1. Unsupervised Learning

In [1]:

```
%matplotlib inline
import scipy
import numpy as np
import itertools
import matplotlib.pyplot as plt
import time
```

1. Generating the data

First, we will generate some data for this problem. Set the number of points $N = 400$, their dimension $D = 2$, and the number of clusters $K = 2$, and generate data from the distribution $p(x|z = k) = \mathcal{N}(\mu_k, \Sigma_k)$. Sample 200 data points for $k = 1$ and 200 for $k = 2$, with

$$\mu_1 = \begin{bmatrix} 0.1 \\ 0.1 \end{bmatrix}, \mu_2 = \begin{bmatrix} 6.0 \\ 0.1 \end{bmatrix} \text{ and } \Sigma_1 = \Sigma_2 = \begin{bmatrix} 10 & 7 \\ 7 & 10 \end{bmatrix}$$

Here, $N = 400$. Since you generated the data, you already know which sample comes from which class. Run the cell in the IPython notebook to generate the data.

In [179]:

```
# TODO: Run this cell to generate the data
num_samples = 400
cov = np.array([[1., .7], [.7, 1.]]) * 10
mean_1 = [.1, .1]
mean_2 = [6., .1]

x_class1 = np.random.multivariate_normal(mean_1, cov, num_samples // 2)
x_class2 = np.random.multivariate_normal(mean_2, cov, num_samples // 2)
xy_class1 = np.column_stack((x_class1, np.zeros(num_samples // 2)))
xy_class2 = np.column_stack((x_class2, np.ones(num_samples // 2)))
data_full = np.row_stack([xy_class1, xy_class2])
np.random.shuffle(data_full)
data = data_full[:, :2]
labels = data_full[:, 2]
```

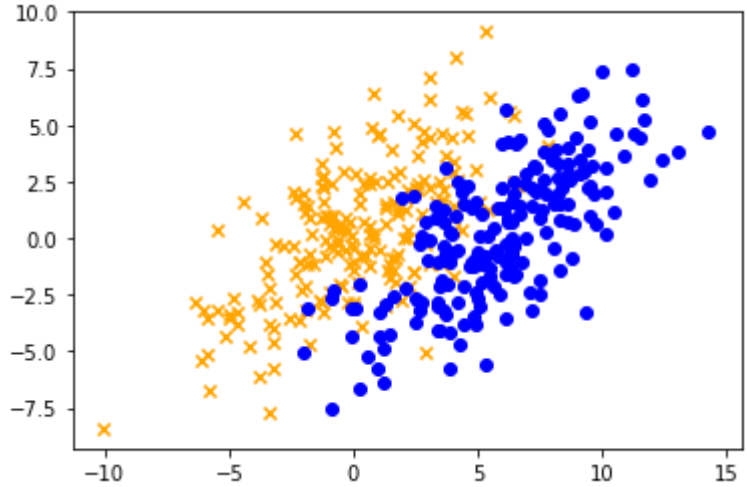
Make a scatter plot of the data points showing the true cluster assignment of each point using different color codes and shape (x for first class and circles for second class):

In [180]:

```
# TODO: Make a scatterplot for the data points showing the true cluster assignments of each
plt.scatter(xy_class1[:, 0], xy_class1[:, 1], marker='x', c='orange')
plt.scatter(xy_class2[:, 0], xy_class2[:, 1], marker='o', c='blue')
```

Out[180]:

```
<matplotlib.collections.PathCollection at 0x1ddc7e845f8>
```



2. Implement and Run K-Means algorithm

Now, we assume that the true class labels are not known. Implement the k-means algorithm for this problem. Write two functions: `km_assignment_step` , and `km_refitting_step` as given in the lecture (Here, `km_` means k-means). Identify the correct arguments, and the order to run them. Initialize the algorithm with

$$\hat{\mu}_1 = \begin{bmatrix} 0.0 \\ 0.0 \end{bmatrix}, \hat{\mu}_2 = \begin{bmatrix} 1.0 \\ 1.0 \end{bmatrix}$$

and run it until convergence. Show the resulting cluster assignments on a scatter plot either using different color codes or shape or both. Also plot the cost vs. the number of iterations. Report your misclassification error.

In [181]:

```
def cost(data, R, Mu):
    N, D = data.shape
    K = Mu.shape[1]
    J = 0
    for k in range(K):
        J += np.dot(np.linalg.norm(data - np.array([Mu[:, k], ] * N), axis=1)**2, R[:, k])
    return J
```

In [182]:

```
# TODO: K-Means Assignment Step
def km_assignment_step(data, Mu):
    """ Compute K-Means assignment step

    Args:
        data: a NxD matrix for the data points
        Mu: a DxK matrix for the cluster means locations

    Returns:
        R_new: a NxK matrix of responsibilities
    """

    # Fill this in:
    N, D = data.shape # Number of datapoints and dimension of datapoint
    K = Mu.shape[1] # number of clusters
    r = np.zeros((N, K))

    for k in range(K):
        r[:, k] = np.linalg.norm(data - np.array([Mu[:, k], ] * N), axis=1)**2

    arg_min = np.argmin(r, axis=1) # argmax/argmin along dimension 1
    R_new = np.zeros((N, K)) # Set to zeros/ones with shape (N, K)
    R_new[range(N), arg_min] = 1 # Assign to 1
    return R_new
```

In [183]:

```
# TODO: K-means Refitting Step
def km_refitting_step(data, R, Mu):
    """ Compute K-Means refitting step.

    Args:
        data: a NxD matrix for the data points
        R: a NxK matrix of responsibilities
        Mu: a DxK matrix for the cluster means locations

    Returns:
        Mu_new: a DxK matrix for the new cluster means locations
    """
    N, D = data.shape # Number of datapoints and dimension of datapoint
    K = Mu.shape[1]  # number of clusters
    Mu_new = np.zeros((D, K))
    for k in range(K):
        Rk = R[:, k]
        Mu_new[:, k] = np.dot(Rk, data) / np.sum(Rk)
    return Mu_new
```

In [215]:

```
# TODO: Run this cell to call the K-means algorithm
N, D = data.shape
K = 2
max_iter = 100
class_init = np.random.binomial(1., .5, size=N)
R = np.vstack([class_init, 1 - class_init]).T

Mu = np.zeros([D, K])
Mu[:, 1] = 1.
R.T.dot(data), np.sum(R, axis=0)

costs = []
assign_time = 0;
refit_time = 0;

for it in range(max_iter):
    assign_start = time.time()
    R = km_assignment_step(data, Mu)
    assign_time += time.time() - assign_start

    refit_start = time.time()
    Mu = km_refitting_step(data, R, Mu)
    refit_time += time.time() - refit_start

    c = cost(data, R, Mu)
    costs.append(c)
    print(it, c)

class_1 = np.where(R[:, 0])
class_2 = np.where(R[:, 1])
```

```
0 5613.351836728001
1 5318.994008627165
2 5246.14970346552
3 5233.715352943092
4 5229.082541507936
5 5227.82423246978
6 5226.8118522851
7 5226.420112289794
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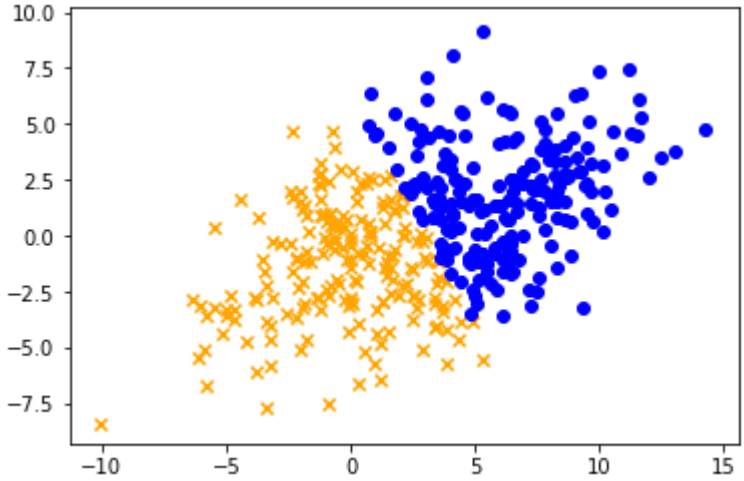
In [216]:

```
# TODO: Make a scatterplot for the data points showing the K-Means cluster assignments of e
plt.scatter(data[class_1, 0], data[class_1, 1], marker='x', c='orange')
plt.scatter(data[class_2, 0], data[class_2, 1], marker='o', c='blue')
```

Out[216]:

```
<matplotlib.collections.PathCollection at 0x1ddc932dda0>
```



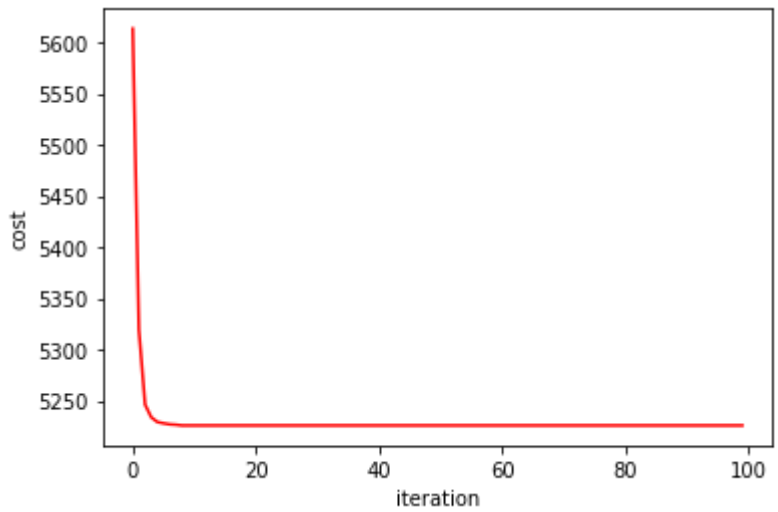
In [217]:

```
plt.xlabel("iteration")
plt.ylabel("cost")

plt.plot(range(max_iter), costs, color='red')
```

Out[217]:

```
[<matplotlib.lines.Line2D at 0x1ddc9392358>]
```



In [218]:

```
expected = np.argmax(R, axis=1)
total_mis = np.sum(np.abs(labels - expected))

print("The misclassification error is {}.".format(total_mis / data.shape[0]))
```

```
The misclassification error is 0.2475.
```

In [220]:

```
print("The total assignment time is {}.".format(assign_time))
print("The total refitting time is {}.".format(refit_time))
```

```
The total assignment time is 0.10485982894897461.
The total refitting time is 0.006563663482666016.
```

3. Implement EM algorithm for Gaussian mixtures

Next, implement the EM algorithm for Gaussian mixtures. Write three functions: `log_likelihood` , `gm_e_step` , and `gm_m_step` as given in the lecture. Identify the correct arguments, and the order to run them. Initialize the algorithm with means as in Qs 2.1 k-means initialization, covariances with $\hat{\Sigma}_1 = \hat{\Sigma}_2 = I$, and $\hat{\pi}_1 = \hat{\pi}_2$.

In addition to the update equations in the lecture, for the M (Maximization) step, you also need to use this following equation to update the covariance Σ_k:

$$\hat{\Sigma}_k = \frac{1}{N_k} \sum_{n=1}^{N} r_k^{(n)} (\mathbf{x}^{(n)} - \hat{\mu}_k)(\mathbf{x}^{(n)} - \hat{\mu}_k)^\top$$

Run the algorithm until convergence and show the resulting cluster assignments on a scatter plot either using different color codes or shape or both. Also plot the log-likelihood vs. the number of iterations. Report your misclassification error.

In [221]:

```
def normal_density(x, mu, Sigma):
    return np.exp(-.5 * np.dot(x - mu, np.linalg.solve(Sigma, x - mu))) \
        / np.sqrt(np.linalg.det(2 * np.pi * Sigma))
```

In [222]:

```
def log_likelihood(data, Mu, Sigma, Pi):
    """ Compute log likelihood on the data given the Gaussian Mixture Parameters.

    Args:
        data: a NxD matrix for the data points
        Mu: a DxK matrix for the means of the K Gaussian Mixtures
        Sigma: a list of size K with each element being DxD covariance matrix
        Pi: a vector of size K for the mixing coefficients

    Returns:
        L: a scalar denoting the log likelihood of the data given the Gaussian Mixture
    """
    # Fill this in:
    N, D = data.shape # Number of datapoints and dimension of datapoint
    K = Mu.shape[1] # number of mixtures
    L, T = 0., 0.

    for n in range(N):
        for k in range(K):
            # Compute the likelihood from the k-th Gaussian weighted by the mixing coeffici
            T += Pi[k] * normal_density(data[n], Mu[:, k], Sigma[k])

        L += np.log(T)

    return L
```

In [223]:

```
# TODO: Gaussian Mixture Expectation Step
def gm_e_step(data, Mu, Sigma, Pi):
    """ Gaussian Mixture Expectation Step.

    Args:
        data: a NxD matrix for the data points
        Mu: a DxK matrix for the means of the K Gaussian Mixtures
        Sigma: a list of size K with each element being DxD covariance matrix
        Pi: a vector of size K for the mixing coefficients

    Returns:
        Gamma: a NxK matrix of responsibilities
    """
    # Fill this in:
    N, D = data.shape # Number of datapoints and dimension of datapoint
    K = Mu.shape[1] # number of mixtures
    Gamma = np.zeros((N, K)) # zeros of shape (N,K), matrix of responsibilities

    for n in range(N):
        for k in range(K):
            Gamma[n, k] = Pi[k] * normal_density(data[n], Mu[:, k], Sigma[k])

        Gamma[n, :] /= np.sum(Gamma[n, :]) # Normalize by sum across second dimension (mixt

    return Gamma
```

In [224]:

```
# TODO: Gaussian Mixture Maximization Step
def gm_m_step(data, Gamma):
    """ Gaussian Mixture Maximization Step.

    Args:
        data: a NxD matrix for the data points
        Gamma: a NxK matrix of responsibilities

    Returns:
        Mu: a DxK matrix for the means of the K Gaussian Mixtures
        Sigma: a list of size K with each element being DxD covariance matrix
        Pi: a vector of size K for the mixing coefficients
    """
    # Fill this in:
    N, D = data.shape # Number of datapoints and dimension of datapoint
    K = Gamma.shape[1] # number of mixtures
    Nk = np.sum(Gamma, axis=0) # Sum along first axis
    Mu = np.zeros((D, K))
    Sigma = [None] * K

    for k in range(K):
        Mu[:, k] = (1 / Nk[k]) * np.dot(Gamma[:, k], data)
        func = data - np.array([Mu[:, k]] * N)
        res = np.column_stack([Gamma[:, k]] * D)
        Sigma[k] = (1 / Nk[k]) * np.dot((res * func).T, func)

    Pi = Nk / np.sum(Nk)

    return Mu, Sigma, Pi
```

In [225]:

```
# TODO: Run this cell to call the Gaussian Mixture EM algorithm
N, D = data.shape
K = 2
Mu = np.zeros([D, K])
Mu[:, 1] = 1.
Sigma = [np.eye(2), np.eye(2)]
Pi = np.ones(K) / K
Gamma = np.zeros([N, K]) # Gamma is the matrix of responsibilities

max_iter  = 200
logs = []
e_time = 0;
m_time = 0;

for it in range(max_iter):
    e_start = time.time()
    Gamma = gm_e_step(data, Mu, Sigma, Pi)
    e_time += time.time() - e_start

    m_start = time.time()
    Mu, Sigma, Pi = gm_m_step(data, Gamma)
    m_time += time.time() - m_start

    logs.append(log_likelihood(data, Mu, Sigma, Pi))
    # This function makes the computation longer, but good for debugging
    # print(it, log_likelihood(data, Mu, Sigma, Pi))

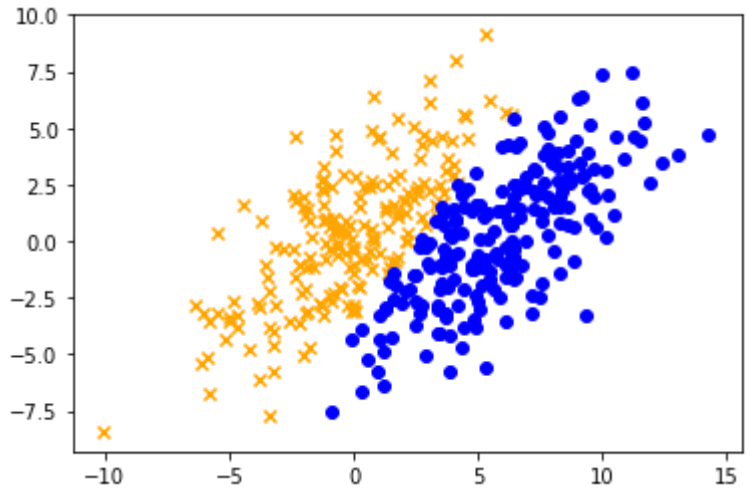
class_1 = np.where(Gamma[:, 0] >= .5)
class_2 = np.where(Gamma[:, 1] >= .5)
```

In [226]:

```
# TODO: Make a scatterplot for the data points showing the Gaussian Mixture cluster assignm
plt.scatter(data[class_1, 0], data[class_1, 1], marker='x', c='orange')
plt.scatter(data[class_2, 0], data[class_2, 1], marker='o', c='blue')
```

Out[226]:

```
<matplotlib.collections.PathCollection at 0x1ddc93f9da0>
```



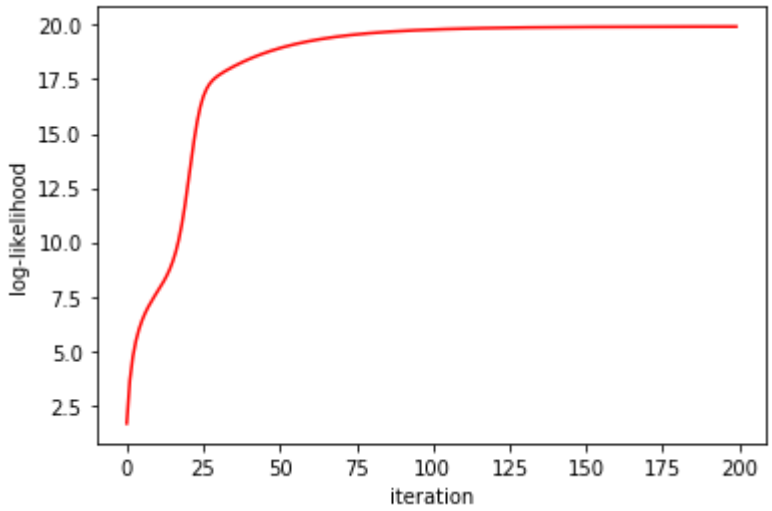
In [227]:

```
plt.xlabel("iteration")
plt.ylabel("log-likelihood")

plt.plot(range(max_iter), logs, color='red')
```

Out[227]:

```
[<matplotlib.lines.Line2D at 0x1ddc94623c8>]
```



In [228]:

```
expected = np.argmax(Gamma, axis=1)
total_mis = np.sum(np.abs(labels - expected))

print("The misclassification error is {}.".format(total_mis / data.shape[0]))
```

```
The misclassification error is 0.1.
```

In [230]:

```
print("The total E-step time is {}.".format(e_time))
print("The total M-step time is {}.".format(m_time))
```

```
The total E-step time is 11.905616760253906.
The total M-step time is 0.1569352149963379.
```

4. Comment on findings + additional experiments

Comment on the results:

- Compare the performance of k-Means and EM based on the resulting cluster assignments.
- Compare the performance of k-Means and EM based on their convergence rate. What is the bottleneck for which method?
- Experiment with 5 different data realizations (generate new data), run your algorithms, and summarize your findings. Does the algorithm performance depend on different realizations of data?

Answer

- By comparing the resulting cluster assignments, we can see that EM performs much better than k-Means. Also the k-Means' misclassification error is often 2-3 times of the EM's misclassification error, which shows that EM is more accurate.
- By comparing the convergence rates, we can see that EM takes a much longer time to converge. For k-Means, the bottleneck is the assignment step. For EM, the bottleneck is the E-step.
- The algorithm performance varies for different realizations of data. But EM always performs better than k-Means.

2. Reinforcement Learning

There are 3 files:

1. `maze.py` : defines the `MazeEnv` class, the simulation environment which the Q-learning agent will interact in.
2. `qlearning.py` : defines the `qlearn` function which you will implement, along with several helper functions. Follow the instructions in the file.
3. `plotting_utils.py` : defines several plotting and visualization utilities. In particular, you will use `plot_steps_vs_iters` , `plot_several_steps_vs_iters` , `plot_policy_from_q`

In [2]:

```
from qlearning import qlearn
from maze import MazeEnv, ProbabilisticMazeEnv
from plotting_utils import plot_steps_vs_iters, plot_several_steps_vs_iters, plot_policy_fr
```

In [ ]:

```
# %load qlearning.py
import numpy as np
import math
import copy


def qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, \
           use_softmax_policy, init_beta=None, k_exp_sched=None):
    """ Runs tabular Q learning algorithm for stochastic environment.

    Args:
        env: instance of environment object
        num_iters (int): Number of episodes to run Q-learning algorithm
        alpha (float): The learning rate between [0,1]
        gamma (float): Discount factor, between [0,1)
        epsilon (float): Probability in [0,1] that the agent selects a random move instead
                selecting greedily from Q value
        max_steps (int): Maximum number of steps in the environment per episode
        use_softmax_policy (bool): Whether to use softmax policy (True) or Epsilon-Greedy (
        init_beta (float): If using stochastic policy, sets the initial beta as the paramet
        k_exp_sched (float): If using stochastic policy, sets hyperparameter for exponentia
            on beta

    Returns:
        q_hat: A Q-value table shaped [num_states, num_actions] for environment with num_st
            number of states (e.g. num rows * num columns for grid) and num_actions number
            actions (e.g. 4 actions up/down/left/right)
        steps_vs_iters: An array of size num_iters. Each element denotes the number
            of steps in the environment that the agent took to get to the goal
            (capped to max_steps)
    """
    action_space_size = env.num_actions
    state_space_size = env.num_states
    q_hat = np.zeros(shape=(state_space_size, action_space_size))
    steps_vs_iters = np.zeros(num_iters)

    for i in range(num_iters):
        # TODO: Initialize current state by resetting the environment
        curr_state = env.reset()
        num_steps = 0
        done = False

        # TODO: Keep looping while environment isn't done and less than maximum steps
        while not done and num_steps < max_steps:
            num_steps += 1

            # Choose an action using policy derived from either softmax Q-value
            # or epsilon greedy
            if use_softmax_policy:
                assert(init_beta is not None)
                assert(k_exp_sched is not None)
                # TODO: Boltzmann stochastic policy (softmax policy)
                # Call beta_exp_schedule to get the current beta value
                beta = beta_exp_schedule(init_beta, i, k_exp_sched)
                action = softmax_policy(q_hat, beta, curr_state)
            else:
                # TODO: Epsilon-greedy
                action = epsilon_greedy(q_hat, epsilon, curr_state, \
                                        action_space_size)
```

```
            # TODO: Execute action in the environment and observe the next state, reward, a
            next_state, reward, done = env.step(action)

            # TODO: Update Q_value
            if next_state != curr_state:
                new_value = reward + gamma * max(q_hat[next_state]) - \
                q_hat[curr_state, action]
                # TODO: Use Q-learning rule to update q_hat for the curr_state and action:
                # i.e., Q(s,a) <- Q(s,a) + alpha*[reward + gamma * max_a'(Q(s',a')) - Q(s,a
                q_hat[curr_state, action] += alpha * new_value

                # TODO: Update the current state to be the next state
                curr_state = next_state

        steps_vs_iters[i] = num_steps

    return q_hat, steps_vs_iters


def epsilon_greedy(q_hat, epsilon, state, action_space_size):
    """ Chooses a random action with p_rand_move probability,
    otherwise choose the action with highest Q value for
    current observation

    Args:
        q_hat: A Q-value table shaped [num_rows, num_col, num_actions] for
            grid environment with num_rows rows and num_col columns and num_actions
            number of possible actions
        epsilon (float): Probability in [0,1] that the agent selects a random
            move instead of selecting greedily from Q value
        state: A 2-element array with integer element denoting the row and column
            that the agent is in
        action_space_size (int): number of possible actions

    Returns:
        action (int): A number in the range [0, action_space_size-1]
            denoting the action the agent will take
    """
    # TODO: Implement your code here
    # Hint: Sample from a uniform distribution and check if the sample is below
    # a certain threshold
    rand_prob = np.random.uniform()

    if rand_prob < epsilon:
        action = np.random.randint(action_space_size)
    else:
        values = q_hat[state]
        max_value = max(values)
        actions = [i for i in range(action_space_size) if values[i] == max_value]
        action = np.random.choice(actions)

    return action


def softmax_policy(q_hat, beta, state):
    """ Choose action using policy derived from Q, using
    softmax of the Q values divided by the temperature.

    Args:
        q_hat: A Q-value table shaped [num_rows, num_col, num_actions] for
```

```
            grid environment with num_rows rows and num_col columns
        beta (float): Parameter for controlling the stochasticity of the action
        state: A 2-element array with integer element denoting the row and column
            that the agent is in

    Returns:
        action (int): A number in the range [0, action_space_size-1]
            denoting the action the agent will take
    """
    # TODO: Implement your code here
    # Hint: use the stable_softmax function defined below
    x = beta * q_hat[state]
    softmax = stable_softmax(x, 0)
    action = np.random.choice(q_hat.shape[1], p=softmax)

    return action


def beta_exp_schedule(init_beta, iteration, k=0.1):
    beta = init_beta * np.exp(k * iteration)
    return beta


def stable_softmax(x, axis=2):
    """ Numerically stable softmax:
    softmax(x) = e^x /(sum(e^x))
               = e^x / (e^max(x) * sum(e^x/e^max(x)))

    Args:
        x: An N-dimensional array of floats
        axis: The axis for normalizing over.

    Returns:
        output: softmax(x) along the specified dimension
    """
    max_x = np.max(x, axis, keepdims=True)
    z = np.exp(x - max_x)
    output = z / np.sum(z, axis, keepdims=True)

    return output
```

1. Basic Q Learning experiments

(a) Run your algorithm several times on the given environment. Use the following hyperparameters:

1. Number of episodes = 200
2. Alpha (α) learning rate = 1.0
3. Maximum number of steps per episode = 100. An episode ends when the agent reaches a goal state, or uses the maximum number of steps per episode
4. Gamma (γ) discount factor = 0.9
5. Epsilon (ϵ) for ϵ-greedy = 0.1 (10% of the time). Note that we should "break-ties" when the Q-values are zero for all the actions (happens initially) by essentially choosing uniformly from the action. So now you have two conditions to act randomly: for epsilon amount of the time, or if the Q values are all zero.

In [6]:

```
# TODO: Fill this in
num_iters = 200
alpha = 1.0
gamma = 0.9
epsilon = 0.1
max_steps = 100
use_softmax_policy = False

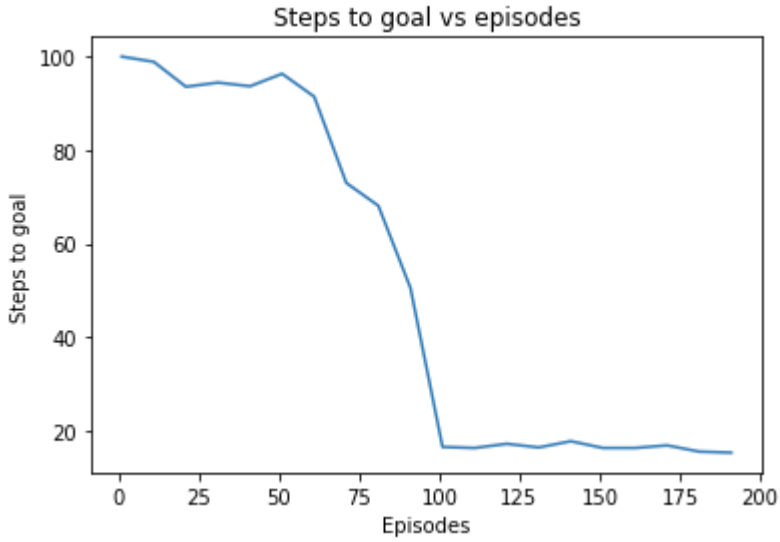
# TODO: Instantiate the MazeEnv environment with default arguments
env = MazeEnv()

# TODO: Run Q-learning:
q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, \
                               max_steps, use_softmax_policy)
```

Plot the steps to goal vs training iterations (episodes):

In [7]:

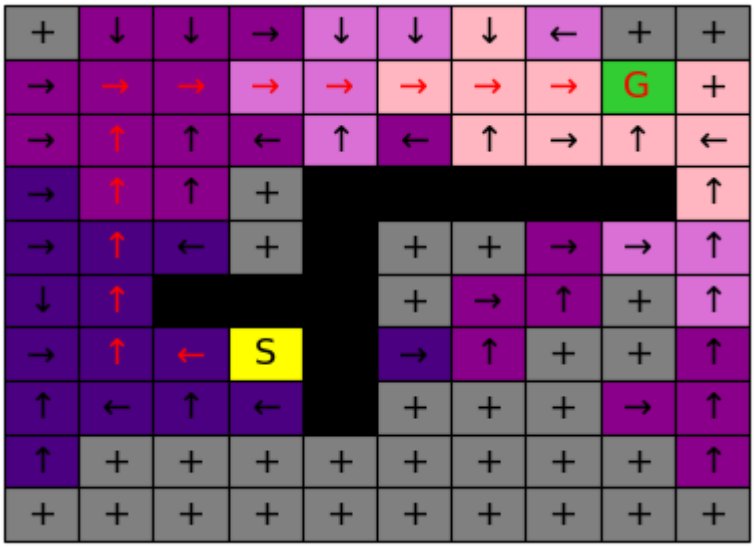
```
# TODO: Plot the steps vs iterations
plot_steps_vs_iters(steps_vs_iters)
```



Visualize the learned greedy policy from the Q values:

In [8]:

```
# TODO: plot the policy from the Q value
plot_policy_from_q(q_hat, env)
```




```
<Figure size 720x720 with 0 Axes>
```

Summary

- Given a goal location, the algorithm returns the optimal path to the goal location. But it cannot always give the shortest path, more iterations or a higher epsilon may be needed.

(b) Run your algorithm by passing in a list of 2 goal locations: (1,8) and (5,6). Note: we are using 0-indexing, where (0,0) is top left corner. Report on the results.

In [9]:

```
# TODO: Fill this in (same as before)
num_iters = 200
alpha = 1.0
gamma = 0.9
epsilon = 0.1
max_steps = 100
use_softmax_policy = False

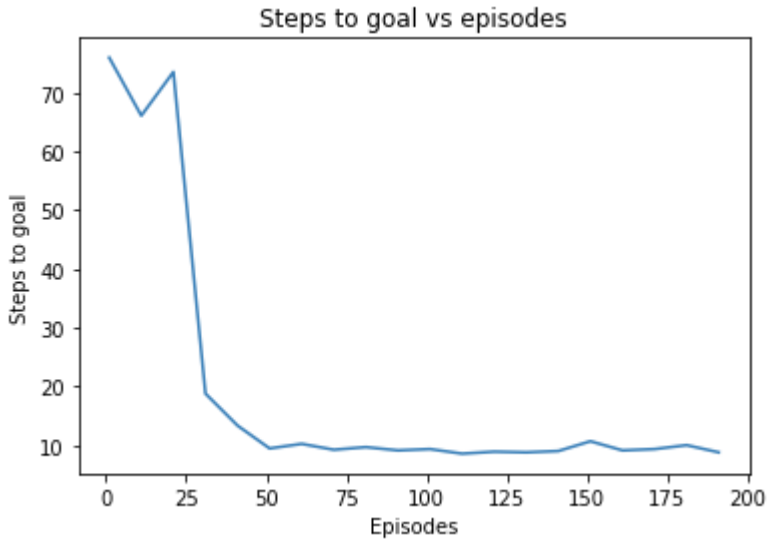
# TODO: Set the goal
goal_locs = [[1, 8], [5, 6]]
env = MazeEnv(goals=goal_locs)

# TODO: Run Q-learning:
q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, \
                               max_steps, use_softmax_policy)
```

Plot the steps to goal vs training iterations (episodes):

In [10]:

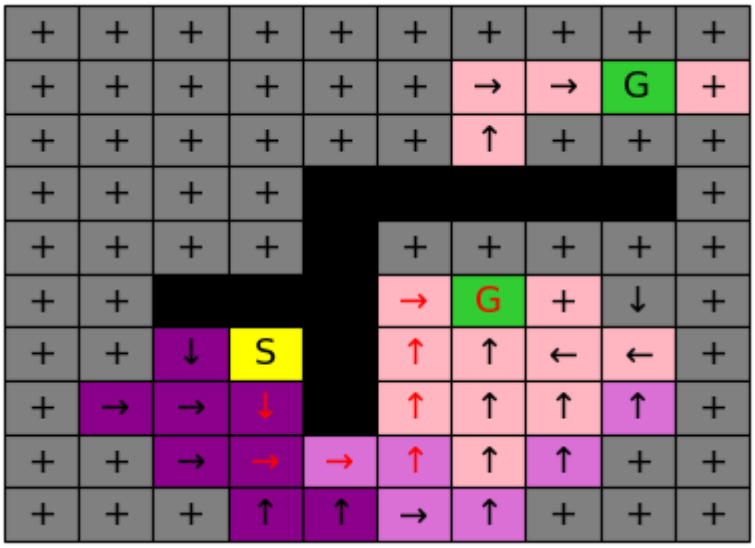
```
# TODO: Plot the steps vs iterations
plot_steps_vs_iters(steps_vs_iters)
```



Plot the steps to goal vs training iterations (episodes):

In [11]:

```
# TODO: plot the policy from the Q values
plot_policy_from_q(q_hat, env)
```




```
<Figure size 720x720 with 0 Axes>
```

Summary

- Given two different goal locations, the algorithm returns the shortest path to the nearest goal location.

2. Experiment with the exploration strategy, in the original environment

(a) Try different ϵ values in ϵ-greedy exploration: We asked you to use a rate of ϵ=10%, but try also 50% and 1%. Graph the results (for 3 epsilon values) and discuss the costs and benefits of higher and lower exploration rates.

In [18]:

```
# TODO: Fill this in (same as before)
num_iters = 200
alpha = 1.0
gamma = 0.9
max_steps = 100
use_softmax_policy = False

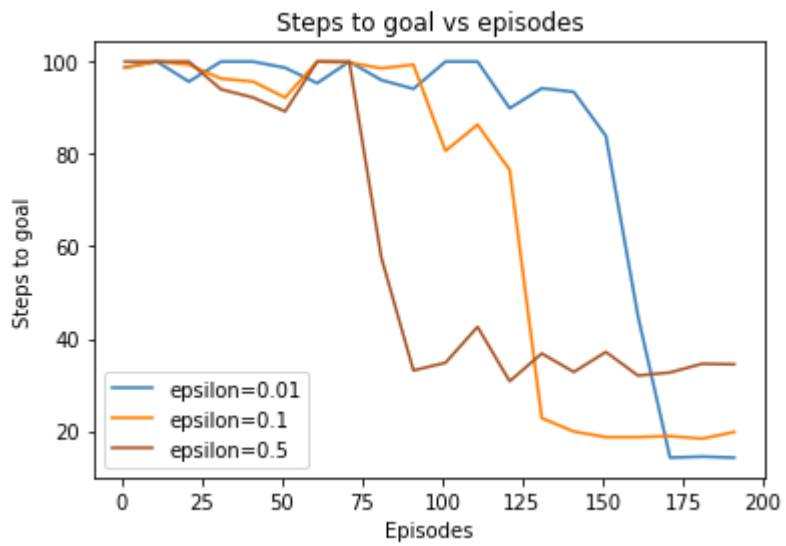
# TODO: set the epsilon lists in increasing order:
epsilon_list = [0.01, 0.1, 0.5]

env = MazeEnv()

steps_vs_iters_list = []
for epsilon in epsilon_list:
    q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, \
                                   max_steps, use_softmax_policy)
    steps_vs_iters_list.append(steps_vs_iters)
```

In [19]:

```
# TODO: Plot the results
label_list = ["epsilon={}".format(eps) for eps in epsilon_list]
plot_several_steps_vs_iters(steps_vs_iters_list, label_list)
```



Summary

- Given different epsilons. The algorithm's performance varies a lot.
- For higher exploration rate, the steps drops much earlier. But when it converges, the steps is the highest.
- For lower exploration rate, the steps drops very late. But when it converges, the steps is the lowest.

(b) Try exploring with policy derived from **softmax of Q-values** described in the Q learning lecture. Use the values of $\beta \in \{1, 3, 6\}$ for your experiment, keeping β fixed throughout the training.

In [38]:

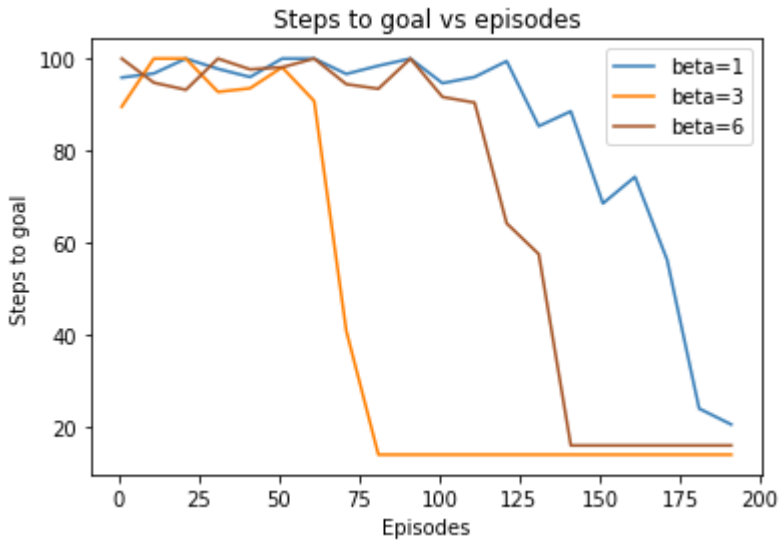
```
# TODO: Fill this in for Static Beta with softmax of Q-values
num_iters = 200
alpha = 1.0
gamma = 0.9
epsilon = 0.1
max_steps = 100

# TODO: Set the beta
beta_list = [1, 3, 6]
use_softmax_policy = True
k_exp_schedule = 0.0 # (float) choose k such that we have a constant beta during training

env = MazeEnv()
steps_vs_iters_list = []
for beta in beta_list:
    q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, max_steps, \
                                   use_softmax_policy, beta, k_exp_schedule)
    steps_vs_iters_list.append(steps_vs_iters)
```

In [39]:

```
label_list = ["beta={}".format(beta) for beta in beta_list]
# TODO:
plot_several_steps_vs_iters(steps_vs_iters_list, label_list)
```



Summary

- Using softmax policy. Given different betas and keep beta fixed throughout the training. The algorithm's performance varies a lot.
- Beta that is higher or lower takes longer time to converge and the steps when converges is higher. Beta = 3 seems to a better choice.

(c) Instead of fixing the $\beta = \beta_0$ to the initial value, we will increase the value of β as the number of episodes t increase:

$$\beta(t) = \beta_0 e^{kt}$$

That is, the β value is fixed for a particular episode. Run the training again for different values of $k \in \{0.05, 0.1, 0.25, 0.5\}$, keeping $\beta_0 = 1.0$. Compare the results obtained with this approach to those obtained with a static β value.

In [56]:

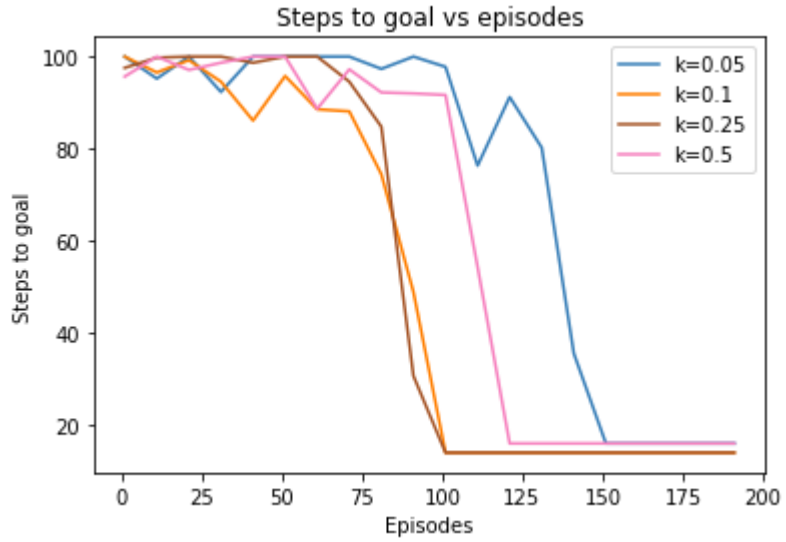
```
# TODO: Fill this in for Dynamic Beta
num_iters = 200
alpha = 1.0
gamma = 0.9
epsilon = 0.1
max_steps = 100

# TODO: Set the beta
beta = 1.0
use_softmax_policy = True
k_exp_schedule_list = [0.05, 0.1, 0.25, 0.5]
env = MazeEnv()

steps_vs_iters_list = []
for k_exp_schedule in k_exp_schedule_list:
    q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, epsilon, \
                                   max_steps, use_softmax_policy, beta, k_exp_schedule)
    steps_vs_iters_list.append(steps_vs_iters)
```

In [57]:

```
# TODO: Plot the steps vs iterations
label_list = ["k={}".format(k_exp_schedule) for k_exp_schedule in k_exp_schedule_list]
plot_several_steps_vs_iters(steps_vs_iters_list, label_list)
```



Summary

- Given different ks. The algorithm's performance varies.
- k that is higher or lower takes longer time to converge and the steps when converges is higher. k = 0.25 seems to a better choice.
- Comparing the results obtained with this approach to those obtained with a static beta value (e.g. beta = 1 from the previous graph): increasing the value of beta as the number of episodes t increase has a better performance.

3. Stochastic Environments

(a) Make the environment stochastic (uncertain), such that the agent only has a 95% chance of moving in the chosen direction, and has a 5% chance of moving in some random direction.

In [ ]:

```
# %load maze.py
import numpy as np
import copy
import math

ACTION_MEANING = {
    0: "UP",
    1: "RIGHT",
    2: "LEFT",
    3: "DOWN",
}

SPACE_MEANING = {
    1: "ROAD",
    0: "BARRIER",
    -1: "GOAL",
}


class MazeEnv:

    def __init__(self, start=[6, 3], goals=[[1, 8]]):
        """Deterministic Maze Environment"""

        self.m_size = 10
        self.reward = 10
        self.num_actions = 4
        self.num_states = self.m_size * self.m_size

        self.map = np.ones((self.m_size, self.m_size))
        self.map[3, 4:9] = 0
        self.map[4:8, 4] = 0
        self.map[5, 2:4] = 0

        for goal in goals:
            self.map[goal[0], goal[1]] = -1

        self.start = start
        self.goals = goals
        self.obs = self.start

    def step(self, a):
        """ Perform a action on the environment

            Args:
                a (int): action integer

            Returns:
                obs (list): observation list
                reward (int): reward for such action
                done (int): whether the goal is reached
        """
        done, reward = False, 0.0
        next_obs = copy.copy(self.obs)

        if a == 0:
            next_obs[0] = next_obs[0] - 1
        elif a == 1:
            next_obs[1] = next_obs[1] + 1
```

```
        elif a == 2:
            next_obs[1] = next_obs[1] - 1
        elif a == 3:
            next_obs[0] = next_obs[0] + 1
        else:
            raise Exception("Action is Not Valid")

        if self.is_valid_obs(next_obs):
            self.obs = next_obs

        if self.map[self.obs[0], self.obs[1]] == -1:
            reward = self.reward
            done = True

        state = self.get_state_from_coords(self.obs[0], self.obs[1])

        return state, reward, done

    def is_valid_obs(self, obs):
        """ Check whether the observation is valid

            Args:
                obs (list): observation [x, y]

            Returns:
                is_valid (bool)
        """

        if obs[0] >= self.m_size or obs[0] < 0:
            return False

        if obs[1] >= self.m_size or obs[1] < 0:
            return False

        if self.map[obs[0], obs[1]] == 0:
            return False

        return True

    @property
    def _get_obs(self):
        """ Get current observation
        """
        return self.obs

    @property
    def _get_state(self):
        """ Get current observation
        """
        return self.get_state_from_coords(self.obs[0], self.obs[1])

    @property
    def _get_start_state(self):
        """ Get the start state
        """
        return self.get_state_from_coords(self.start[0], self.start[1])

    @property
    def _get_goal_state(self):
        """ Get the start state
        """
```

```
        goals = []
        for goal in self.goals:
            goals.append(self.get_state_from_coords(goal[0], goal[1]))
        return goals

    def reset(self):
        """ Reset the observation into starting point
        """
        self.obs = self.start
        state = self.get_state_from_coords(self.obs[0], self.obs[1])
        return state

    def get_state_from_coords(self, row, col):
        state = row * self.m_size + col
        return state

    def get_coords_from_state(self, state):
        row = math.floor(state/self.m_size)
        col = state % self.m_size
        return row, col


class ProbabilisticMazeEnv(MazeEnv):
    """ (Q2.3) Hints: you can refer the implementation in MazeEnv
    """

    def __init__(self, goals=[[2, 8]], p_random=0.05):
        """ Probabilistic Maze Environment

            Args:
                goals (list): list of goals coordinates
                p_random (float): random action rate
        """

        super().__init__(goals=goals)
        self.p_random = p_random

    def step(self, a):
        rand_prob = np.random.uniform()

        if rand_prob < self.p_random:
            a = np.random.randint(self.num_actions)

        return super().step(a)
```

(b) Change the learning rule to handle the non-determinism, and experiment with different probability of environment performing random action $p_{rand} \in \{0.05, 0.1, 0.25, 0.5\}$ in this new rule. How does performance vary as the environment becomes more stochastic?

Use the same parameters as in first part, except change the alpha (α) value to be **less than 1**, e.g. 0.5.

In [6]:

```
# TODO: Use the same parameters as in the first part, except change alpha
num_iters = 200
alpha = 0.5
gamma = 0.9
epsilon = 0.1
max_steps = 100
use_softmax_policy = False

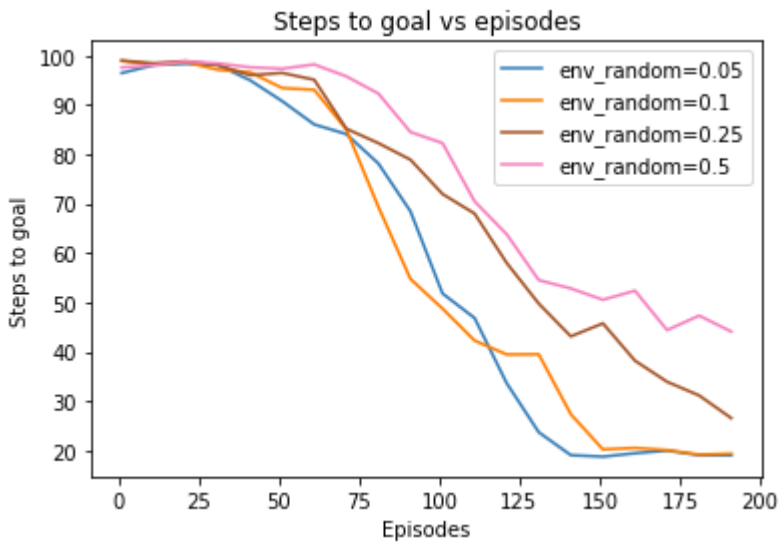
# Set the environment probability of random
env_p_rand_list = [0.05, 0.1, 0.25, 0.5]

steps_vs_iters_list = []
for env_p_rand in env_p_rand_list:
    # Instantiate with ProbabilisticMazeEnv
    env = ProbabilisticMazeEnv(p_random=env_p_rand)

    # Note: We will repeat for several runs of the algorithm to make the result less noisy
    avg_steps_vs_iters = np.zeros(num_iters)
    for i in range(10):
        q_hat, steps_vs_iters = qlearn(env, num_iters, alpha, gamma, \
                                       epsilon, max_steps, use_softmax_policy)
        avg_steps_vs_iters += steps_vs_iters
    avg_steps_vs_iters /= 10
    steps_vs_iters_list.append(avg_steps_vs_iters)
```

In [7]:

```
label_list = ["env_random={}".format(env_p_rand) for env_p_rand in env_p_rand_list]
plot_several_steps_vs_iters(steps_vs_iters_list, label_list)
```



Summary

- Given different env_random(s). The algorithm's performance varies slightly.
- As the environment becomes more stochastic, the algorithm performs worse. A very small env_random seems to better a choice.

3. Did you complete the course evaluation?

Yes.

UNIVERSITY OF TORONTO

Your List of Course Evaluations to Complete

Task Owner: Zhuozi Zou
Project Title: FAS Fall 2019 Undergrad
Category: Arts and Science
Subcategory: 2019 Fall

Subject	Due date	Status
Intro to Soft Eng CSC301H1-F-LEC0101	Friday, December 6, 2019	Completed
Intro Machine Learning CSC311H1-F-LEC0201	Friday, December 6, 2019	Completed
Prog Languages CSC324H1-F-LEC0101	Friday, December 6, 2019	Completed
Operating Systems CSC369H1-F-LEC5101	Friday, December 6, 2019	Completed

Mobile Version | Standard Version

Powered by blue

In [ ]:

```

```