Does Symbolic Knowledge Prevent Adversarial Fooling?

Stefano Teso

University of Trento stefano.teso@unitn.it

Abstract

Arguments in favor of injecting symbolic knowledge into neural architectures abound. When done right, constraining a sub-symbolic model can substantially improve its performance, sample complexity, interpretability, and can prevent it from predicting invalid configurations. Focusing on deep probabilistic (logical) graphical models – i.e., constrained joint distributions whose parameters are determined (in part) by neural nets based on low-level inputs – we draw attention to an elementary but unintended consequence of symbolic knowledge: that the resulting constraints can propagate the negative effects of adversarial examples.

Introduction

Deep probabilistic (logical) graphical models (dPGMs) tie together a sub-symbolic level that processes low-level inputs with a symbolic level that handles logical and probabilistic inference, see for instance (De Raedt et al. 2019). The two levels are often implemented with k neural networks and one probabilistic (logical) graphical models, respectively. Prominent examples of dPGMs include DeepProbLog (Manhaeve et al. 2018) and "neural" extensions of Markov Logic (Lippi and Frasconi 2009; Marra and Kuželka 2019). In this preliminary investigation, we show with a concrete toy example that fooling a single neural network with an adversarial example (Szegedy et al. 2013; Biggio and Roli 2018) can corrupt the state of multiple output variables. We develop an intuition of this phenomenon and show that it occurs despite the model being probabilistic and regardless of whether the symbolic knowledge is factually correct.

Deep Probabilistic-Logical Models

We restrict ourselves to deep Bayesian networks (dBNs), i.e., directed dPGMs stripped of their logical component. (Our arguments do transfer to other dPGMs and deep statistical-relational models too.) These models are Bayesian networks where some conditional distributions are

Copyright © 2020, Association for the Advancement of Artificial Intelligence (www.aaai.org). All rights reserved.

implemented as neural networks feeding on low-level inputs, and (roughly speaking) correspond to ground Deep-ProbLog models.

Let us illustrate them with a restricted version of the addition example from (Manhaeve et al. 2018): the goal is to recognize the digits $x_1, x_2 \in \{1, ..., 4\}$ appearing in two MNIST images z_1 and z_2 , knowing that the digits satisfy the constraint $\varphi = (x_1 + x_2 = 5)$. Notice that the only valid predictions are (1, 4), (2, 3), (3, 2), (4, 1).

Let $x=(x_1,x_2)$ and $z=(z_1,z_2)$. Our dBN for this problem defines a joint distribution $\mathbb{P}(x\,|\,z;\varphi)$ built on the conditionals $\mathbb{P}(x_1\,|\,z_1)$, $\mathbb{P}(x_2\,|\,z_2)$ and on φ . In particular, the probability of the event $X_i=x_i$ is implemented as a ConvNet with a softmax output layer applied to z_i . The dBN is consistent with the symbolic knowledge φ in that it ensures that the joint distribution satisfies $\mathbb{P}(x\,|\,z;\varphi)=0$ for all $x\not\models\varphi$. This is achieved by taking an unconstrained joint distribution $\mathbb{P}(x\,|\,z)=\prod_i\mathbb{P}(x_i\,|\,z_i)$ and constraining it:

$$\mathbb{P}(x \mid z; \varphi) = \mathbb{P}(x \mid z) \mathbb{1} \left\{ x \models \varphi \right\} / Z \tag{1}$$

Here $Z = \sum_{x \models \varphi} \mathbb{P}(x \mid z)$ is a normalization constant and the sum runs over all x's consistent with φ . A joint prediction is obtained via maximum a-posteriori (MAP) inference (Koller and Friedman 2009):

$$F(z) = \operatorname{argmax}_{x} \mathbb{P}(x \mid z; \varphi) \tag{2}$$

If no symbolic knowledge φ was given, the most likely outputs would simply be $(f_1(z_1), f_2(z_2))$, where:

$$f_i(z_i) = \operatorname{argmax}_{x_i} \mathbb{P}(x_i \mid z_i) \tag{3}$$

Finally, we use the same ConvNet for both images, and let $f = f_1 = f_2$.

Adversarial Examples and Constraints

Consider a pair of images z representing a 1 and a 4, respectively, and let the ConvNet output the following conditional probabilities:

$$\mathbb{P}(X_1 \mid z_1) = (0.9, 0.1, 0, 0) \tag{4}$$

$$\mathbb{P}(X_2 \mid z_2) = \left(\frac{1}{4} - \frac{\epsilon}{3}, \frac{1}{4} - \frac{\epsilon}{3}, \frac{1}{4} - \frac{\epsilon}{3}, \frac{1}{4} + \epsilon\right) \tag{5}$$

for some small ϵ , e.g., 0.001. Although the second image is rather uninformative, the unconstrained dPGM gets both

digits right, with joint probability ≈ 0.226 (by Eq. 3) and so does the constrained classifier, with probability ≈ 0.9 (Eq. 2). In this case, the symbolic knowledge boosts the confidence of the model, a desirable and expected result.

Now, perturbing z_i by δ_i shifts the conditional distribution output by the ConvNet from $\mathbb{P}(x_i \mid z_i)$ to $\mathbb{P}(x_i \mid z_i + \delta_i)$ and hence changes the probabilities assigned to the possible outcomes X_i . Intuitively, a perturbation is adversarial if it is at the same time imperceptible and it forces MAP inference to output a wrong configuration. In other words, assuming that z_i is classified correctly, $z_i + \delta_i$ is adversarial if $f(z_i + \delta_i) \neq f(z_i)$ and $\|\delta_i\|$ is "small" for some norm $\|\cdot\|$.

It is well known that neural networks are often susceptible to rather eye-catching adversarial perturbations that can alter their output by arbitrary amounts (Szegedy et al. 2013; Biggio and Roli 2018). Thus it is not too far fetched to imagine a perturbation δ_1 that induces the following conditional distribution on the first digit:

$$\mathbb{P}(X_1 \mid z_1 + \delta_1) = (0.1, 0.9, 0, 0) \tag{6}$$

Now, it can be readily verified that this perturbation forces the unconstrained dBN to predict (2,4) with joint probability ≈ 0.226 (which is symmetrical to the above case). Clearly this model is fooled by the adversarial image into making a mistake on x_1 , but the damage is limited to the first digit: x_2 is still predicted correctly.

However (2,4) does violate the symbolic knowledge φ , while the constrained dBN is forced to output a valid prediction, namely the most likely configuration out of $\{(1,4),(2,3),(3,2),(4,1)\}$. Given the above conditional distributions and ϵ , the constrained dBN outputs (2,3) with probability ≈ 0.9 . This prediction is definitely consistent with φ , but now both digits are classified wrongly.

Discussion

The toy example above illustrates the perhaps elementary but seemingly neglected fact that symbolic knowledge can propagate the negative effects of adversarial examples. This occurs because the model trades off predictive loss in exchange for satisfying a hard constraint.

While our example is decidedly toy, it is easy to see that the same phenomenon could occur in relevant sensitive applications. The phenomenon is also likely to transfer to undirected dPGMs like deep extensions of Markov Logic Networks (Lippi and Frasconi 2009; Marra and Kuželka 2019).

We make a couple of important remarks. First, depending on the structure of the symbolic knowledge, fooling a single neural networks in the dPGM may perturb any subset of output variables. Thus, seeking robustness of a single network is not enough and all k networks must be robustified. Second, this may not be enough either: if an adversary manages to fool a robustified neural network – even by random luck – the effects of fooling will still cascade across the model. Thus the dPGM $as\ a\ whol\ e$ must be made robust, in the sense that all CPTs appearing in it – not only the ConvNets – must be made robust. Finally, it may be the case that access to the symbolic knowledge might help attackers in designing minimal targeted attacks that induce any target variable.

Adversarial examples in dPGMs can be understood through the lens of sensitivity analysis for directed (Chan and Darwiche 2002) and undirected probabilistic graphical models (Chan and Darwiche 2005); see especially (Chan and Darwiche 2006). These works show how to constrain a probabilistic graphical model to ensure that the probabilities of different queries are sufficiently far apart. These constraints could be injected into standard adversarial training routines for neural networks to encourage global robustness of the dPGM. Of course, robust training of complex dPGMs is likely to be computationally challenging. Algebraic model counting in the sensitivity semiring might prove useful in tackling this computational challenge (Kimmig, Van den Broeck, and De Raedt 2017).

References

[Biggio and Roli 2018] Biggio, B., and Roli, F. 2018. Wild patterns: Ten years after the rise of adversarial machine learning. *Pattern Recognition* 84.

[Chan and Darwiche 2002] Chan, H., and Darwiche, A. 2002. When do numbers really matter? *Journal of artificial intelligence research* 17.

[Chan and Darwiche 2005] Chan, H., and Darwiche, A. 2005. Sensitivity analysis in Markov networks. In *International Joint Conference on Artificial Intelligence*, volume 19.

[Chan and Darwiche 2006] Chan, H., and Darwiche, A. 2006. On the robustness of most probable explanations. In *Proceedings of the Twenty-Second Conference on Uncertainty in Artificial Intelligence*. AUAI Press.

[De Raedt et al. 2019] De Raedt, L.; Manhaeve, R.; Dumancic, S.; Demeester, T.; and Kimmig, A. 2019. Neuro-Symbolic= Neural + Logical + Probabilistic. In NeSy'19@ IJCAI, the 14th International Workshop on Neural-Symbolic Learning and Reasoning.

[Kimmig, Van den Broeck, and De Raedt 2017] Kimmig, A.; Van den Broeck, G.; and De Raedt, L. 2017. Algebraic model counting. *Journal of Applied Logic* 22.

[Koller and Friedman 2009] Koller, D., and Friedman, N. 2009. *Probabilistic graphical models: principles and techniques*.

[Lippi and Frasconi 2009] Lippi, M., and Frasconi, P. 2009. Prediction of protein β -residue contacts by markov logic networks with grounding-specific weights. *Bioinformatics* 25(18).

[Manhaeve et al. 2018] Manhaeve, R.; Dumancic, S.; Kimmig, A.; Demeester, T.; and De Raedt, L. 2018. Deep-ProbLog: Neural probabilistic logic programming. In *Advances in Neural Information Processing Systems*.

[Marra and Kuželka 2019] Marra, G., and Kuželka, O. 2019. Neural Markov Logic Networks. *arXiv preprint arXiv:1905.13462*.

[Szegedy et al. 2013] Szegedy, C.; Zaremba, W.; Sutskever, I.; Bruna, J.; Erhan, D.; Goodfellow, I.; and Fergus, R. 2013. Intriguing properties of neural networks. *arXiv preprint arXiv:1312.6199*.