Boost or Rest: How Caffeine and Sleep Influence Problem-Solving

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1 Abstract

The study investigates the influence of caffeine intake and nap duration on problem-solving skills among islanders aged 18 to 28. By utilizing a Two-Way Randomized Block Design, participants were assigned to one of three caffeine treatments (high, low, placebo) and one of two nap durations (15 minutes, 60 minutes). Participants first completed a pre-survey to eliminate outliers and then took a pre-treatment problem-solving test to establish baseline scores. Following random assignment, participants received their respective treatments, which involved taking a nap first and then consuming the designated coffee type, followed by a 30-minute waiting period. Post-treatment problem-solving abilities were measured using the same test.

Our analysis, based on blocking by collection time (morning or evening), included boxplots, interaction plots, ANOVA, and TukeyHSD. The results revealed that while caffeine levels did not significantly impact problem-solving scores, nap duration did have significance. Specifically, a longer nap duration consistently resulted in higher problem-solving scores compared to a shorter nap. Additionally, the interaction between caffeine intake and nap duration was not significant, although certain group contrasts indicated notable differences. The findings suggest that while caffeine may not enhance cognitive performance in problem-solving tasks, sufficient sleep does, highlighting the importance of sleep in cognitive functioning.

2 Introduction

As college students, we frequently encounter the dilemma of whether to prioritize sleep or allocate additional time to studying before exams. This common situation raises questions about the optimal balance between rest and cognitive performance, particularly in problem-solving tasks that are crucial for academic success.

Existing literature highlights the significant effects of both caffeine and sleep on cognitive functions. Caffeine, the world's most commonly used psychoactive drug, is consumed at an average rate of 70 mg daily across the globe. Upon ingestion, it takes about 30 minutes to reach its peak level within the body, exerting its influence by blocking adenosine A_1 and A_2 receptors in both the peripheral and central nervous systems[1]. This blockage leads to increased neural activity, resulting in cognitive benefits such as enhanced alertness, improved vigilance and motor performance, enhanced concentration and attentional focus, and elevated mood [2].

On the other hand, sleep, including short naps, has been shown to offer significant cognitive benefits. Brief naps (5-15 minutes) provide almost immediate benefits that last for 1-3 hours, while longer naps (over 30 minutes) may cause short-term grogginess upon waking but lead to enhanced cognitive performance for several hours afterward [3]. Research indicates that sleep, even in the form of a nap, can potentiate the solution of problems that involve logical reasoning [4].

Additionally, the time of data collection can impact cognitive performance, as cognitive functions can vary based on the time of day. Studies have shown that caffeine offers particular advantages for memory during students' non-optimal times of day, such as early morning [5]. Therefore, our study accounts for this by blocking data collection times to control for these potential variations.

Given these insights, our study aims to investigate the combined effects of caffeine intake and nap duration on problem-solving skills. By examining how different levels of caffeine and varying nap durations impact cognitive performance, we hope to provide valuable insights that can inform our decision-making processes and help optimize study routines for better academic outcomes.

Our research questions are:

1. What effect will coffee (zero caffeine vs decaffeinated vs caffeinated) and nap duration (short vs long) have on the problem-solving abilities of the islanders?

2. Are these effects significant?

3. Is there a significant interaction between the coffee and nap duration?

3 Methods

3.1 Participants

The participants were selected from among islanders. The population sampling process begins with selecting cities. We assigned numbers to 27 cities, ranging from 1 to 27. Using R, we generated a 14×12 matrix containing random integers between 1 and 27, with each column representing one of the treatment combinations. Participants, aged 18-28, were then chosen from the corresponding cities based on the matrix numbers. It's important to note that while this sampling method is not entirely random due to the varying population sizes in each city, the assignment of participants to treatment conditions is random.

3.2 Design

The study employs a Two-Way Randomized Block Design to examine the effects of caffeine and napping on problem-solving abilities. This design allows for controlling variability due to different times of day (morning and evening). The parameters for the design are detailed as follows:

Response Variable

The primary outcome of interest is the Basic Problem Solving Test Score, which is represented as a percentage. The test lasts for 20 minutes and measures the participants' problem-solving abilities under different treatment conditions.

Treatments

The study involves two treatment factors:

Caffeine Intake (Treatment 1):

- High: 250 ml of caffeinated coffee.
- Low: 250 ml of decaffeinated coffee.
- Placebo: 250 ml of water.

Nap Duration (Treatment 2):

- Long: 60-minute nap.
- Short: 15-minute nap.

Blocking Variable

To control for potential variations in cognitive performance due to the time of day, the study will be blocked by time:

- Morning
- Evening

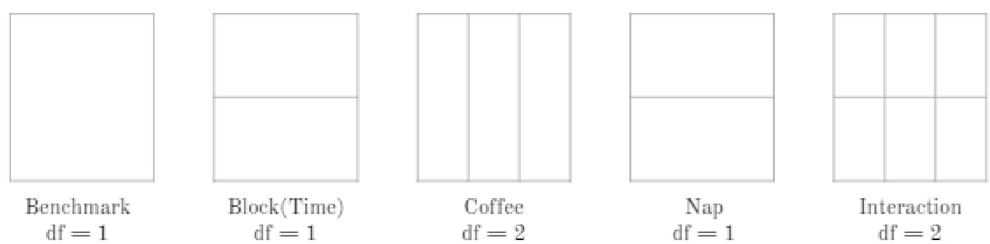

Figure 1: Factor Diagram

3.3 Procedure 1 (Sample Size)

We decided to use a power of 0.8, which is the probability that we will correctly reject the null hypothesis when it is false. We used a significance level of 0.05, which is the probability of incorrectly rejecting the null hypothesis. We used an effect size of 0.25, which is a way of quantifying the difference between the groups. Using G-Power, we determined that the sample size required is 158, based on the factor with the largest degree of freedom (2). However, to have a balanced design, we will have a sample size of 168, so that each group will have 14 samples.

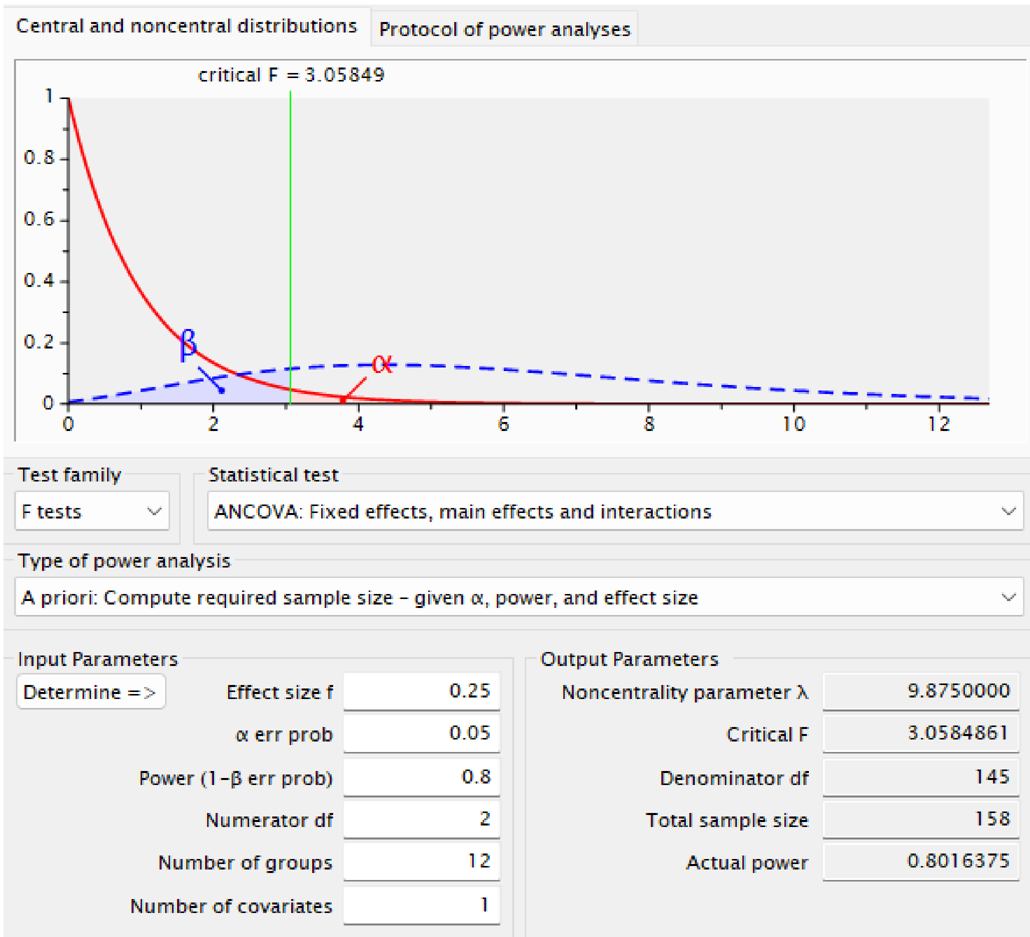

Figure 2: G-Power

3.4 Procedure 2 (Data Description)

After selecting the individuals within each randomly selected city, we first presented each islander with survey questions to collect the islander's name, city, sex, age, and well-being in terms of factors that pertain to sleep, tiredness, and energy consisting of these questions:

1. How many hours did you sleep last night?
2. How many minutes did you spend doing vigorous physical activity in the last seven days?

3. On a scale from 1 to 10, how energetic do you feel right now?

4. On a scale from 1 to 10, how tired do you feel right now?

5. Did you drink coffee today?

We divided the data collection times into two segments: morning (6am - 12pm) and evening (6pm-12am) per each treatment combination. We administered the pre-treatment basic problem-solving test which presents the score as a percentage. Then, we gave the designated treatment levels to each islander (nap level, followed by caffeine level), waited 30 minutes for the caffeine effects to kick in, and then administered the same problem-solving test for the post-test. Lastly, we collected the score difference for the pre-test and post-test, and recorded the data in a table in the following format:

Name	City	Sex	Age	Time of Day	Survey Questions	Pre-test	Nap Duration	Coffee	Post-test

Figure 3: Data Collection Format

4 Result Analysis

4.1 Boxplot

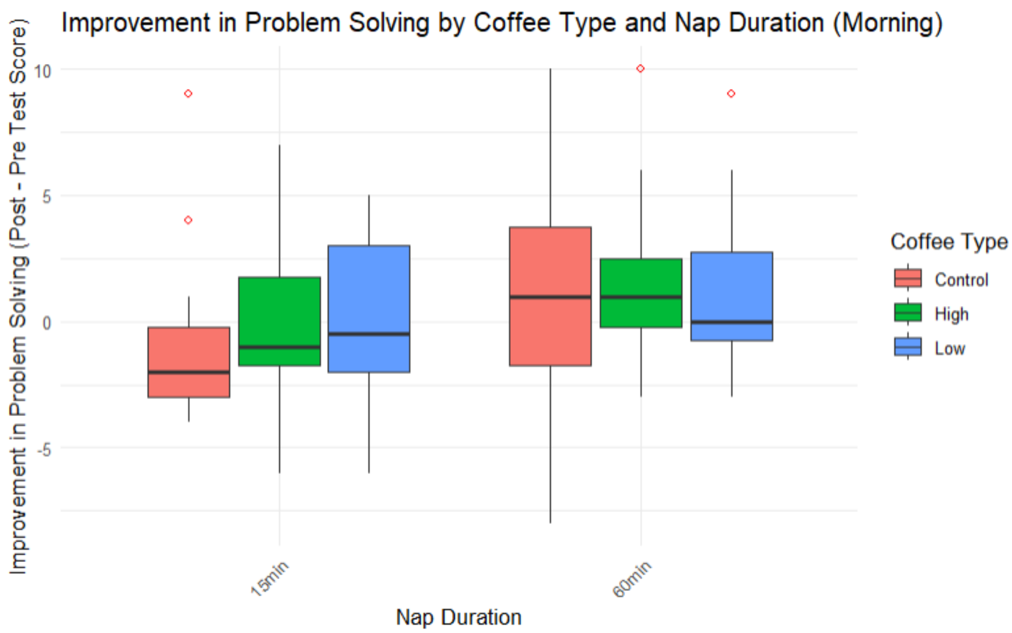

Figure 4: Side-by-side Boxplots Clustered by Coffee Type (Morning)

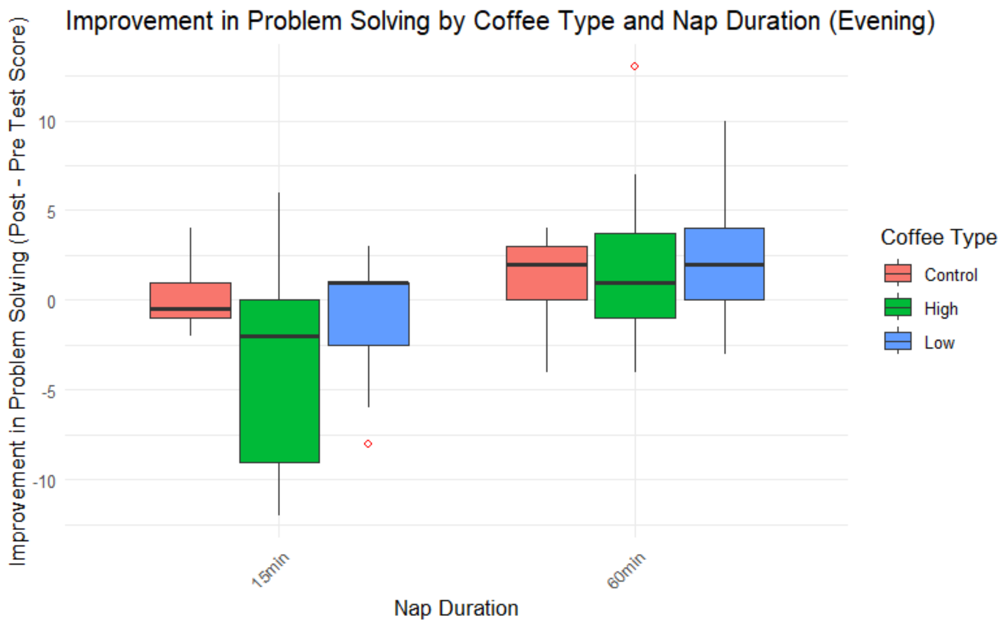

Figure 5: Side-by-side Boxplots Clustered by Coffee Type (Evening)

First, we utilized box plots to visualize the effect of different factors on our response variables. We used three colors to represent three different coffee types: control group, high-caffeine coffee, and low-caffeine coffee. Since the time of day (morning or evening) is a blocking variable, we created separate plots for the different groups based on the time. The horizontal line in the middle of the box is the median of the data set. Our analysis did not reveal a clear effect of coffee type on problem-solving scores within these comparisons. Consequently, we may not be able to reject the null hypothesis that coffee types have no effect.

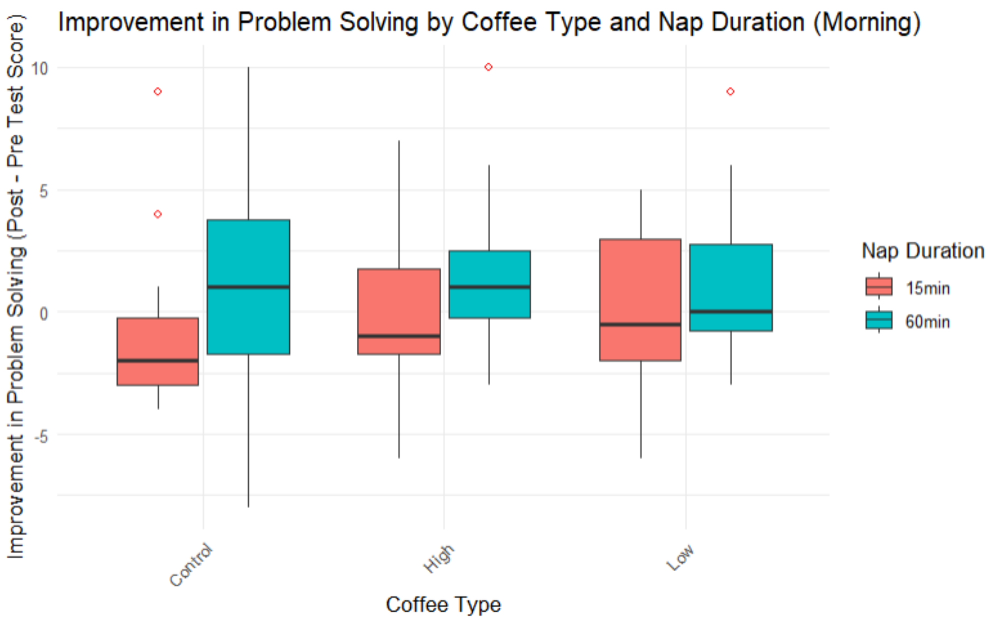

Figure 6: Side-by-side Box plots Clustered by Nap Duration (Morning)

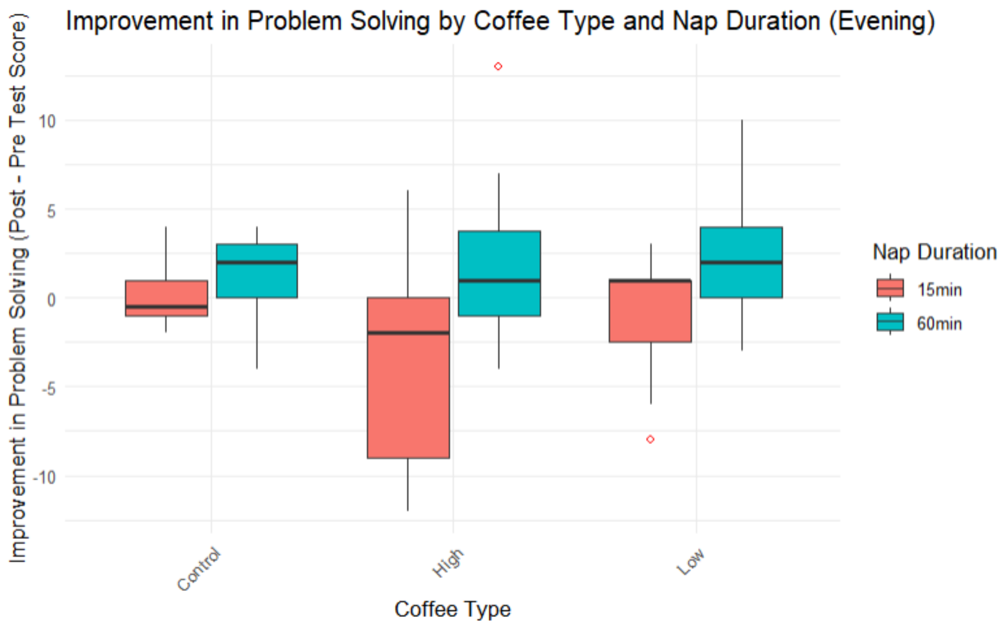

Figure 7: Side-by-side Box plots Clustered by Nap Duration (Evening)

We used two colors to distinguish between the two nap durations: 15 minutes and 60 minutes. Given that time of day (morning or evening) is a blocking variable, we presented the data in two separate plots for each time. The horizontal line within each box plot represents the median of the data set. Our analysis reveals that, across comparable conditions, the group with a 60-minute nap consistently exhibits higher median and mean problem-solving scores than the group with a 15-minute nap.

4.2 Interaction plot

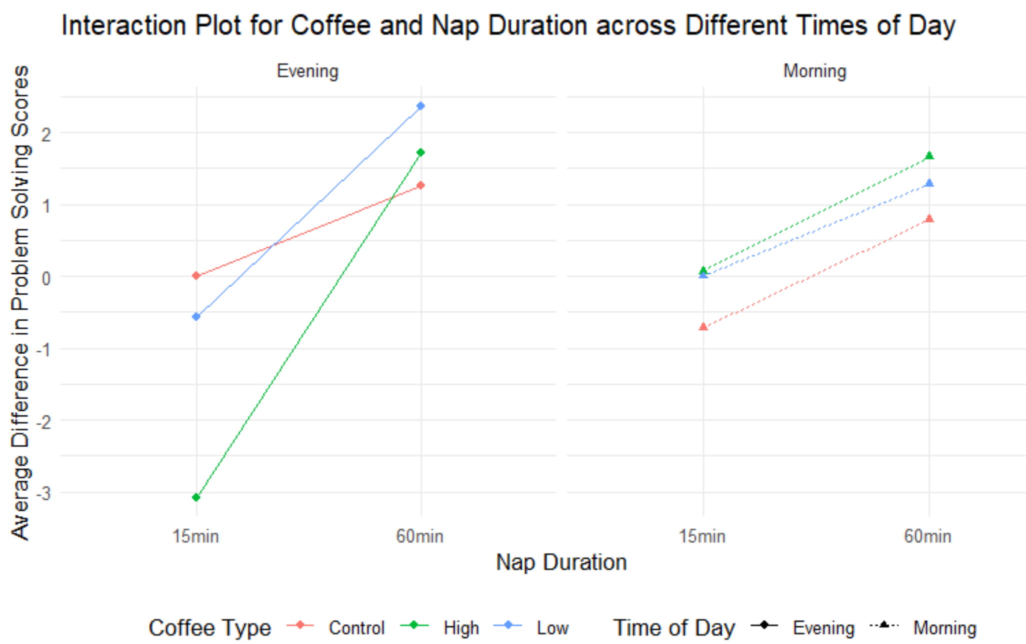

Figure 8: Interaction Plot of Treatments And The Response Variable

The interaction plot (Figure 8) illustrates the effects of coffee type and nap duration on problem-solving scores, with time of day as a blocking variable. The average difference in problem-solving scores is plotted against nap duration for each combination of coffee type and time of day. As shown in Figure 8, most lines exhibit similar slopes, suggesting that most interaction terms for this model might not be significant. However, it is noteworthy that the combination of high coffee consumption and longer nap durations significantly improves problem-solving scores, particularly in the evening. The time of day also affects the impact, with evening naps showing greater benefits in terms of score improvements across all coffee types.

4.3 ANOVA

	Df	Sum Sq	Mean Sq	F value	Pr(>F)
Nap	1	198.952732	198.952732	14.1738420	0.0002332
Coffee	2	11.941570	5.970785	0.4253722	0.6542588
Time.of.Day	1	1.405862	1.405862	0.1001568	0.7520515
Nap:Coffee	2	22.281365	11.140683	0.7936874	0.4539355
Residuals	161	2259.894661	14.036613	NA	NA

Figure 9: Two-way ANOVA table with Blocking and Interaction

We use the Analysis of Variance (ANOVA) table to test the significance of the full model. The F-value for the effect of nap duration is notably high at 14.1738420, with a very low p-value of 0.0002332, indicating a statistically significant effect of nap duration on problem-solving scores. Conversely, the effect of different coffee types on problem-solving scores has a p-value of 0.6542588, suggesting no statistically significant effect of coffee levels on the scores. Additionally, the blocking variable (time of day) and the interaction between different coffee types and nap durations are not statistically significant, with p-values of 0.7520615 and 0.4539355, respectively, at a significance level of 0.05. Moreover, the adjusted R^2 value for the full model is quite low at 0.06028, indicating that the model explains only a small portion of the variance in problem-solving scores. This finding supports our box plot that nap duration has an effect on the difference in problem-solving scores.

4.4 TukeyHSD

	diff	lwr	upr	p adj
60min-15min	2.176612	1.034885	3.318339	0.0002332

Figure 10: TukeyHSD result of the factor Nap Duration

Following the ANOVA analysis, we determined that the nap duration factor is significant. As there are only two levels for this factor, the adjusted p-value obtained using the Tukey HSD test is identical to the p-value reported in the ANOVA table.

	diff	lwr	upr	p adj
60min:Control-15min:Control	1.3642486	-1.9381672	4.6666644	0.8405186
15min:High-15min:Control	-0.3836453	-3.7159117	2.9486212	0.9994574
60min:High-15min:Control	3.1896621	-0.1426044	6.5219285	0.0692512
15min:Low-15min:Control	0.0449262	-3.2873403	3.3771926	1.0000000
60min:Low-15min:Control	2.1539478	-1.1783187	5.4862142	0.4280894
15min:High-60min:Control	-1.7478939	-5.1077817	1.6119940	0.6646530
60min:High-60min:Control	1.8254135	-1.5344744	5.1853013	0.6215850
15min:Low-60min:Control	-1.3193224	-4.6792103	2.0405654	0.8672476
60min:Low-60min:Control	0.7896992	-2.5701887	4.1495870	0.9841751
60min:High-15min:High	3.5733073	0.1840749	6.9625397	0.0322999
15min:Low-15min:High	0.4285714	-2.9606610	3.8178038	0.9991439
60min:Low-15min:High	2.5375930	-0.8516394	5.9268255	0.2627941
15min:Low-60min:High	-3.1447359	-6.5339683	0.2444965	0.0857387
60min:Low-60min:High	-1.0357143	-4.4249467	2.3535181	0.9505352
60min:Low-15min:Low	2.1090216	-1.2802108	5.4982540	0.4721859

Figure 11: TukeyHSD result of the interaction term

Although our ANOVA analysis indicated that the interaction term was not statistically significant, we utilized the Tukey HSD test to identify if any specific group differences were statistically significant. As observed in the interaction plot, there is only one set of significant contrasts (line 10): "participants who drank high caffeine in the morning and took naps for 15 minutes compared to those who took naps for 60 minutes". The mean difference in scores between these two groups is 3.5793, with a p-value of 0.0323. This indicates that longer naps might be more beneficial for cognitive performance in the context of high caffeine intake.

4.5 Validity Checking

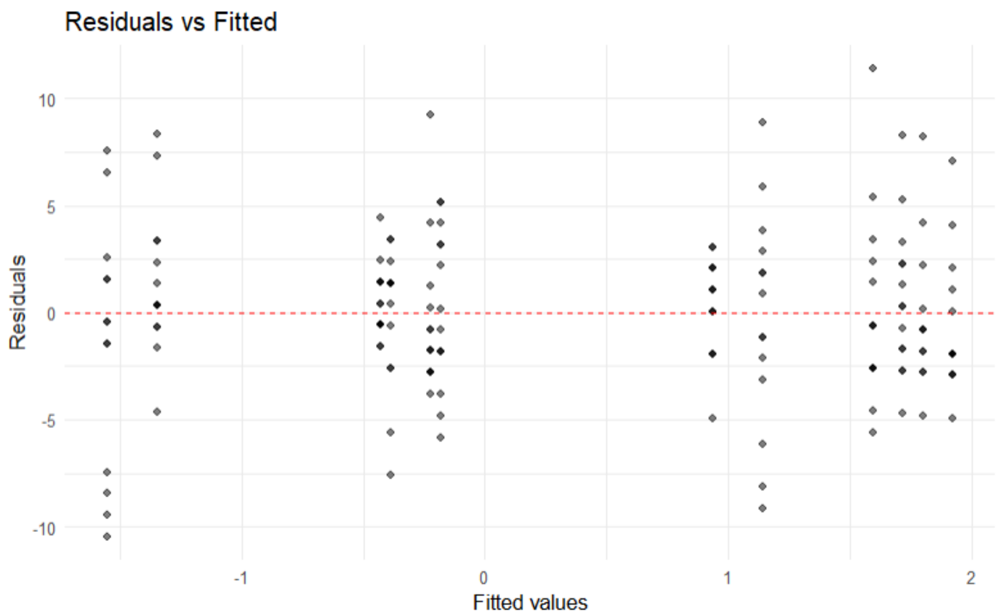

Figure 12: Residual Plot of the Model

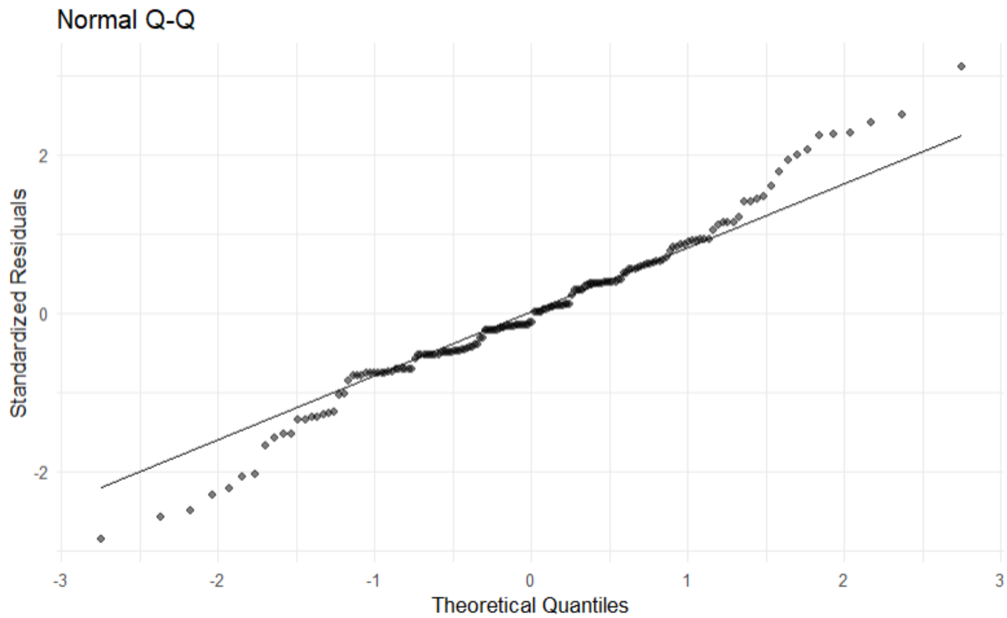

Figure 13: QQ Plot of the Model

In the residual plots, since the red line and the dots on the plot don't have any apparent pattern, the average of residuals is 0 and the constant variance assumption holds. Although there are few outliers, the distribution of the residuals is normal. Therefore, we can conclude that our model is valid and no further transformations are necessary.

5 Discussion

5.1 Conclusion

The final model can be summarized as $y_{ik} = \mu + \tau_i + \beta_k + \lambda_{ijk}$, with an R^2 value of 0.06921, where

- y_{ik}: The observed response variable for the i-th level of nap duration and the k-th level of coffee type.
- μ: The overall mean of the basic problem-solving test score.
- τ_i: The effect of the i-th level of nap duration.

- β_k: The effect of the k-th level of coffee type.

- λ_{ijk}: The interaction effect between the i-th level of nap duration and the k-th level of coffee type.

The ANOVA suggests that the factor nap has a statistically significant effect on problem-solving scores, while coffee does not. Nap has a p-value less than 0.05 of 0.0002332. Furthermore, TukeyHSD indicates that there is a significant difference between the 60-minute high caffeine treatment and the 15-minute nap high caffeine treatment as it has a p-value less than 0.05 of 0.0322999. Therefore the data suggests that napping may make a beneficial difference in problem-solving.

The validity of the final model is supported by the residual plots and the QQ plot. However, it is important to note a limitation of the model: the R^2 value is relatively low. This suggests that while the model is valid, it explains only a small portion of the variance in the response variable. Further research and additional variables may be needed to improve the explanatory power of the model.

5.2 Further Questions

After discovering that the factor nap has a significant effect on problem-solving abilities, several future research questions arise:

- Which is more beneficial for other cognitive tests: power naps or long naps?

- Would the outcome be different if a long-term study is conducted to assess the effects of napping and/or caffeine intake on academic performance?

- How does napping and/or caffeine intake affect other cognitive functions?

- What additional insights can be gained from this dataset, given that we have both survey data and treatment data?

6 References

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