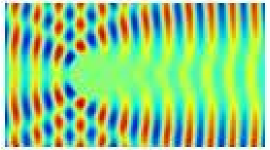


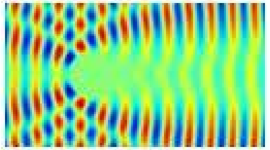
VT EM Waves Qualifying Exam Oral Presentation

John Hodge

11/16/16

Presentation Outline

i. Submarine Communicating with an Overhead Surveillance Aircraft
ii. Detection Trades & Analysis of F-XY Fighter Aircraft Against Enemy Radar
iii. Transmission Line Single-Stub Impedance Matching
iv. RCS of a Spinning Dumbbell Target Consisting of Two Small PEC Spheres
v. Radiated Fields and Power Radiated From Rotating Bar Magnet

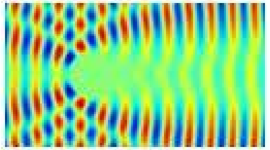


Question #1: Submarine Communicating with an Overhead Aircraft

Given Parameter	Value	Unit
Airplane Altitude	10	km
Airplane Power Transmitted	200	kW
Operating Frequency	20	kHz
Submarine Receiver Sensitivity	1	µV/m
Airplane Transmit Antenna Gain	0	dBi

Major Assumptions:

- Assuming the water is seawater with the following material properites: ε_r = 81, μ_r = 0.999991, Conductivity σ = 4 siemens/m [Obtained material properties from HFSS material library and FoAE]
- Assuming the impedance at the submarine receiver is 50 Ohms in order to convert the Rx sensitivity from an E-field strength (V/m) to a power level (W) that can be added to the RF communications link budget
- Assuming the Tx and Rx antennas are polarization matched
- Assuming that the submarine has a perfectly isotropic Rx VLF antenna to receive the signal
- Assuming the isotropic radiator has perfect efficiency (efficiency factor = 1)



Question #1: Calculate Properties of Seawater

Permittivity of Seawater: $\varepsilon = \varepsilon_r \varepsilon_0 - j\frac{\sigma}{\omega}$ = (81)*(8.854e-12 F/m) – j*(4 siemens/m)/(2 π *20000 Hz) = 7.17e-10 – j*3.18e-05 F/m

$\frac{\varepsilon''}{\varepsilon'} = \frac{\sigma}{\omega\varepsilon} = \frac{\sigma}{\omega\varepsilon_r\varepsilon_0} = \frac{4\ S/m}{(2*\pi*20000\ Hz)(81)(8.854e{-12}\frac{F}{m})} = 4.44e4 \gg 1$-> Good Conductor

$\alpha = \sqrt{\pi f \mu \sigma} = \sqrt{\pi(20000\ Hz)(4\pi * 10^{-7}\ H/m)(4\ S/m)} =$ 0.562 Np/m

$\beta = \ \alpha =$ 0.562 rad/m

$\eta_{seawater} = (1+j)\frac{\alpha}{\sigma} = \ (1+j)\frac{0.562\ \text{Np/m}}{4\ S/m}$ = (1+j)*0.1405 Ω (Ohms)

$\Gamma = \frac{\eta_{seawater} - \eta_{air}}{\eta_{seawater} + \eta_{air}} = \frac{(1+j)*0.1405 - 377}{(1+j)*0.1405 + 377}$ = -0.9993 + j*0.0007

Reflected Power @ Air-Seawater Interface: $|\Gamma|^2$ = 0.9985 = -0.0065 dB

Transmitted Power @ Air-Seawater Interface: T = 1 - $|\Gamma|^2$ = 1 - 0.9985 = 0.0015 = -28.24 dB

Transmitted E-Field: T_E = 1 + Γ = 1 + (-0.9993 + j*0.0007) = 7.0e-04 + j*7.0e-04 = 9.90e-04e^{j45^o}

Table 7-1: Expressions for α, β, η_c, u_p, and λ for various types of media.

	Any Medium	Lossless Medium ($\sigma = 0$)	Low-loss Medium ($\varepsilon''/\varepsilon' \ll 1$)	Good Conductor ($\varepsilon''/\varepsilon' \gg 1$)	Units
$\alpha =$	$\omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right]\right]^{1/2}$	0	$\frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\pi f\mu\sigma}$	(Np/m)
$\beta =$	$\omega\left[\frac{\mu\varepsilon'}{2}\left[\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}+1\right]\right]^{1/2}$	$\omega\sqrt{\mu\varepsilon}$	$\omega\sqrt{\mu\varepsilon}$	$\sqrt{\pi f\mu\sigma}$	(rad/m)
$\eta_c =$	$\sqrt{\frac{\mu}{\varepsilon'}}\left(1-j\frac{\varepsilon''}{\varepsilon'}\right)^{-1/2}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$\sqrt{\frac{\mu}{\varepsilon}}$	$(1+j)\frac{\alpha}{\sigma}$	(Ω)
$u_p =$	ω/β	$1/\sqrt{\mu\varepsilon}$	$1/\sqrt{\mu\varepsilon}$	$\sqrt{4\pi f/\mu\sigma}$	(m/s)
$\lambda =$	$2\pi/\beta = u_p/f$	u_p/f	u_p/f	u_p/f	(m)

Notes: $\varepsilon' = \varepsilon$; $\varepsilon'' = \sigma/\omega$; in free space, $\varepsilon = \varepsilon_0$, $\mu = \mu_0$; in practice, a material is considered a low-loss medium if $\varepsilon''/\varepsilon' = \sigma/\omega\varepsilon < 0.01$ and a good conducting medium if $\varepsilon''/\varepsilon' > 100$.

Conductive Seawater is Very Lossy Which Inhibits Radio Wave Propagation

Question #1: Calculated Max Depth of Communication Between Directly Overhead Aircraft and Submarine

$$A_{SA} = 4\pi R^2 = 4\pi(10{,}000\ m)^2 = 1.257 * 10^9\ m^2$$

Power Density = P_D= P_t/A_{SA} = (200,000 W)/($1.257 * 10^9\ m^2$) = 1.591*10^{-4} W/m^2

Calculate Transmitted E-Field at the Air-Seawater Interface on the Air Side:

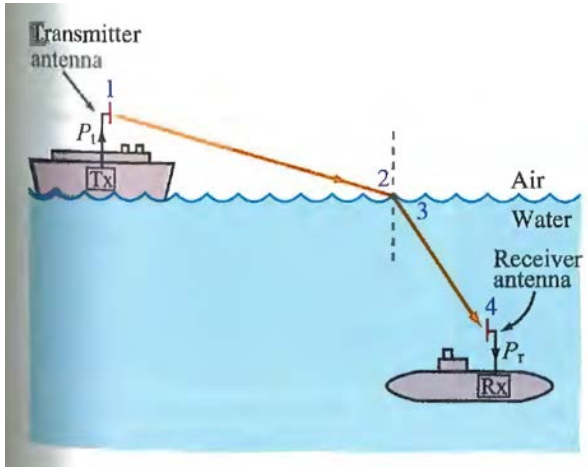
$$P_D = 1.591 * 10^{-4}\, W/m^2 = \frac{E^2}{\eta_{air}} = \frac{E^2}{377\Omega}\text{; E = 0.245 V/m}$$

Exponential Decay of E-Field Strength as Signal Propagates Through Lossy Seawater:

$$|E^t| = |\mathrm{T}_E E_0^i e^{-\alpha d}|$$

Can use this equation to calculate depth (d) at which $|E^t|$ = 1 µV/m = 10^{-6} V/m

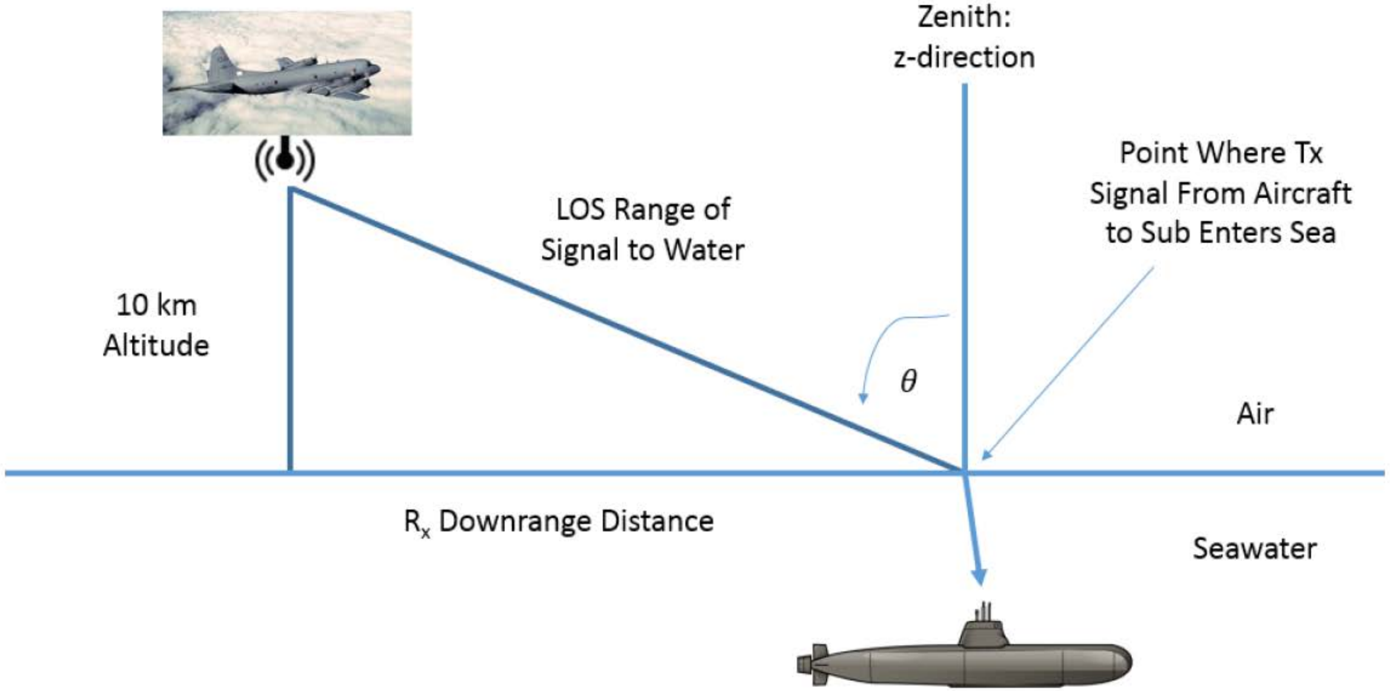
$$|E^t| = 10^{-6}\text{ V/m} = |(9.90 * 10^{-4})(0.245\ \mathrm{V/m})e^{-(0.562\ \mathrm{Np/m})(d)}| \text{ -> d = 9.77m}$$



*Note: This problem has a Tx airplane in the sky transmitting to the submarine rather than a ship, however, the general concept and method of calculation is similar

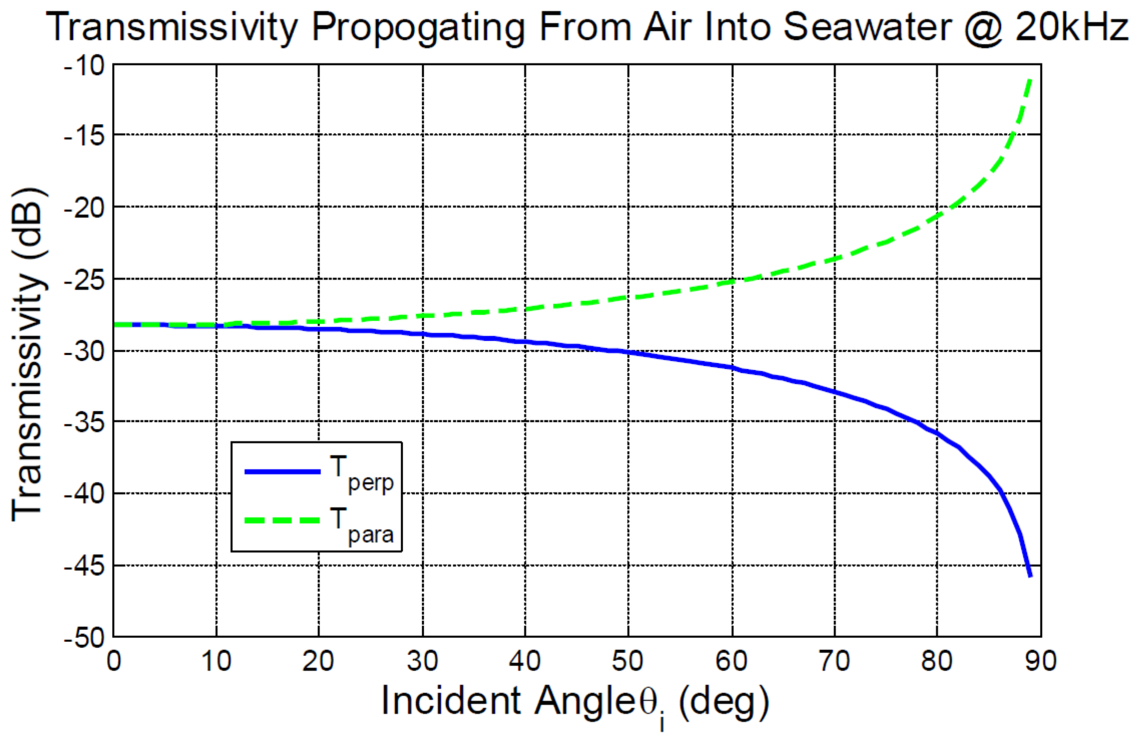
Reflection @ Air-Sea Interface & Heavy Propagation Loss Limit Underwater Comms Depth

Question #1: Diagram of Communication Between Overhead Aircraft and Underwater Submarine



Signal Must Pass Between Heavily Mismatched Air-Sea Interface; Signal Decays Quickly as it Propagates Through Lossy Conductive Seawater

Question #1: Calculate Reflected & Transmitted Fields at Air-Seawater Interface as Function of Incident Angle and Polarization

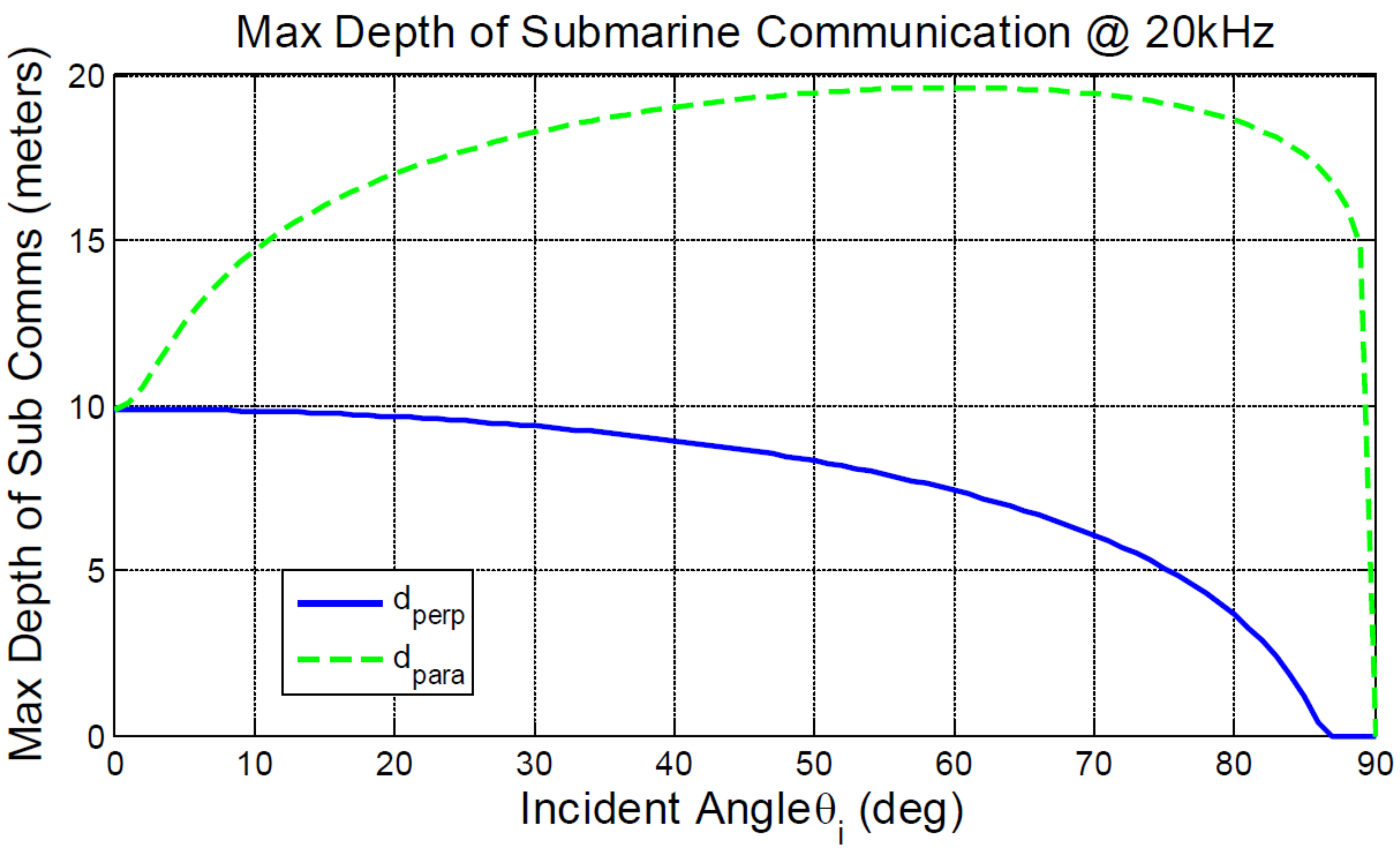


Property	Normal Incidence $\theta_i = \theta_t = 0$	Perpendicular Polarization	Parallel Polarization
Reflection coefficient	$\Gamma = \dfrac{\eta_2 - \eta_1}{\eta_2 + \eta_1}$	$\Gamma_\perp = \dfrac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$	$\Gamma_\parallel = \dfrac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$
Transmission coefficient	$\tau = \dfrac{2\eta_2}{\eta_2 + \eta_1}$	$\tau_\perp = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t}$	$\tau_\parallel = \dfrac{2\eta_2 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i}$
Relation of Γ to τ	$\tau = 1 + \Gamma$	$\tau_\perp = 1 + \Gamma_\perp$	$\tau_\parallel = (1 + \Gamma_\parallel) \dfrac{\cos\theta_i}{\cos\theta_t}$
Reflectivity	$R = \lvert\Gamma\rvert^2$	$R_\perp = \lvert\Gamma_\perp\rvert^2$	$R_\parallel = \lvert\Gamma_\parallel\rvert^2$
Transmissivity	$T = \lvert\tau\rvert^2 \left(\dfrac{\eta_1}{\eta_2}\right)$	$T_\perp = \lvert\tau_\perp\rvert^2 \dfrac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}$	$T_\parallel = \lvert\tau_\parallel\rvert^2 \dfrac{\eta_1 \cos\theta_t}{\eta_2 \cos\theta_i}$
Relation of R to T	$T = 1 - R$	$T_\perp = 1 - R_\perp$	$T_\parallel = 1 - R_\parallel$

Notes: (1) $\sin\theta_t = \sqrt{\mu_1\varepsilon_1/\mu_2\varepsilon_2}\,\sin\theta_i$; (2) $\eta_1 = \sqrt{\mu_1/\varepsilon_1}$; (3) $\eta_2 = \sqrt{\mu_2/\varepsilon_2}$; (4) for nonmagnetic media, $\eta_2/\eta_1 = n_1/n_2$.

Parallel Polarized Incident Fields Increase Comm Depth w/ Scan; Perpendicularly Polarized Incident Fields Decrease Comm Depth w/ Scan

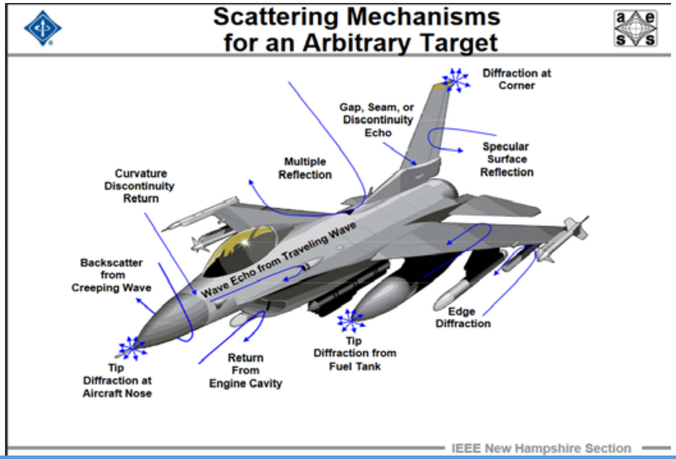
Question #1: Calculate Max Communication Depth of Submarine as Function of Aircraft Angle



Best Performance w/ Aircraft Angle 60 Degrees Off Zenith w/ Parallel E-Field Polarization

Question #2: Detection Trades & Analysis of F-XY Fighter Aircraft Against Enemy Radar

Parameter	Value	Unit
Frequency	3	GHz
Antenna Gain	38	dBi
Receiver Noise Figure (Rx NF)	6	dB
Receiver Bandwidth (Rx BW)	750	kHz
System Losses	13	dB
Antenna Transmit Loss (L_{Ant_Tx})	2	dB
$SNR_{Detection\_Min}$	8.2	dB

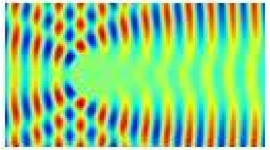


Radar Performance Blake Chart (Implementing Radar Range Equation):

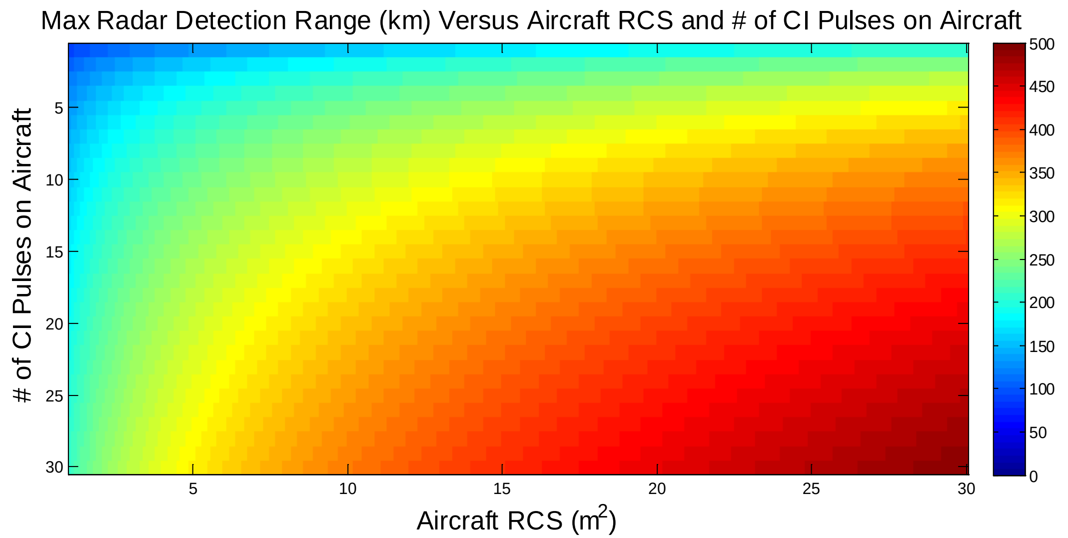
Numerator					
Quantity	Unit	Linear	dB - Gain	dB - Losses	
Pt	W	800000	59.0		
Gt	dBi		38.0		
Gr	dBi		38.0		
lambda^2	m^2	0.01	-20.0		
RCS	m^2	6	7.8		
# of Pulses		10	10.0		
Denominator					
(4*pi)^3		1984.40171		33.0	
k	w/Hz*K	1.38E-23		-228.6	
To	K	290		24.6	
B	Hz	7.50E+05		58.8	
NF	dB			6.0	
Ls	dB			13.0	
L_Tx_Ant	dB			2.0	
SNR	dB			8.2	
Totals			132.8	-83.1	
R^4 (dB)					215.9

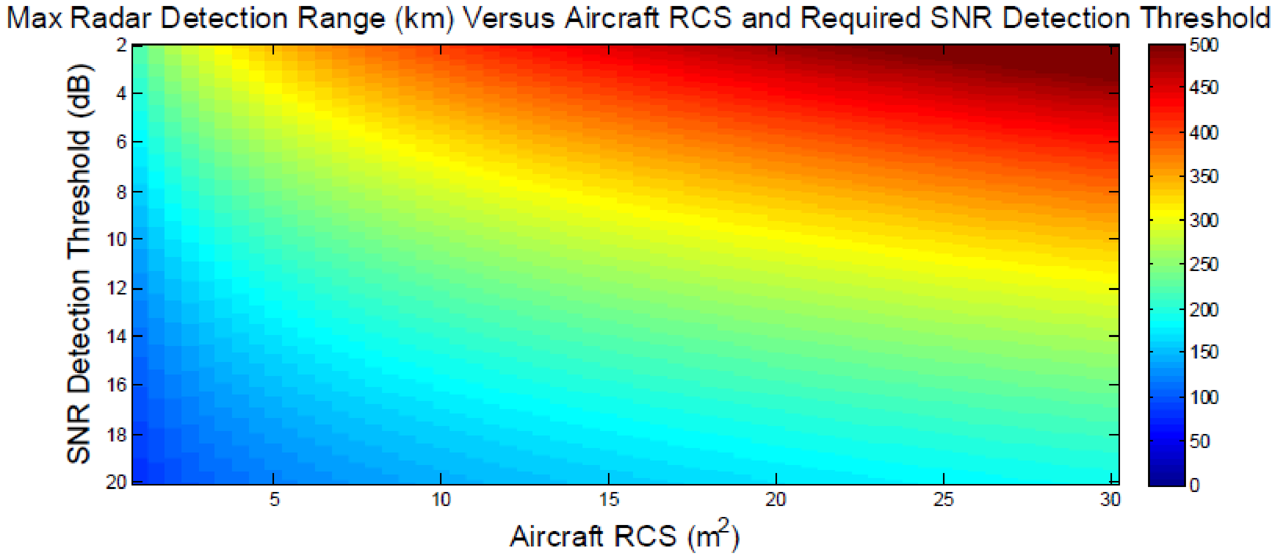
R^4 (m^4)	3.85721E+21
Range (m)	249211.6
Range (km)	249.2
Range (Nm)	134.6
Range (miles)	154.9

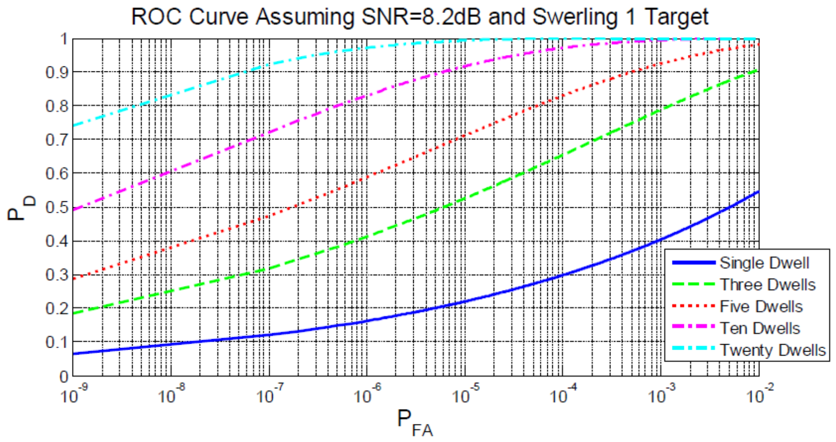
Powerful Enemy Early Warning Radar Can Detect Fighter Aircraft at Significant Ranges



Question #2: Radar Detection Parametric Trades & Analysis



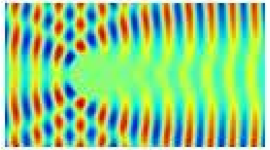




Parameter	Value
Estimated Radar Pulse Repetition Frequency (PRF)	~500 Hz (Low PRFs are typically used for long-range surveillance and search radars)
Effective Radar Antenna Aperture Size	5 m^2
Approx 3dB Antenna Beamwidth	3.1 degrees
Range Resolution	2 km
Unambiguous Range	300 km
Unambiguous Doppler Velocity	12.5 m/s

*Note: Some of these ambiguities can likely be overcome using signal processing techniques

Detectable Range of Fighter Aircraft on Enemy Radar is Dependent Mission Factors & Environment



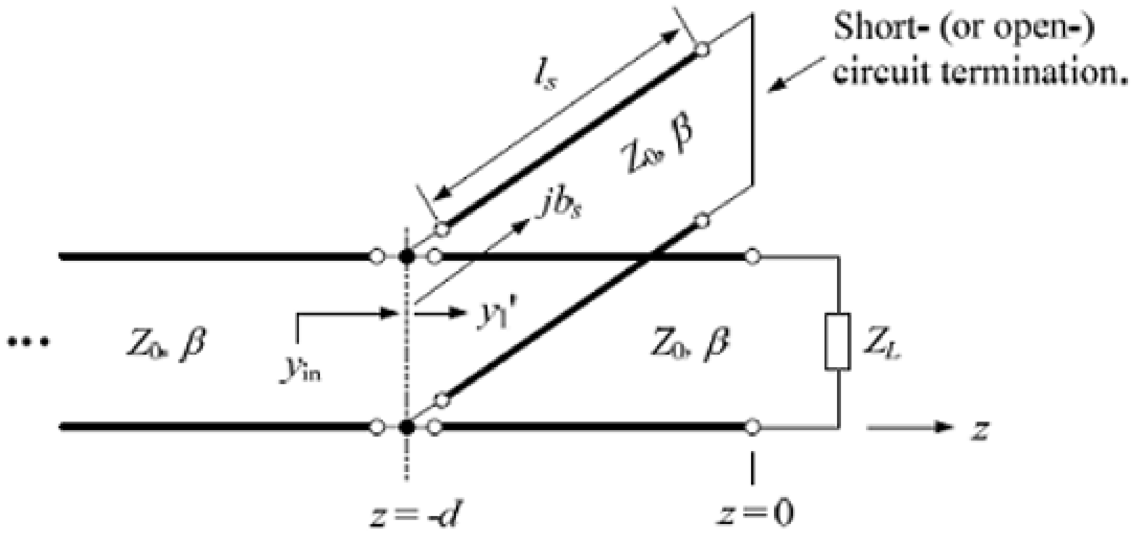
Question #2: Conclusions

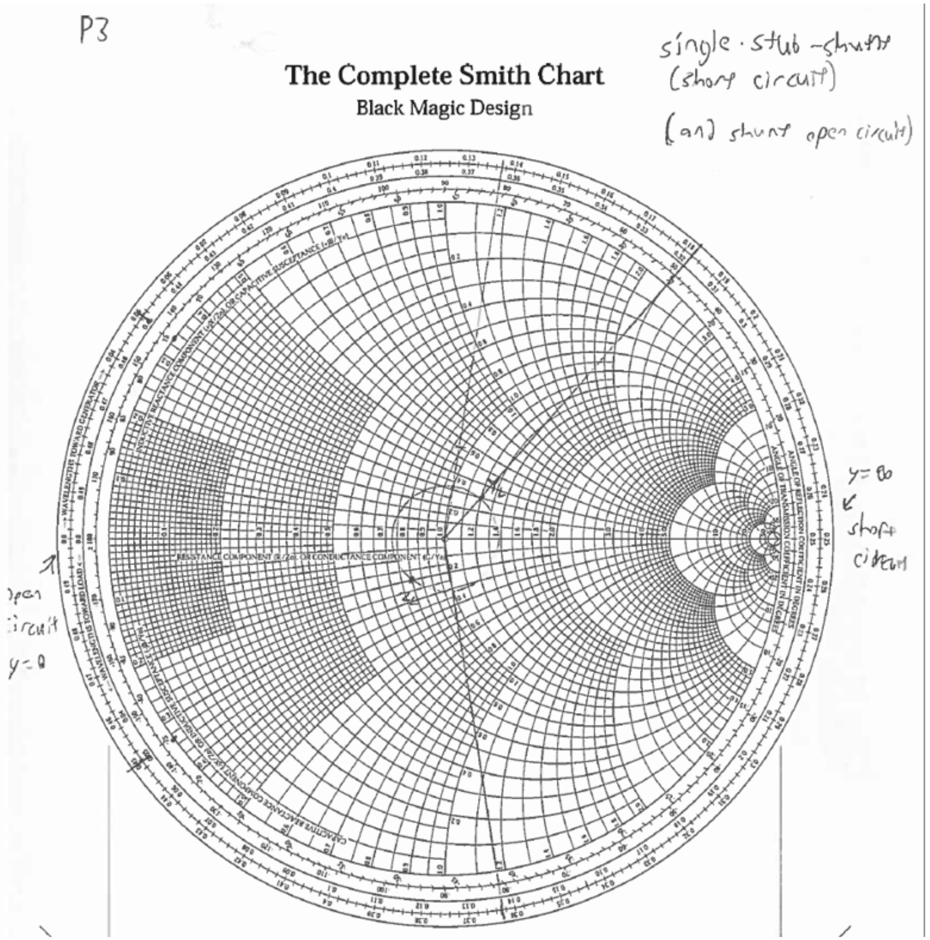
- A number of factors, including environmental factors, determine the range at which our F-XY fighter jet will first be detected by the 9SXX radar system protecting the enemy facility of interest
- Flying-in with a low RCS configuration will do a lot to reduce the radar detectability of the fighter aircraft and increase the maximum range of radar detection
- Given reasonable estimates and the operation of the search mode on the 9SXX radar, the F-XY aircraft could reasonably get within 150-200km without being detected by the enemy radar, however, using more conservative estimates, prepare for potential detection within 300km
- Only takes one RCS spike at a given angle and the enemy radar can switch into track mode, use coherent integration and increase the detectable range to potentially ~350km or more
- Given current technologies and military capabilities, stand-off jamming techniques can be used to temporarily trick the enemy radar or flood it with random noise, which will allow the aircraft to travel much closer to the target on an attack run without being detected by the enemy radar

Question #3: Transmission Line Single-Stub Impedance Matching

Quantity	Value	Units
Z_0	50	Ohms
Z_L	40 – j10	Ohms
f	800	MHz
v_p	c	m/s

Diagram of a Single-Stub RF Impedance Matching Circuit of a Load





$$Z_{in}(l) = Z_0 \frac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)}$$

Short Stub: $Z_{in}(l) = jZ_0 \tan(\beta l)$

Open Stub: $Z_{in}(l) = -jZ_0 \cot(\beta l)$

Results Using Smith Chart:
Used Gradient Optimizer to Fine-Tune Smith Chart Values for Best Matc

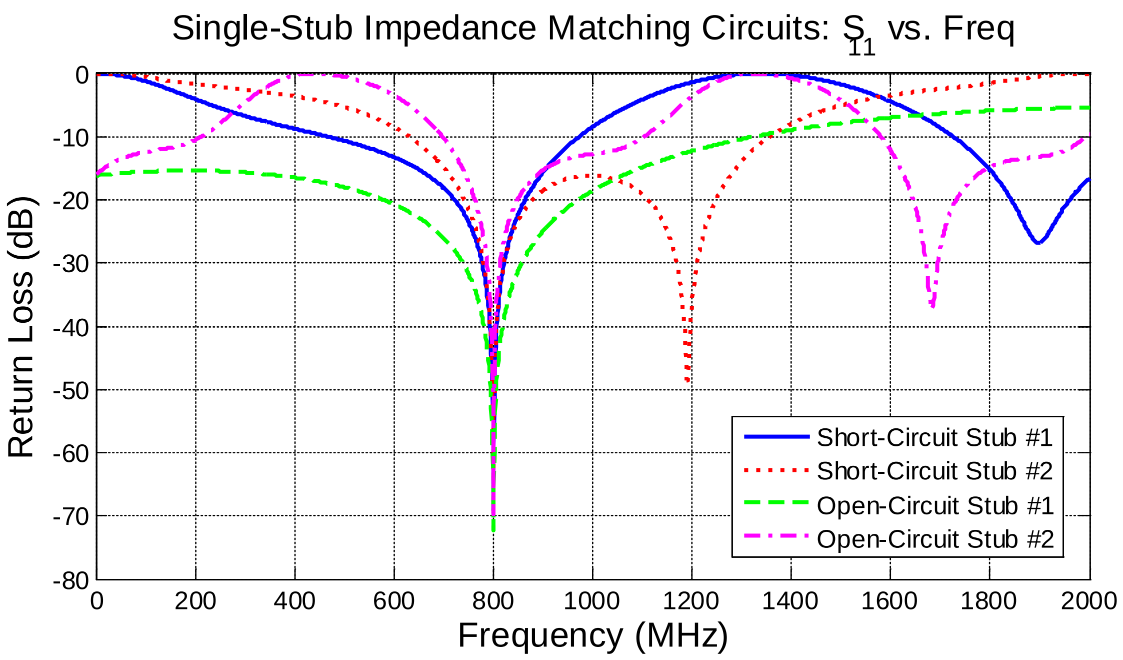
Question #3: Transmission Line Single-Stub Impedance Matching

Results Using Smith Chart:

Variable	Parallel Short-Circuit Stub (m)	Parallel Open-Circuit Stub (m)
d_1	0.067	0.067
d_2	0.168	0.168
l_1	0.111	0.019
l_2	0.074	0.166

Used Gradient Optimizer to Fine-Tune Smith Chart Values for Best Match at 800 MHz:

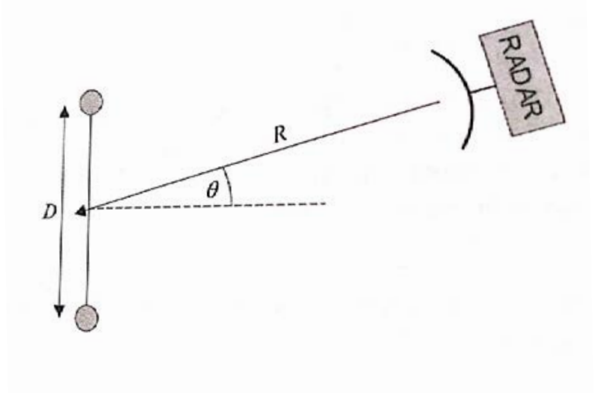
Variable	Parallel Short-Circuit Stub (m)	Parallel Open-Circuit Stub (m)
d_1	0.0689	0.0689
d_2	0.1720	0.1720
l_1	0.1120	0.0183
l_2	0.0755	0.1692

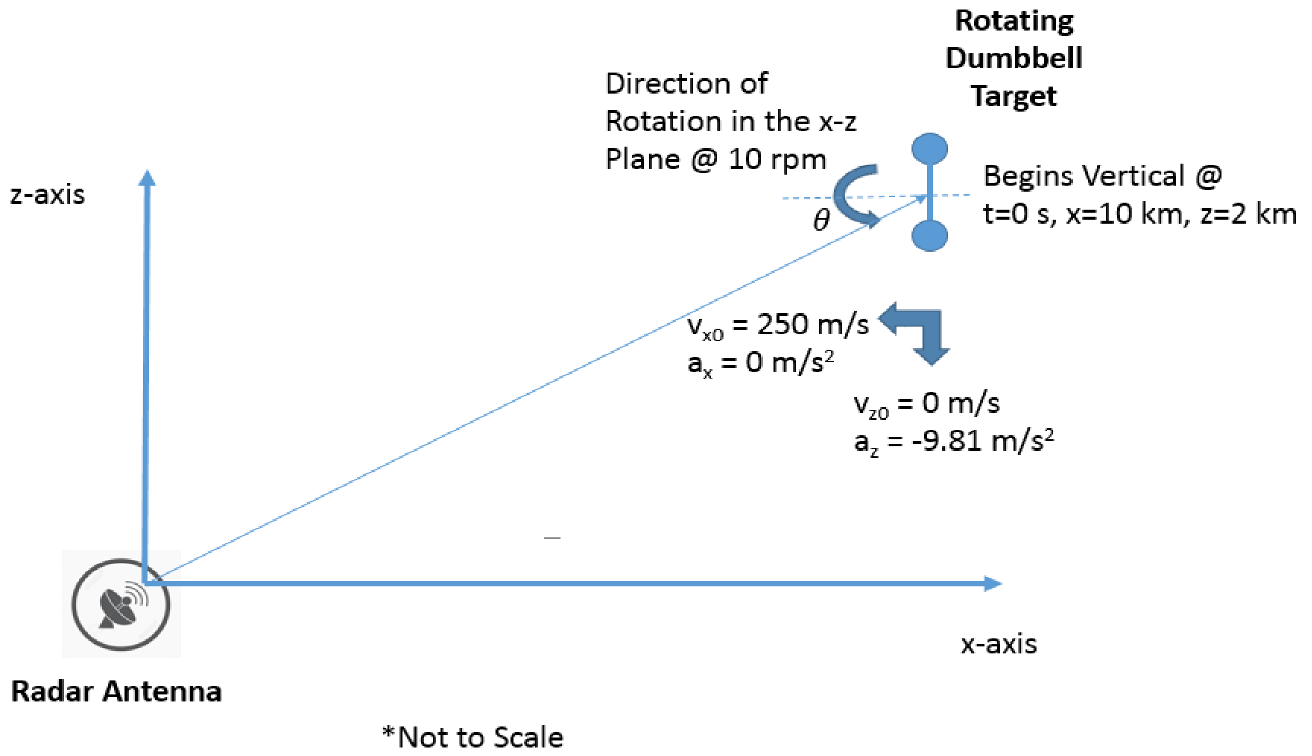


In Most Cases Open-Circuit Stub #1 is Probably Best Single-Stub Impedance Match Design Choice Due to Greater Bandwidth and Smaller Size

Question #4: RCS of a Spinning Dumbbell Target Consisting of Two Small PEC Spheres

Parameter	Value	Units
Diameter of Each Sphere	1	cm
Length of Rod Connecting Spheres	15	cm
Initial Target Height	2	km
Initial Horizontal Velocity	250	m/s
Angular Velocity of Target	10	rpm
Radar Frequency	1	GHz





Rotating Dumbbell Target is Ejected From Aircraft at Given Altitude, Velocity, and Initial Position; Target is Tracked By Ground Radar Until it Impacts the Ground

Question #4: Making Many Assumption (1/2)

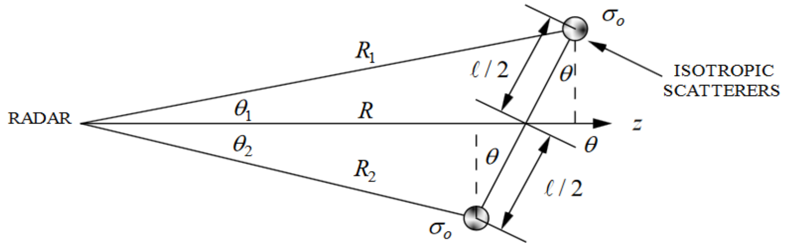
- Both spheres are PEC
- Rod connecting spheres has no RF interaction or scatterings
- Assuming that the acceleration is -9.81 m/s^2
- Assuming that atmosphere has no effect on the motion of the target (as stated in problem) resulting in no aerodynamic drag or other atmospheric forces outside of drag acting on the velocity and motion of the spinning target
- Assuming that the ground is perfectly flat (as stated in problem)
- Radar perfectly tracks the target (as stated in problem) and the antenna beamwidth is wide enough to capture the entire spinning target
- Target rotates at a steady rate (as stated in problem) and remain rotating in the vertical plane throughout the entire trajectory of the target until it hits the ground
- Assuming radar antenna is located at the origin of the coordinate system such that x=0, y=0, z=0
- Assuming that initial position of the spinning target in the relative coordinate system is x=10km, y=0, z=2km
- Assuming that the sky noise temperature and background clutter are either negligible or constant over the field of view of the target along its trajectory and will not have any effect of the radar’s relative received power as it tracks the target

Question #4: Making Many Assumption (2/2)

- Assuming that the center of mass of the target is perfectly midway between the two spheres and that the spheres are rotating around this center of mass
- Radar is a monostatic radar using the same antenna at the same fixed location at the origin for both Tx and Rx
- Target will be within the radar systems required SNR and dynamic range throughout the entire trajectory of the target until it hits the ground
- Assuming that the radar properly accounts for the Doppler shift in received power and that the electronics have a steady response over the band of interest, such that Doppler shift of the received signal of the target does not affect the received power
- Assuming that the spheres are vertically oriented at t = 0 and that the angular rotation of the spheres is in the direction that the top sphere initially rotations towards the radar while the bottom sphere rotates away from the radar
- The distance between the centers of the two spheres is d = 15cm, meaning that the connecting rod extends to the center of the spheres [In Appendix E, I add resulting plots for the case where d = 16 cm (15 cm rod + 2*0.5cm sphere radius). Was not completely sure how the geometry of this hypothetical target should be interpreted, however, results are similar]

Question #4: Calculate Effective Radar Cross-Section of Spinning Target as a Function of Observation Angle

Consider the RCS obtained from two isotropic scatterers (approximated by spheres).



Law of cosines:

$$R_1 = \sqrt{R^2 + (\ell/2)^2 - 2R(\ell/2)\cos(\theta + \pi/2)} = R\sqrt{1 + (\ell/2R)^2 + 2(\ell/2R)\sin\theta}$$

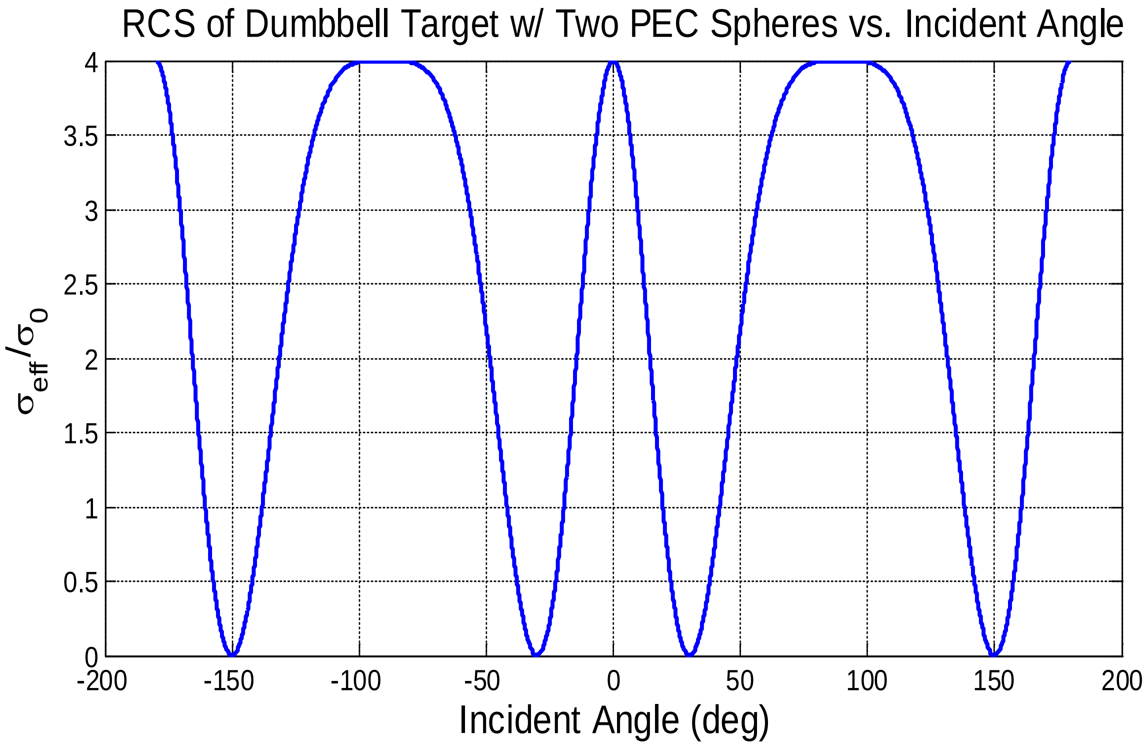
$$R_2 = \sqrt{R^2 + (\ell/2)^2 - 2R(\ell/2)\cos(\theta - \pi/2)} = R\sqrt{1 + (\ell/2R)^2 - 2(\ell/2R)\sin\theta}$$

Let $\alpha = \ell\sin\theta/R$ and note that

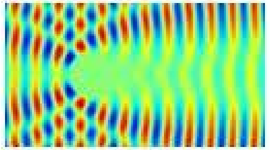
$$(1 \pm \alpha)^{1/2} = 1 \pm \frac{1}{2}\alpha \underbrace{\mp \frac{3}{8}\alpha^2 \pm \cdots}_{\text{NEGLECT SINCE } \alpha \ll 1}$$

$$R_1 \approx R + \frac{l}{2}\sin\theta\,;\; R_2 \approx R - \frac{l}{2}\sin\theta$$

$$\sigma_{eff} = 4\pi R^2|E_s|^2 = (4\pi R^2)\left(\frac{\sigma_0\cos(kl\sin\theta)^2}{\pi R^2}\right) = 4\sigma_0\cos(kl\sin\theta)^2$$



RCS of Dumbbell Target Oscillates as Radar Reflections From the PEC Spheres Come In and Out of Phase; Max RCS is 4x Value of Single Sphere



Question #4: Calculate the RCS of the Single PEC Sphere Given Size and Frequency

Calculation of the RCS for the Single PEC Sphere (σ_0):

$$f = 1\,GHz = 10^9\,Hz; \rightarrow \lambda = 0.3 \text{ meters}$$

Radius of single PEC sphere: d = 1cm = 0.01m -> a = r_{sphere} = d/2 = 0.005m

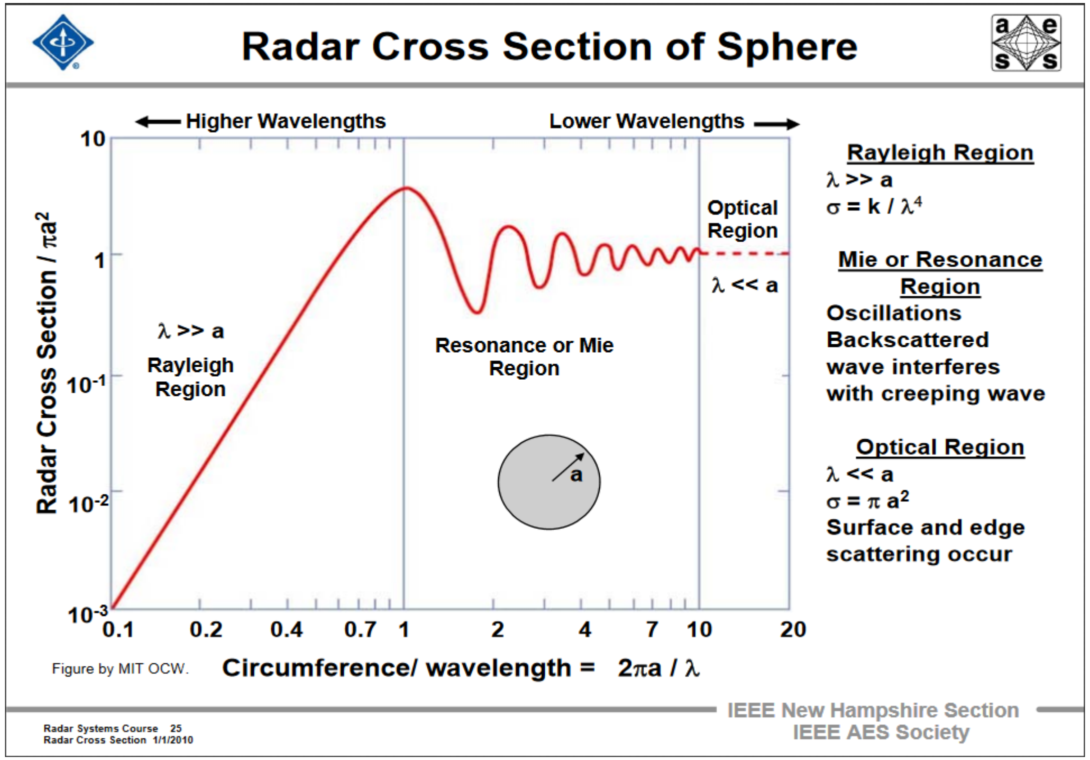
$$ka = \left(\frac{2\pi}{\lambda}\right)(0.005m) = \left(\frac{2\pi}{0.3m}\right)(0.005m) = 0.10472$$

$$\frac{a}{\lambda} = \frac{0.005m}{0.3m} = 0.0167 \ll 0.1 \text{ -> Rayleigh Scattering Region}$$

$$\lim_{\frac{a}{\lambda}\to 0} \sigma_{3D} \approx \frac{9\lambda^2}{4\pi}(ka)^6 = \frac{9(0.3m)^2}{4\pi}\left(\left(\frac{2\pi}{0.3m}\right)(0.005m)\right)^6 = 8.5 * 10^{-8}\, m^2 \propto \frac{1}{\lambda^4}$$

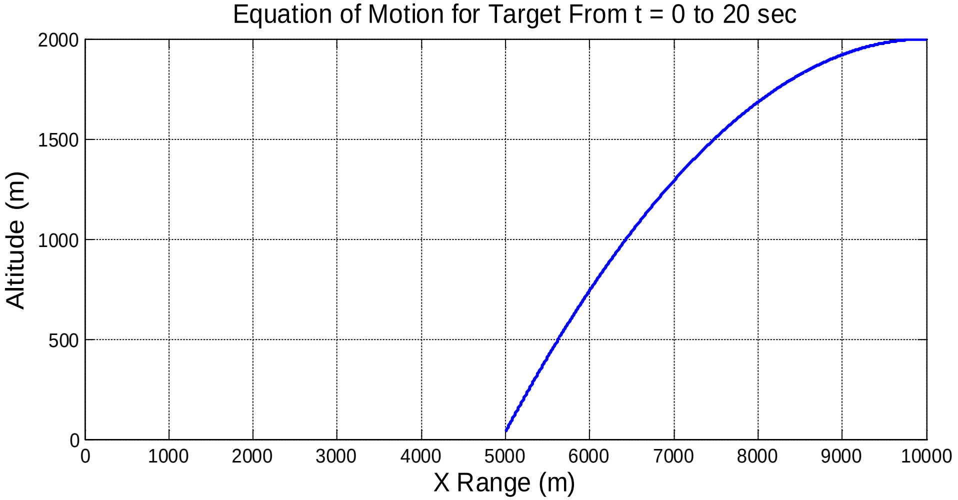
Using ECE 5106 HW Matlab Script w/ Spherical Multipole Expansion:

RCS of a single sphere with radius of a = 0.005m at f = 1 GHz: σ = 3.1566e-8 m^2 = -75.0 dBsm



RCS of Electrically Small PEC Falls Off Rapidly w/ Size Due to Rayleigh Scattering

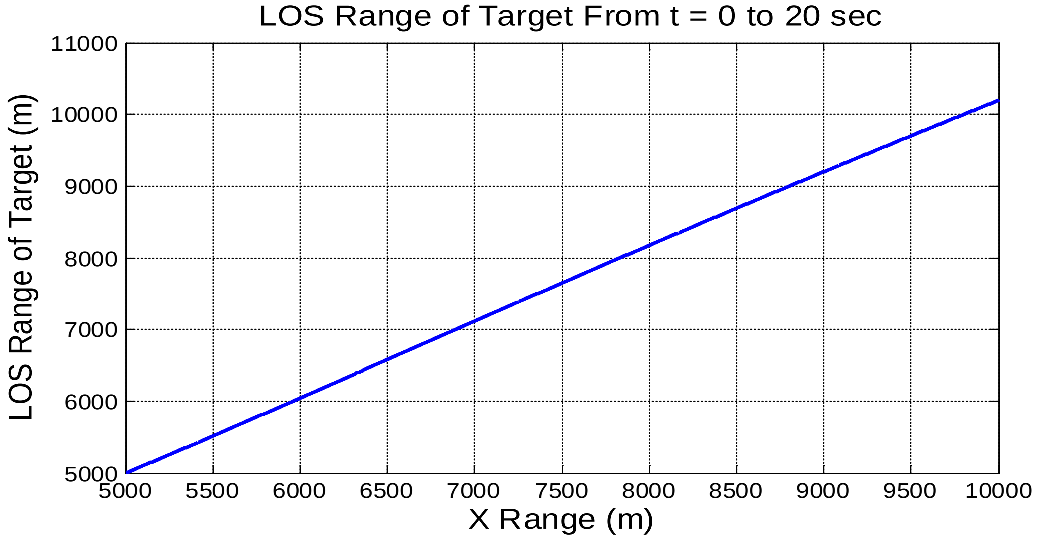
Question #4: Use Kinematics from Intro Physics to Calculate Trajectory of Spinning Target's Center of Mass (CoM) Over Time

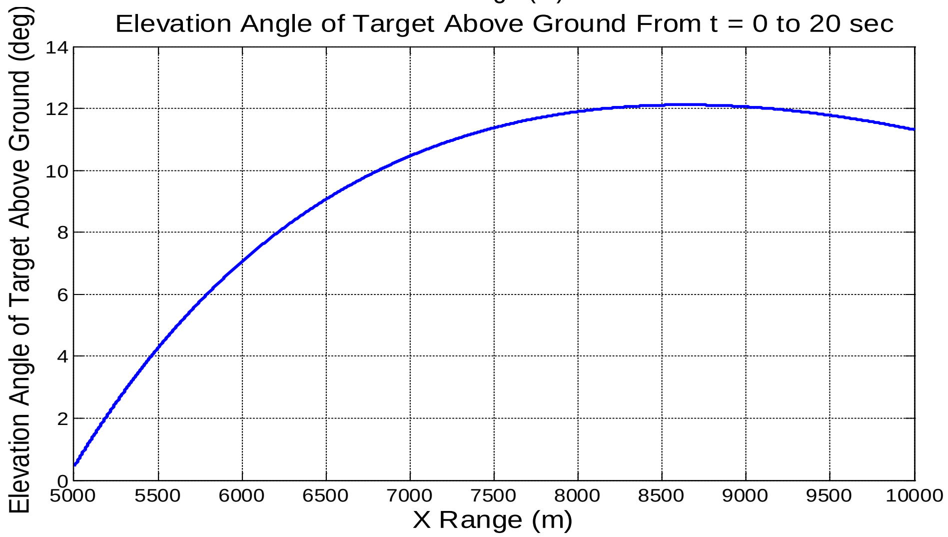


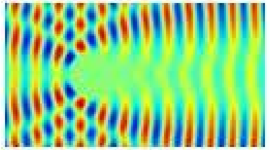
Equation of Motion for Target CoM:

$$pos = (v_{horizontal})(t)\,\overrightarrow{e_x} + (h_0 - \frac{1}{2}gt^2)\,\overrightarrow{e_z}$$

$$= \left(250\frac{m}{s}\right)(t)\,\overrightarrow{e_x} + (2000 - \frac{1}{2}\left(-9.81\frac{m}{s^2}\right)t^2)\,\overrightarrow{e_z}$$



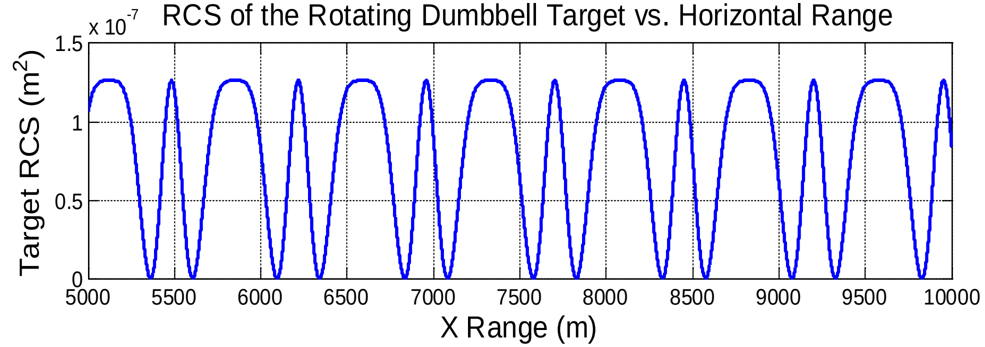


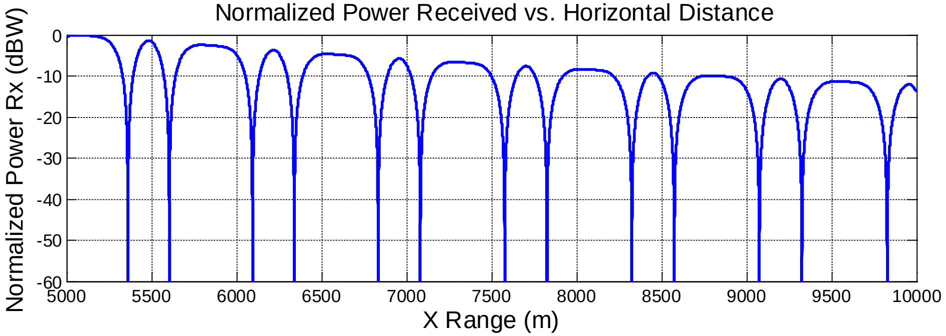


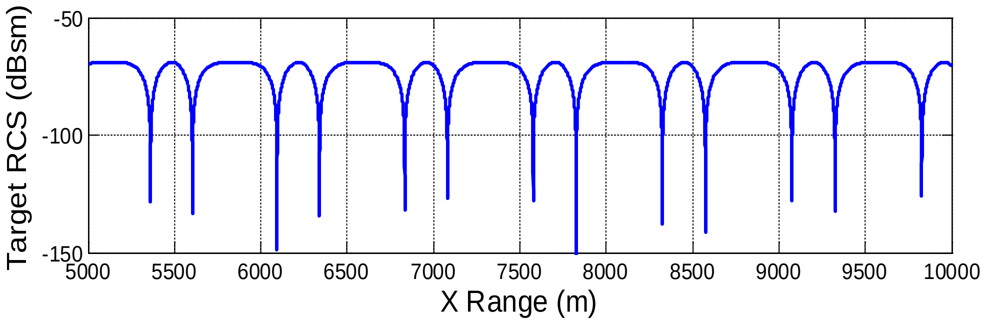
Question #4: Plot Spinning Target RCS and Normalized Reflected Power Received vs. Horizontal Distance Between Radar and Target

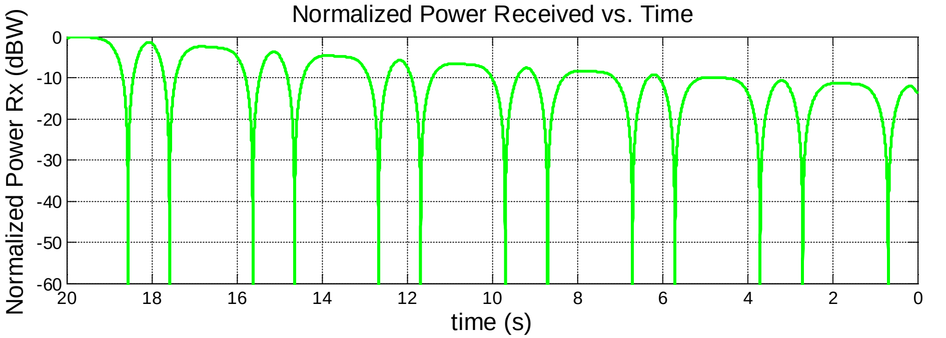
$$\theta(t) = \tan^{-1}\frac{z(t)}{x(t)} - \omega_{target}t$$

$$P_r = \frac{P_t G_t G_r \lambda^2 \sigma(t)}{(4\pi)^3 R(t)^4} = \frac{P_t G_t G_r \lambda^2 \sigma(x,z)}{(4\pi)^3 R(x,z)^4}$$





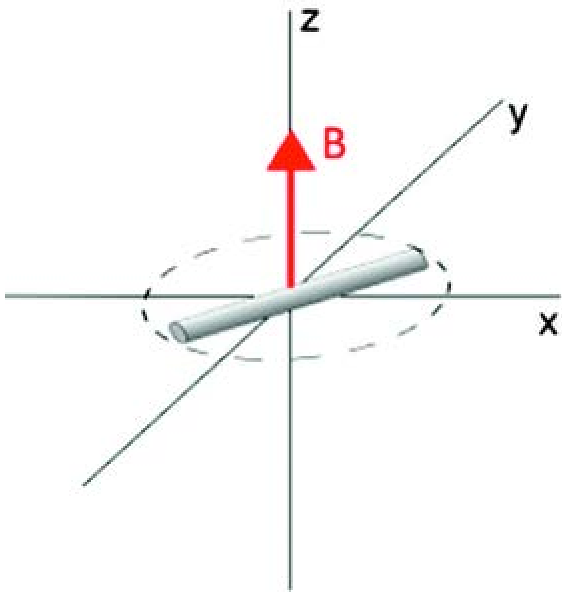


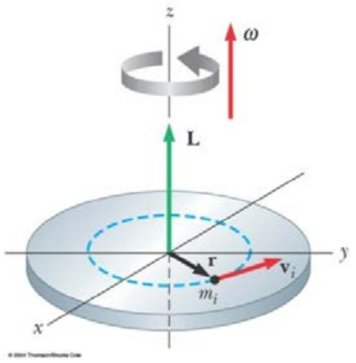


Power Received From Spinning Target Comes In & Out of Fades as Target Rotates;
Power Received Gradually Increases as Distance to Radar Decreases

Question #5: Radiated Fields and Power Radiated From Rotating Bar Magnet

Question: Consider a permanent magnet of strength of B_0 and length of L in the xy plane can rotate around its center (z axis) with an angular frequency of omega. Find the electric and magnetic field distributions in the reactive near-zone. Use far-field approximation to find the radiated power.



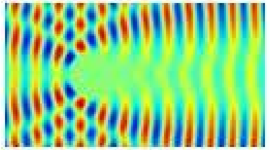


Advice From Professor Manteghi:

What you described about Griffith's method shows that he is using far-field approximation. Basically close to the rotating dipole you cannot make that assumption. I can point you to Larmor Potential which formulate the radiation from charge with arbitrary motion and then use duality. People in the field of astronomy have faced rotating magnet problem for the case of pulsars. You can check their works. They originally called it the Liénard-Wiechert Potentials.

Assumptions:

- Rotation (angular velocity) of bar magnet is slow enough that we can assume $\gamma \cong 1$ meaning that this is a non-relativistic problem since $v \ll c$ and $\gamma = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \cong 1$
- Assuming that the bar magnetic has a negligible thickness



Question #5: Initially Attempted to Solve as Two Oscillating Magnetic Dipoles Out of Phase by 90 Degrees

Original Approach using equations from Chapter 11 of Griffiths:

$$\boldsymbol{m} = m_0\left[\cos\left(\omega\left(t-\frac{r'}{c}\right)\right)\hat{x}+\sin\left(\omega\left(t-\frac{r'}{c}\right)\right)\hat{y}\right]$$

$$r' = \sqrt{r^2+\left(\frac{L}{2}\right)-2r\left(\frac{L}{2}\right)\cos\psi} = \sqrt{r^2+\left(\frac{L}{2}\right)-rL\sin\theta\cos\varphi'}$$

$$\cos\psi = \sin\theta\cos\varphi'$$

$$\boldsymbol{A}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\boldsymbol{r}}{|\boldsymbol{r}|^3}$$

$$\boldsymbol{B}\,(\boldsymbol{r}) = \nabla\times\boldsymbol{A}(\boldsymbol{r}) = \nabla\times\left(\frac{\mu_0}{4\pi}\frac{\boldsymbol{m}\times\boldsymbol{r}}{|\boldsymbol{r}|^3}\right) = \frac{\mu_0}{4\pi}\left(\frac{3\boldsymbol{r}(\boldsymbol{m}\cdot\boldsymbol{r})}{|\boldsymbol{r}|^5}-\frac{\boldsymbol{m}}{|\boldsymbol{r}|^3}\right)$$

$$\psi(\boldsymbol{r}) = \frac{\boldsymbol{m}\cdot\boldsymbol{r}}{|\boldsymbol{r}|^3}$$

Using this approach, I was able to calculate the Radiate Power for the Rotating Magnetic Dipole to be:

$$\langle P_{magnetic}\rangle = \frac{\mu_0 m_0^2\omega_0^4}{6\pi c^3}$$

This result is twice the value of a single oscillating magnetic dipole as stated in Griffiths equation 11.40 on pg. 454:

$$\langle P_{magnetic}\rangle = \frac{\mu_0 m_0^2\omega_0^4}{12\pi c^3}$$

Assumptions Made By Griffiths in This Approach Do Not Appear to Accurately Capture Near-Fields; Appears that Different Approach is Needed

Question #5: Use Liénard-Wiechert Potentials and Position Vector to Calculate Scalar and Vector Potentials

In Jackson Electrodynamics Section 14.1 on pg.663 on Liénard-Wiechert Potentials and Fields for a Point Charge, the following quantities are defined:

$\hat{n}$ is a unit vector in the direction of $\vec{x} - \vec{r}(\tau)$ (where $\vec{x}$ is the observation point) and $\vec{\beta} = \frac{\vec{v}(\tau)}{c}$ where $\vec{v}(\tau) = (\frac{d}{dt})\vec{r}(\tau)$

$$R = |\vec{\boldsymbol{x}} - \vec{\boldsymbol{r}}(\tau)|$$

Scalar Potential: $\Phi(\vec{\mathbf{x}}, \mathrm{t}) = \left[\frac{e}{(1 - \vec{\boldsymbol{\beta}} \cdot \hat{\boldsymbol{n}})\, R}\right]_{ret}$

Vector Potential: $\mathbf{A}(\vec{\mathbf{x}}, \mathrm{t}) = \left[\frac{e\vec{\boldsymbol{\beta}}}{(1 - \vec{\boldsymbol{\beta}} \cdot \hat{\boldsymbol{n}})\, R}\right]_{ret}$

Using this result for our position vector:

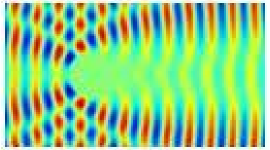
$$\vec{\boldsymbol{r}}(t) = a[\hat{x}\cos(\omega_0 t) + \hat{y}\sin(\omega_0 t)]$$

$$\vec{\boldsymbol{v}}(t) = \left(\frac{d}{dt}\right)\vec{r}(t) = (a\omega_0)[-\hat{x}\sin(\omega_0 t) + \hat{y}\cos(\omega_0 t)]$$

$\vec{\boldsymbol{\beta}} = \frac{\vec{\boldsymbol{v}}(\tau)}{c} = (\frac{a\omega_0}{c})[-\hat{x}\sin(\omega_0 t) + \hat{y}\cos(\omega_0 t)]$ (velocity term)

$\dot{\boldsymbol{\beta}} = (\frac{d}{dt})\vec{\boldsymbol{\beta}} = (\frac{-a\omega_0^2}{c})[\hat{x}\cos(\omega_0 t) + \hat{y}\sin(\omega_0 t)]$ (acceleration term)





Question #5: Choose Observation Point to Calculate Liénard-Wiechert Potentials

Observation Point: $\vec{\boldsymbol{x}} = z\hat{z}$

*Note: This observation point is independent of the charge's position, since it is always at the same distance from a point on the z-axis.

Additionally: $\vec{\boldsymbol{x}} = z\hat{z} = (r\ \cos\theta)\hat{z}$ since $z = r\ \cos\theta$

$$\vec{\boldsymbol{x}} - \vec{\boldsymbol{r}}(\tau) = z\hat{z} - a[\hat{x}\cos(\omega_0 t) + \hat{y}\sin(\omega_0 t)] = z\hat{z} - \hat{x}(a)\cos(\omega_0 t) - \hat{y}(a)\sin(\omega_0 t)$$

$$R = |\vec{\boldsymbol{x}} - \vec{\boldsymbol{r}}(\tau)| = \sqrt{z^2 + a^2}$$

$$\hat{\boldsymbol{n}} = \frac{z\hat{z} - \hat{x}(a)\cos(\omega_0 t) - \hat{y}(a)\sin(\omega_0 t)}{\sqrt{z^2 + a^2}}$$

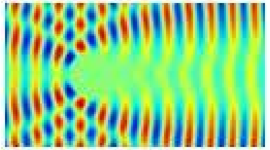
$$\vec{\boldsymbol{\beta}} \cdot \hat{\boldsymbol{n}} = \left(\frac{a\omega_0}{c\sqrt{z^2+a^2}}\right)[(a)\cos(\omega_0 t)\sin(\omega_0 t) - (a)\cos(\omega_0 t)\sin(\omega_0 t)] = 0$$

$$\Phi(\vec{\mathbf{x}}, \mathrm{t}) = \left[\frac{e}{(1 - 0)\,R}\right]_{ret} = \left[\frac{e}{\sqrt{z^2 + a^2}}\right]_{ret}$$

$$\mathbf{A}(\vec{\mathbf{x}}, \mathrm{t}) = \left[\frac{e\vec{\boldsymbol{\beta}}}{(1 - \vec{\boldsymbol{\beta}} \cdot \hat{\boldsymbol{n}})\,R}\right]_{ret} = \left[\left(\frac{ea\omega_0}{c\sqrt{z^2 + a^2}}\right)[-\hat{x}\sin(\omega_0 t) + \hat{y}\cos(\omega_0 t)]\right]_{ret}$$

$$\mathbf{B}(\vec{\mathbf{x}}, \mathrm{t}) = \nabla\, x\, \mathbf{A}(\vec{\mathbf{x}}, \mathrm{t})$$

$$\mathbf{E}(\vec{\mathbf{x}}, \mathrm{t}) = -\nabla\Phi(\vec{\mathbf{x}}, \mathrm{t}) - \frac{1}{c}\frac{\mathrm{d}}{\mathrm{dt}}\mathbf{A}(\vec{\mathbf{x}}, \mathrm{t})$$



Question #5: Use Liénard–Wiechert Potentials to Calculate Reactive Near-Fields Due to Moving Electric Charge

To take a more rigorous approach, I also attempt to calculate the near-fields using the more general approach using the Liénard–Wiechert potential field equations (14.13-14.14) in Section 14.1 of Jackson as follows:

$$\mathbf{E}(\mathbf{r},t) = q\left(\frac{\mathbf{n}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\mathbf{n})^3R^2}\right)_{\text{ret}} + \frac{q}{c}\left(\frac{\mathbf{n}\times[(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}]}{(1-\boldsymbol{\beta}\cdot\mathbf{n})^3R}\right)_{\text{ret}}$$

and

$$\mathbf{B} = \mathbf{n}\times\mathbf{E},$$

Since the magnetic rod is rotating very slowly compared to the speed of light, we can assume $\gamma = 1$ as noted above for this nonrelativistic accelerated charge problem. As Jackson notes, this equation naturally splits the fields into "velocity fields," which are independent of acceleration and "acceleration fields," which are linearly dependent on $\dot{\boldsymbol{\beta}}$. Jackson also notes the velocity fields are essentially static fields, which fall off as R^{-2}, whereas the acceleration fields are typical radiation fields, both E and H being transverse for the radius vector and varying as R^{-1}.

Question #5: Use Liénard–Wiechert Potentials to Calculate Electric Field in the Reactive Near-Field

Now that we have calculated both the velocity fields and acceleration fields for the rotating electric charge problem, we will add the two terms together to calculate the electric, which encompasses the near-fields, since we have not made any far-field only assumptions to this point:

$$\mathbf{E}(\vec{\mathbf{x}},\mathrm{t}) = \mathbf{E}_{\boldsymbol{v}}(\vec{\mathbf{x}},\mathrm{t}) + \mathbf{E}_{\boldsymbol{a}}(\vec{\mathbf{x}},\mathrm{t}) = e\left[\frac{\hat{\boldsymbol{n}}-\vec{\boldsymbol{\beta}}}{\gamma^2(1-\vec{\boldsymbol{\beta}}\cdot\hat{\boldsymbol{n}})^3 R^2}\right]_{ret} + \frac{e}{c}\left[\frac{\hat{\boldsymbol{n}}\; x\;\{(\hat{\boldsymbol{n}}-\vec{\boldsymbol{\beta}})\boldsymbol{x}\;\dot{\boldsymbol{\beta}}\}}{\gamma^2(1-\vec{\boldsymbol{\beta}}\cdot\hat{\boldsymbol{n}})^3 R}\right]_{ret}$$

x-component:

$$[\mathbf{E}(\vec{\mathbf{x}},\mathrm{t})]_{\boldsymbol{x}} = \left(\frac{ae}{c^2}\right)\left[\left(\frac{\omega_0(a^2\omega_0+c)}{(z^2+a^2)}\right)\sin(\omega_0 t) + \left(\frac{\omega_0^2 z^2 - c^2}{(z^2+a^2)^{3/2}}\right)\cos(\omega_0 t)\right]$$

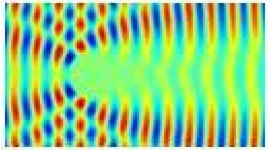
y-component:

$$[\mathbf{E}(\vec{\mathbf{x}},\mathrm{t})]_{\boldsymbol{y}} = \left(\frac{ae}{c^2}\right)\left[-\left(\frac{\omega_0(a^2\omega_0+c)}{(z^2+a^2)}\right)\cos(\omega_0 t) + \left(\frac{\omega_0^2 z^2 - c^2}{(z^2+a^2)^{3/2}}\right)\sin(\omega_0 t)\right]$$

z-component:

$$[\mathbf{E}(\vec{\mathbf{x}},\mathrm{t})]_{\boldsymbol{z}} = \frac{ez(a^2\omega_0^2+c^2)}{c^2\,(z^2+a^2)^{3/2}}$$





Question #5: Use Result From Electric Field to Calculate the Magnetic B-Field

Now knowing the result for the electric field generated by the rotating electric charge in the near-field, it is now possible to calculate the **B** field using equation 14.13 from Jackson as follows:

$$\boldsymbol{B} = \ [\hat{\boldsymbol{n}}\, x\, \boldsymbol{E}]_{ret}$$

x-component:

$$B_x = \left(\frac{ae}{c^2}\right)\left[\left(\frac{z\omega_0(a^2\omega_0 + c)}{(z^2+a^2)^{3/2}}\right)\cos(\omega_0 t) - \left(\frac{\omega_0^2 z}{(z^2+a^2)}\right)\sin(\omega_0 t)\right]$$

y-component:

$$B_y = \left(\frac{ae}{c^2}\right)\left[\left(\frac{z\omega_0(a^2\omega_0 + c)}{(z^2+a^2)^{3/2}}\right)\sin(\omega_0 t) + \left(\frac{\omega_0^2 z}{(z^2+a^2)}\right)\cos(\omega_0 t)\right]$$

z-component:

$$B_z = \left(\frac{ae}{c^2}\right)\left(\frac{a\omega_0(a^2\omega_0 + c)}{(z^2+a^2)^{3/2}}\right)$$

Question #5: Use EM Duality Principle to Transform Fields Due to Electric Charge to Fields Due to Magnetic Charge

Now that we have calculated the E and B fields in the near-reactive zone for the rotating electric dipole problem, we can now use the duality theorem to convert our results from electric charge to magnetic charge for the fields insert constants for the rotating bar magnet in this problem. Using duality, we will make the following transformations using the duality transformations listed in Griffiths Electrodynamics equation 7.68 with $\alpha = 90\ deg$ as follows:

$$\vec{E}^{'} = c\vec{B}$$

$$\vec{B}^{'} = -\frac{\vec{E}}{c}$$

$$q_m^{'} = -cq_e \rightarrow\ m^{'} = -ce$$ (to keep our notation consistent)

(a) Show that Maxwell's equations with magnetic charge (Eq. 7.43) are invariant under the **duality transformation**

$$\left.\begin{aligned} \mathbf{E}' &= \mathbf{E}\cos\alpha + c\mathbf{B}\sin\alpha, \\ c\mathbf{B}' &= c\mathbf{B}\cos\alpha - \mathbf{E}\sin\alpha, \\ cq_e' &= cq_e\cos\alpha + q_m\sin\alpha, \\ q_m' &= q_m\cos\alpha - cq_e\sin\alpha, \end{aligned}\right\} \quad (7.68)$$

where $c \equiv 1/\sqrt{\epsilon_0\mu_0}$ and α is an arbitrary rotation angle in "**E**/**B**-space." Charge and current densities transform in the same way as q_e and q_m. [This means, in particular, that if you know the fields produced by a configuration of *electric* charge, you can immediately (using $\alpha = 90°$) write down the fields produced by the corresponding arrangement of *magnetic* charge.]

Question #5: Use Duality Principle to Transform Electric Charge B-Field to Magnetic Charge E-Field

Taking the fields from the rotating electric charge above, we can apply these duality transformations and calculate the fields for the rotating magnetic charge:

$$\left(\frac{ae}{c^2}\right) \to c\left(\frac{a(-m/c)}{c^2}\right) = -\left(\frac{a\,m}{c^2}\right)$$

Convert B-Field to E-Field:

x-component:

$$E_x^{'} = cB_x = -\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{a\,m}{c^2}\right)\left[\left(\frac{z\omega_0(a^2\omega_0 + c)}{(z^2+a^2)^{3/2}}\right)\cos(\omega_0 t) - \left(\frac{\omega_0^2 z}{(z^2+a^2)}\right)\sin(\omega_0 t)\right]$$

y-component:

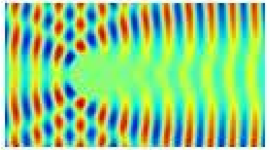
$$E_y^{'} = cB_y = -\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{a\,m}{c^2}\right)\left[\left(\frac{z\omega_0(a^2\omega_0 + c)}{(z^2+a^2)^{3/2}}\right)\sin(\omega_0 t) + \left(\frac{\omega_0^2 z}{(z^2+a^2)}\right)\cos(\omega_0 t)\right]$$

z-component:

$$E_z^{'} = cB_z = -\left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{a\,m}{c^2}\right)\left(\frac{a\omega_0(a^2\omega_0 + c)}{(z^2+a^2)^{3/2}}\right)$$

where $z = (r\ \cos\theta)$ and $a = L/2$





Question #5: Use Duality Principle to Transform Electric Charge E-Field to Magnetic Charge B-Field

Similarly, convert E-Field to B-Field:

$$\left(\frac{ae}{c^2}\right) \rightarrow -\frac{1}{c}\left(\frac{a(-m/c)}{c^2}\right) = \left(\frac{a\,m}{c^4}\right)$$

Note that $B_x = [\mathbf{B}(\vec{\mathbf{x}}, \mathrm{t})]_{\boldsymbol{x}}$ and vice versa.

x-component:

$$B_x^{'} = -\frac{1}{c}[\mathbf{E}(\vec{\mathbf{x}}, \mathrm{t})]_{\boldsymbol{x}} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{a\,m}{c^4}\right)\left[\left(\frac{\omega_0(a^2\omega_0 + c)}{(z^2 + a^2)}\right)\sin(\omega_0 t) + \left(\frac{\omega_0^2 z^2 - c^2}{(z^2 + a^2)^{3/2}}\right)\cos(\omega_0 t)\right]$$

y-component:

$$B_y^{'} = -\frac{1}{c}[\mathbf{E}(\vec{\mathbf{x}}, \mathrm{t})]_{\boldsymbol{y}} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{a\,m}{c^4}\right)\left[-\left(\frac{\omega_0(a^2\omega_0 + c)}{(z^2 + a^2)}\right)\cos(\omega_0 t) + \left(\frac{\omega_0^2 z^2 - c^2}{(z^2 + a^2)^{3/2}}\right)\sin(\omega_0 t)\right]$$

z-component:

$$B_z^{'} = -\frac{1}{c}[\mathbf{E}(\vec{\mathbf{x}}, \mathrm{t})]_{\boldsymbol{z}} = \left(\frac{1}{4\pi\epsilon_0}\right)\left(\frac{mz(a^2\omega_0^2 + c^2)}{c^4\,(z^2 + a^2)^{3/2}}\right)$$

where $z = (r\,\cos\theta)$ and $a = L/2$

Question #5: Relate Magnetic Dipole Moment Back to the Strength and Length of the Rotating Bar Magnet

Next, need to relate the strength of the permanent magnet B_0 and length L to the magnetic moment value of m:

$$\oint \vec{B} \cdot dl = \mu_0 I_{enc}$$

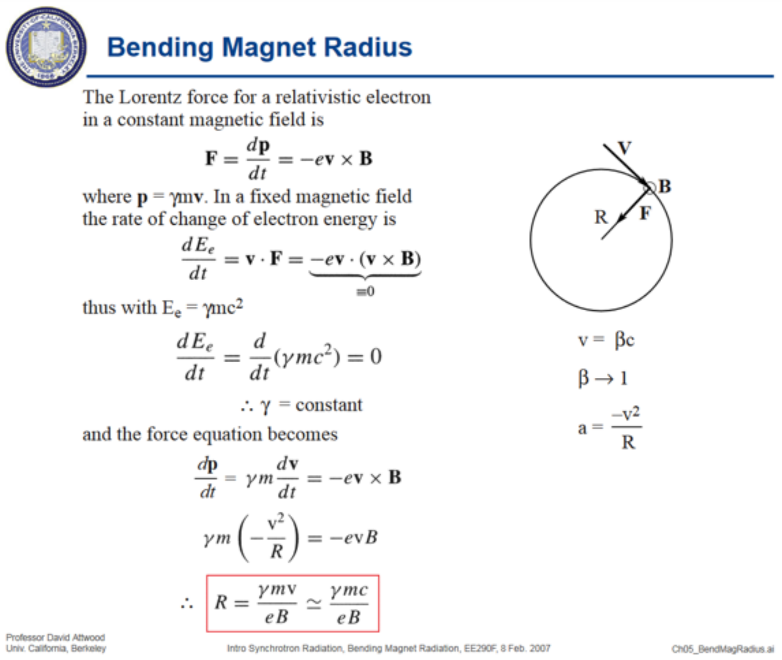
Griffiths equation 5.87 states that the magnetic field of a magnetic dipole can be written as:

$$\boldsymbol{B}_{dip}(\boldsymbol{r}) = \frac{\mu_0}{4\pi}\frac{1}{r^3}[3(\boldsymbol{m} \cdot \hat{r})\hat{r} - \boldsymbol{m}]$$

Where $\boldsymbol{m} = m_0 \widehat{m}$

$$B_0 = \frac{\mu_0}{4\pi}\frac{2m}{r^3} = \frac{\mu_0}{4\pi}\frac{2m}{\left(\frac{L}{2}\right)^3} \rightarrow m = \frac{\pi B_0 L^3}{4\mu_0}$$

If the mass of the magnet is known, it is possible to express the relation between length and B-field strength as shown below as shown by Attwood (http://people.eecs.berkeley.edu/~attwood/srms/2007/Lec08.pdf):



Bending Magnet Radius

The Lorentz force for a relativistic electron in a constant magnetic field is

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} = -e\mathbf{v} \times \mathbf{B}$$

where $\mathbf{p} = \gamma m\mathbf{v}$. In a fixed magnetic field the rate of change of electron energy is

$$\frac{dE_e}{dt} = \mathbf{v} \cdot \mathbf{F} = \underbrace{-e\mathbf{v} \cdot (\mathbf{v} \times \mathbf{B})}_{\equiv 0}$$

thus with $E_e = \gamma mc^2$

$$\frac{dE_e}{dt} = \frac{d}{dt}(\gamma mc^2) = 0$$

$$\therefore \gamma = \text{constant}$$

and the force equation becomes

$$\frac{d\mathbf{p}}{dt} = \gamma m \frac{d\mathbf{v}}{dt} = -e\mathbf{v} \times \mathbf{B}$$

$$\gamma m \left(-\frac{v^2}{R}\right) = -evB$$

$$\therefore \boxed{R = \frac{\gamma m v}{eB} \simeq \frac{\gamma mc}{eB}}$$

$v = \beta c$

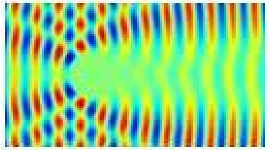
$\beta \rightarrow 1$

$a = \frac{-v^2}{R}$

Professor David Attwood
Univ. California, Berkeley

Intro Synchrotron Radiation, Bending Magnet Radiation, EE290F, 8 Feb. 2007

Ch05_BendMagRadius.ai



Question #5: Use the Larmor Formula to Calculate Time-Averaged Radiated Power (1/3)

Luckily, the radiated power calculation for the spinning bar magnet is a bit easier than that the near field calculation thanks to Larmor's formula found in section 14.2 of the Jackson textbook. According to Jackson, in the far-field and then the particle's velocity is small compared to the speed of light (nonrelativistic), the acceleration E-field reduces to:

$$\boldsymbol{E_a} = \frac{e}{c}\left[\frac{\hat{\boldsymbol{n}} \times (\hat{\boldsymbol{n}} \times \dot{\boldsymbol{\beta}})}{R}\right]_{ret}$$

Poynting Vector: $\boldsymbol{S} = \frac{c}{4\pi}\boldsymbol{E} \times \boldsymbol{H} = \frac{c}{4\pi}|\boldsymbol{E_a}|^2\hat{\boldsymbol{n}}$

When converted to power radiated per solid angle:

$$\frac{dP}{d\Omega} = \frac{c}{4\pi}|R\boldsymbol{E_a}|^2 = \frac{e^2}{4\pi c}\left|\hat{\boldsymbol{n}} \times (\hat{\boldsymbol{n}} \times \dot{\boldsymbol{\beta}})\right|^2$$

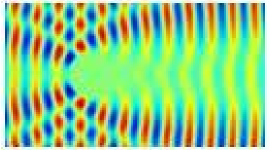
By assuming that θ theta is the angle between the acceleration $\dot{\boldsymbol{v}}\ and\ \hat{\boldsymbol{n}}$ this equation can be re-written as:

$$\frac{dP}{d\Omega} = \frac{e^2}{4\pi c^3}|\,\dot{\boldsymbol{v}}|^2 \sin^2\theta$$

When integrated over the solid angle, this equation neatly becomes:

$$P = \frac{2}{3}\frac{e^2}{c^3}|\,\dot{\boldsymbol{v}}|^2$$





Question #5: Use the Larmor Formula to Calculate Time-Averaged Radiated Power (2/3)

$$P = \frac{2}{3}\frac{e^2}{c^3}|\dot{\boldsymbol{v}}|^2 = \frac{2}{3}\frac{e^2a^2\omega_0^4}{c^3} = \frac{2}{3}\frac{(-m/c)^2\left(\frac{L}{2}\right)^2\omega_0^4}{c^3} = \frac{m^2L^2\omega_0^4}{6c^5} = \frac{\left(\frac{\pi B_0 L^3}{4\mu_0}\right)^2 L^2\omega_0^4}{6c^5} = \frac{\pi^2 B_0^2 L^8\omega_0^4}{96c^5\mu_0^2}$$

$$m = \frac{\pi B_0 L^3}{4\mu_0}$$

Alternatively, since radiated power should be independent of phi since the geometry is azimuthally (rotationally) symmetric, we can choose a more simple $\hat{\boldsymbol{n}}$ the is in the x-z plane (Jackson Section 14.6 pg. 677), such as:

$$\hat{\boldsymbol{n}} = \cos\theta\,\hat{z} + \sin\theta\,\hat{x}$$

$$\boldsymbol{E}_{\boldsymbol{a}} = \frac{e}{c}\left[\frac{\hat{\boldsymbol{n}} \times (\hat{\boldsymbol{n}} \times \dot{\boldsymbol{\beta}})}{R}\right]_{ret}$$

$$= \left(\frac{e}{cR}\right)\left(\frac{-a\omega_0^2}{c}\right)\left(\hat{x}\left(-\cos^2\theta\cos(\omega_0 t)\right) + \hat{y}(-\sin(\omega_0 t)) + \hat{z}(\cos\theta\sin\theta\cos(\omega_0 t)\right)$$

Question #5: Use the Larmor Formula to Calculate Time-Averaged Radiated Power (3/3)

$$\frac{dP}{d\Omega} = \frac{c}{4\pi}|R\boldsymbol{E_a}|^2 = \frac{e^2}{4\pi c}\left|\hat{\boldsymbol{n}} \times (\hat{\boldsymbol{n}} \times \dot{\boldsymbol{\beta}})\right|^2 = \left(\frac{e^2 a^2 \omega_0^4}{4\pi c^3}\right)\left(\cos^2(\omega_0 t)\cos^2\theta + \sin^2(\omega_0 t)\right)$$

Take the time average of the power density function:

$$\langle\frac{dP}{d\Omega}\rangle = \left(\frac{e^2 a^2 \omega_0^4}{4\pi c^3}\right)\left(\frac{1}{2}\cos^2\theta + \frac{1}{1}\right) = \left(\frac{e^2 a^2 \omega_0^4}{8\pi c^3}\right)\left(\cos^2\theta + 1\right)$$

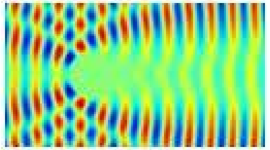
Time average of $\cos^2(\omega_0 t)$ and $\sin^2(\omega_0 t)$ are both ½.

Now we integrate this quantity over the solid angle:

$$P = \iint_{\Phi=0,\theta=0}^{\Phi=2\pi,\theta=\pi}\left(\frac{e^2 a^2 \omega_0^4}{8\pi c^3}\right)\left(\cos^2\theta + 1\right)\sin\theta\, d\theta\, d\Phi$$

$$P = \left(\frac{e^2 a^2 \omega_0^4}{8\pi c^3}\right)(2\pi)\int_{\theta=0}^{\pi}\left(\cos^2\theta + 1\right)\sin\theta\, d\theta\, d\Phi = \left(\frac{e^2 a^2 \omega_0^4}{8\pi c^3}\right)(2\pi)\left(\frac{8}{3}\right) = \frac{2}{3}\frac{e^2 a^2 \omega_0^4}{c^3}$$

This results achieved by integrating the power radiated over solid angle gives us the same result for time-averaged radiated power as the direct equation from Jackson, which we calculated earlier in this problem.



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