STA721 HW1

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3(e) Answer

```r
#input data
X<-matrix(c(1,1,1,1,1,1,1,1,1,0,0,0,0,0,0,1,0,0,0,0,0,0,1,1),
          nrow = 6,byrow = FALSE)
Q<-qr.Q(qr(X))
U<-svd(X)$u
round(Q,7)

##            [,1]       [,2]       [,3]       [,4]
## [1,] -0.4082483 -0.4082483  0.0000000 -0.7415816
## [2,] -0.4082483 -0.4082483  0.0000000  0.0749150
## [3,] -0.4082483 -0.4082483  0.0000000  0.6666667
## [4,] -0.4082483  0.4082483  0.8164966  0.0000000
## [5,] -0.4082483  0.4082483 -0.4082483  0.0000000
## [6,] -0.4082483  0.4082483 -0.4082483  0.0000000

round(U,7)

##            [,1]       [,2]       [,3]       [,4]
## [1,] -0.4490638 -0.3487210  0.1003428  0.8092978
## [2,] -0.4490638 -0.3487210  0.1003428 -0.4983391
## [3,] -0.4490638 -0.3487210  0.1003428 -0.3109587
## [4,] -0.3280033  0.1542044 -0.9320058  0.0000000
## [5,] -0.3791035  0.5529023  0.2248990  0.0000000
## [6,] -0.3791035  0.5529023  0.2248990  0.0000000
```

We see that $Q \neq U$.

3(f) Answer

Calculate the projection matrix use U and Q respectively:

```r
P_svd<-U[,1:3]%*%t(U[,1:3])
P_qr<-Q[,1:3]%*%t(Q[,1:3])
M<-matrix(c(1/3,1/3,1/3,0,0,0,1/3,1/3,1/3,0,0,0,
            1/3,1/3,1/3,0,0,0,0,0,0,1,0,0,0,0,0,
            0,1/2,1/2,0,0,0,0,1/2,1/2),nrow=6)
round(P_svd,7)
```

```
##           [,1]      [,2]      [,3] [,4] [,5] [,6]
## [1,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [2,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [3,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [4,] 0.0000000 0.0000000 0.0000000    1  0.0  0.0
## [5,] 0.0000000 0.0000000 0.0000000    0  0.5  0.5
## [6,] 0.0000000 0.0000000 0.0000000    0  0.5  0.5
```

```r
round(P_qr,7)
```

```
##           [,1]      [,2]      [,3] [,4] [,5] [,6]
## [1,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [2,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [3,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [4,] 0.0000000 0.0000000 0.0000000    1  0.0  0.0
## [5,] 0.0000000 0.0000000 0.0000000    0  0.5  0.5
## [6,] 0.0000000 0.0000000 0.0000000    0  0.5  0.5
```

```r
M
```

```
##           [,1]      [,2]      [,3] [,4] [,5] [,6]
## [1,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [2,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [3,] 0.3333333 0.3333333 0.3333333    0  0.0  0.0
## [4,] 0.0000000 0.0000000 0.0000000    1  0.0  0.0
## [5,] 0.0000000 0.0000000 0.0000000    0  0.5  0.5
## [6,] 0.0000000 0.0000000 0.0000000    0  0.5  0.5
```

The projection matrix obtained from SVD and QR decomposition are the same as M in 1.5.8.

4(c) Answer

The Cholesky decomposition of Y is calculated as follows:

```r
V<-matrix(c(2,0,1,0,3,2,1,2,4),nrow=3)
L<-t(chol(V))
L
```

```
##             [,1]     [,2]    [,3]
## [1,] 1.4142136 0.000000 0.00000
## [2,] 0.0000000 1.732051 0.00000
## [3,] 0.7071068 1.154701 1.47196
```

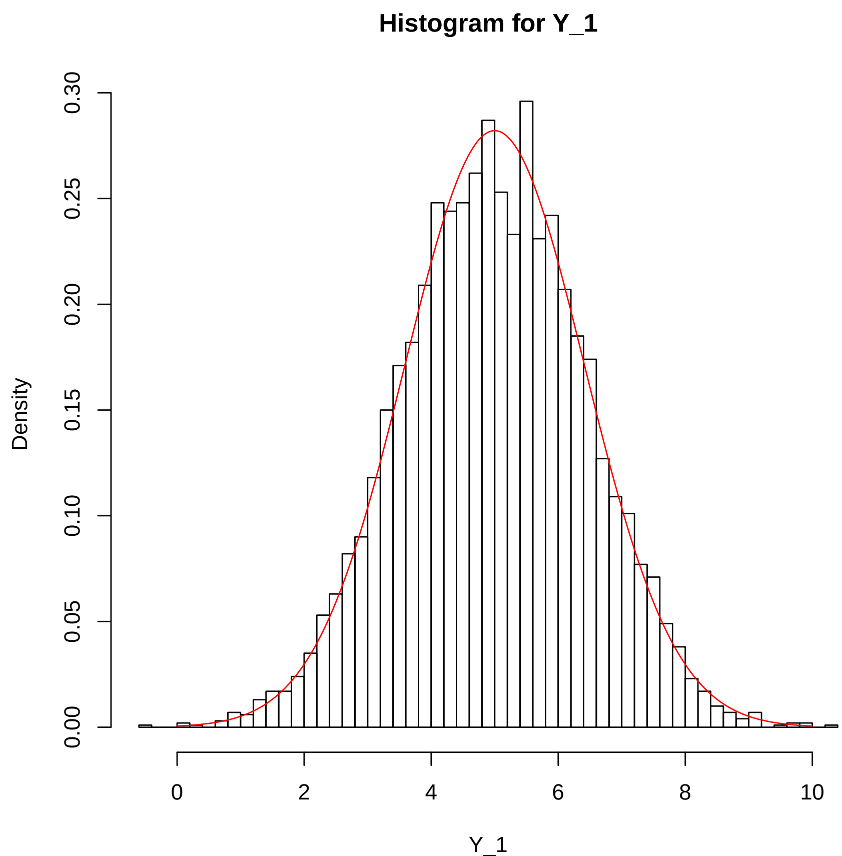
4(d) and 4(e) Answer

Generate 5000 samples of Y:

```
set.seed(123)
Z<-matrix(rnorm(3*5000),nrow=3)
mu<-c(5,6,7)
Y<-apply(Z, 2, function(x){L%*%x+mu})
```

In 1.5.2(a), we have $Y_1 \sim N(5, 2)$. Create a histogram for the marginal distribution of Y_1 and overlay the actual density $N(5, 2)$:

```
hist(Y[1,],freq = FALSE,breaks = 50,
     main = "Histogram for Y_1",xlab = "Y_1")
p<-dnorm(seq(0,10,length.out = 1000),mean = 5,sd = sqrt(2))
lines(seq(0,10,length.out = 1000),p,col='red')
```



The histogram of Y_1 looks like $N(5, 2)$ distribution.

4(f) Answer

We calculate the sample mean, variance and covariance of Z:

```
A<-matrix(c(2,1,1,1,0,1),nrow=2)
b<-matrix(c(-15,-18),ncol=1)
Z_new<-apply(Y, 2, function(x){A%*%x+b})
Z_mean<-apply(Z_new,1,function(x){mean(x)})
Z_variance<-apply(Z_new,1,function(x){var(x)})
Z_covariance<-mean((Z_new[1,]-Z_mean[1])*(Z_new[2,]-Z_mean[2]))
Z_mean

## [1] 0.973299677 0.002561618

Z_variance
```

```
## [1] 11.30531 15.06970

Z_covariance

## [1] 11.25377
```

In 1.5.8, we have $Z \sim N(\mu_Z, \Sigma_Z)$, where $\mu_Z = (1, 0)^T$,

$$\Sigma_Z = \begin{pmatrix} 11 & 11 \\ 11 & 15 \end{pmatrix}. \tag{1}$$

Hence the estimates obtained with simulation are consistent with the results in 1.5.8.