$$E_{cl\_x}^2 = \frac{\beta}{\omega \varepsilon_{cl}^2} A_{cl}^2 e^{j(\omega t - \beta z)} e^{-\kappa_x^2 (x - x_2)} \vec{e}_x$$

$$\vec{M}_{s2} = E_{cl\_x}^2\, \vec{e}_x \times \vec{e}_z$$

$$\vec{e}_x \times \vec{e}_z = -\vec{e}_y$$

$$\vec{M}_{s2} = \left(-\vec{e}_y\right) \frac{\beta}{\omega \varepsilon_{cl}^2} A_{cl}^2 e^{j(\omega t - \beta z)} e^{-\kappa_x^2 (x - x_2)}$$

Summary:

$$\vec{J}_s = \begin{cases} (-\vec{e}_x)\, A_{cl}^1 e^{j(\omega t - \beta z)} e^{\kappa_x^1 (x - x_1)}, & x < 0 \\ (-\vec{e}_x)\{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t - \beta z)}, & 0 \le x \le d \\ (-\vec{e}_x)\, A_{cl}^2 e^{j(\omega t - \beta z)} e^{-\kappa_x^2 (x - x_2)}, & x > d \end{cases}$$

$$\vec{M}_s = \begin{cases} (-\vec{e}_y)\ \dfrac{\beta}{\omega\varepsilon_{cl}^1}\ A_{cl}^1 e^{j(\omega t-\beta z)} e^{\kappa_x^1(x-x_1)}, & x < 0 \\ (-\vec{e}_y)\ \dfrac{\beta}{\omega\varepsilon_{co}} \{A_{co}^1 e^{-jk_x x} + A_{co}^2 e^{+jk_x x}\} e^{j(\omega t-\beta z)}, & 0 \le x \le d \\ (-\vec{e}_y)\ \dfrac{\beta}{\omega\varepsilon_{cl}^2}\ A_{cl}^2 e^{j(\omega t-\beta z)} e^{-\kappa_x^2(x-x_2)}, & x > d \end{cases}$$

Appendix: MATLAB Code Used in Calculations and to Generate Plots

Problem 1:

```
% John Hodge
% ECE 5105 Final Exam Problem 1
% 12/12/15
% TE Three layer film

clear all; clc; close all;

for z = 1:2

    d_1 = 0.01;
    d_step = 0.01;
    d_fin = 10;

    d_len = length(d_1:d_step:d_fin);

    R_arr = zeros(1, d_len);
    T_arr = zeros(1, d_len);

    x = 0;
    for d = d_1:d_step:d_fin; % distance in mm
        x = x + 1;
        d = 10^-3*d;

        theta_i = 60;
        eps_0 = 8.854*10^-12;
        mu_0 = 4*pi*10^-7;

        w = 3*10^11;

        eps_1 = 4.*eps_0;
        eps_3 = 4.*eps_0;

        eps_2A = 1.0*eps_0;
        eps_2B = 8.0.*eps_0;
        eps_2X = [eps_2A, eps_2B];

        eps_2 = eps_2X(z);

        k = w.*sqrt(eps_1.*mu_0);
```

```
    k_1x = (k)./sqrt(1+tand(theta_i).^2);

    beta = tand(theta_i).*k_1x;

    k_2x = sqrt(w.^2.*eps_2.*mu_0 - beta.^2);
    k_3x = sqrt(w.^2.*eps_3.*mu_0 - beta.^2);

    D_1_TE = [[1,1];[k_1x/mu_0, -k_1x/mu_0]];
    D_2_TE = [[1,1];[k_2x/mu_0, -k_2x/mu_0]];
    D_3_TE = [[1,1];[k_3x/mu_0, -k_3x/mu_0]];

    D_2_TE_inv = inv(D_2_TE);
    D_3_TE_inv = inv(D_3_TE);

    P_2_TE = [[exp(-1i.*k_2x.*d), 0]; [0, exp(+1i.*k_2x.*d)]];

    T_TE = [D_3_TE_inv*D_2_TE*P_2_TE]*[D_2_TE_inv*D_1_TE];
    T_TE_inv = inv(T_TE);
    T_TE_inv_11 = T_TE_inv(1,1);
    T_TE_inv_21 = T_TE_inv(2,1);


    R = abs(T_TE_inv_21/T_TE_inv_11).^2;
    R_arr(1,x) = R;

    T = (mu_0*k_3x)./(mu_0*k_1x).*1./abs(T_TE_inv_11).^2;
    T_arr(1,x) = T;


end


d_plot = d_1:d_step:d_fin;

if (z == 1)
    a = 1;
    b = 3;
elseif (z == 2)
    a = 2;
    b = 4;
end

figure(100);
subplot(2,2,a)
plot(d_plot, R_arr)
axis([d_1 d_fin 0 1])
title('Reflected Power vs. Layer Thickness', 'Fontsize', 16)
xlabel('Distance (mm)', 'Fontsize', 16)
ylabel('Reflected Power (R)', 'Fontsize', 16)
set(gca, 'FontSize', 16)

subplot(2,2,b)
plot(d_plot, T_arr)
axis([d_1 d_fin 0 1])
```

```
    title('Transmitted Power vs. Layer Thickness', 'Fontsize', 16)
    xlabel('Distance (mm)', 'Fontsize', 16)
    ylabel('Transmitted Power (T)', 'Fontsize', 16)
    set(gca, 'FontSize', 16)

    figure(200)
    subplot(2,1,a)
    plot(d_plot, R_arr, 'r--', d_plot, T_arr, 'b-')
    axis([d_1 d_fin 0 1])
    if (z == 1)
        title('Case A: Reflected Power vs. Layer Thickness', 'Fontsize', 16)
    elseif (z == 2)
        title('Case B: Reflected Power vs. Layer Thickness', 'Fontsize', 16)
    end
    xlabel('Distance (mm)', 'Fontsize', 16)
    ylabel('Reflected Power (R)', 'Fontsize', 16)
    set(gca, 'FontSize', 16)
    legend('Reflected', 'Transmitted')
    grid on;

    figure(300)
    subplot(2,1,a)
    plot(d_plot, 10*log10(R_arr), 'r--', d_plot, 10*log10(T_arr), 'b-')
    %axis([d_1 d_fin 0 1])
    if (z == 1)
        title('Case A: Reflected Power vs. Layer Thickness', 'Fontsize', 16)
    elseif (z == 2)
        title('Case B: Reflected Power vs. Layer Thickness', 'Fontsize', 16)
    end
    xlabel('Distance (mm)', 'Fontsize', 16)
    ylabel('Reflected Power (R)', 'Fontsize', 16)
    set(gca, 'FontSize', 16)
    legend('Reflected', 'Transmitted')
    grid on;

end
```

Problem 2a:

```
% John Hodge
% ECE 5105 Final Exam Problem 2a
% 12/12/15
% TM Dielectric Slab Wave Guide

clear all; clc; close all;


A_cl_1 = 1; % A/m
d = 0.5 * 10^-6; %m
mu_0 = 4*pi*10^-7; % H/m
lambda = 1.0 * 10^-6; % m
c = 3*10^8; % m/s

w = (2*pi*(c/lambda));
```

```
eps_0 = 8.854 * 10^-12; % F/m
eps_co = 9*eps_0;
eps_cl_1 = 2*eps_0;
eps_cl_2 = 4*eps_0;

beta_i = sqrt(w^2*eps_cl_2*mu_0);
beta_f = sqrt(w^2*eps_co*mu_0);
beta_pts = 1000;

T_TM_11_arr = zeros(1, beta_pts);

x = 0;
for beta = linspace(beta_i, beta_f, beta_pts)
    x = x + 1;

    kappa_x1 = sqrt(beta^2 - w^2*eps_cl_1*mu_0);
    k_x_i = sqrt(w^2*eps_cl_1*mu_0 - beta^2);

    k_x_co = sqrt(w^2*eps_co*mu_0 - beta^2);

    kappa_x2 = sqrt(beta^2 - w^2*eps_cl_2*mu_0);
    k_x_f = sqrt(w^2*eps_cl_2*mu_0 - beta^2);

    D_i_TM = [[1,1]; [k_x_i/eps_cl_1, -k_x_i/eps_cl_1]];
    D_t_TM = [[1,1]; [k_x_f/eps_cl_2, -k_x_f/eps_cl_2]];
    D_co_TM = [[1,1]; [k_x_co./eps_co, -k_x_co./eps_co]];

    D_i_TM_inv = inv(D_i_TM);
    D_t_TM_inv = inv(D_t_TM);
    D_co_TM_inv = inv(D_co_TM);

    P_co_TM = [[exp(-1i*k_x_co*d), 0]; [0, exp(+1i*k_x_co*d)]];

    T_TM = [D_t_TM_inv*D_co_TM*P_co_TM]*[D_co_TM_inv*D_i_TM];

    T_TM_11 = T_TM(1,1);
    T_TM_11_arr(x) = T_TM_11;

    T_TM_inv = inv(T_TM);


end

beta_plot = linspace(beta_i, beta_f, beta_pts);

figure;
plot(beta_plot, abs(T_TM_11_arr))
xlabel('Beta (1/m)', 'Fontsize', 16)
ylabel('abs((T_{TM})_{11})', 'Fontsize', 16)
title('(T_{TM})_{11} vs. Progation Constant', 'Fontsize', 16)
grid on;
set(gca, 'FontSize', 16)

if(max(abs(T_TM_11_arr)) > 10)
```

```
    axis([beta_i beta_f 0 10])
end

figure;
semilogy(beta_plot, abs(T_TM_11_arr))
xlabel('Beta (1/m)', 'Fontsize', 16)
ylabel('abs((T_{TM})_{11})', 'Fontsize', 16)
title('Log Scale: (T_{TM})_{11} vs. Progation Constant', 'Fontsize', 16)
grid on;
set(gca, 'FontSize', 16)
```

Problem 2b:

```
% John Hodge
% ECE 5105 Problem 2b

clear all; clc; close all;

A_cl_1 = 1; % A/m
d = 0.5 * 10^-6; %m
mu_0 = 4*pi*10^-7; % H/m
lambda = 1.0 * 10^-6; % m
c = 3*10^8; % m/s

w = (2*pi*(c/lambda))

eps_0 = 8.854 * 10^-12; % F/m
eps_co = 9*eps_0;
eps_cl_1 = 2*eps_0;
eps_cl_2 = 4*eps_0;
z = 0;
t = 0;
x_1 = 0;
x_2 = d;

beta = 1.7971 * 10^7 % 1/m

kappa_x1 = sqrt(beta^2 - w^2*eps_cl_1*mu_0)
k_x_i = sqrt(w^2*eps_cl_1*mu_0 - beta^2);

k_x_co = sqrt(w^2*eps_co*mu_0 - beta^2)

kappa_x2 = sqrt(beta^2 - w^2*eps_cl_2*mu_0)
k_x_f = sqrt(w^2*eps_cl_2*mu_0 - beta^2);

A_co_1 = 1/2 + 1i.*(9/4).*(kappa_x1/k_x_co)

A_co_2 = 1/2 - 1i.*(9/4).*(kappa_x1/k_x_co)

A_cl_2 = A_co_1.*exp(-1i.*k_x_co.*d) + A_co_2.*exp(+1i.*k_x_co.*d)

x_pts = 30000;
```

```
x = linspace(-0.5, 1, x_pts);

x = x.*(10^-6);

H_cl_1 =  A_cl_1 .* exp(1i.*(w*t - beta*z)) .* exp(kappa_x1.*(x-x_1));

H_co = (A_co_1 .* exp(-1i.* k_x_co .* x) + A_co_2 .* exp(+1i.* k_x_co .* x)) .*
exp(1i.*(w*t - beta*z));

H_cl_2 =  A_cl_2 .* exp(1i.*(w*t - beta*z)) .* exp(-kappa_x2.*(x-x_2));

H_plot = [H_cl_1(1:x_pts./3) H_co(x_pts./3 + 1:2.*x_pts./3) H_cl_2(2.*x_pts./3
+ 1:end)];

figure;
hold on;
plot(x, H_plot)
grid on;
xlabel('Position on x-axis (m)', 'FontSize', 16)
ylabel('H-Field Strength (A/m)', 'FontSize', 16)
title('H_{y} Field Strength in Dielectric Slab Waveguide vs. Position (z=0;
t=0)', 'FontSize', 16)
set(gca, 'FontSize', 16)
hold off;

hold on;
plot([x_1 x_1], [0 14]);
hold off;

hold on;
plot([x_2 x_2], [0 14]);
hold off;

% Plot Poynting Vector vs. x-Position

S_z_1 = (beta./(2.*w.*eps_cl_1)).*real(A_cl_1).^2 .* exp(+2.*kappa_x1.*(x-
x_1));



A_co_comb = (A_co_1 .* exp(-1i.* k_x_co .* x) + A_co_2 .* exp(+1i.* k_x_co .*
x));
A_co_comb_conj = conj(A_co_comb);

S_z_2 = (beta./(2.*w.*eps_cl_1)) .* real(A_co_comb .* A_co_comb_conj);

S_z_3 = (beta./(2.*w.*eps_cl_2)).*real(A_cl_2).^2 .* exp(-2.*kappa_x1.*(x-
x_2));

S_z = [S_z_1(1:x_pts./3) S_z_2(x_pts./3 + 1:2.*x_pts./3) S_z_3(2.*x_pts./3 +
1:end)];

% figure;
% plot(x(1:x_pts./3), S_z_1(1:x_pts./3))
```

```
%
% figure;
% plot(x(2.*x_pts./3 + 1:end), S_z_3(2.*x_pts./3 + 1:end))

figure;
hold on;
plot(x, S_z)
grid on;
xlabel('Position on x-axis (m)', 'FontSize', 16)
ylabel('Poynting Vector Strength (W/m)', 'FontSize', 16)
title('Z-Component of Time-Averaged Poynting Vector vs. Position', 'FontSize',
16)
set(gca, 'FontSize', 16)
hold off;

hold on;
plot([x_1 x_1], [0 max(S_z)]);
hold off;

hold on;
plot([x_2 x_2], [0 max(S_z)]);
hold off;
```