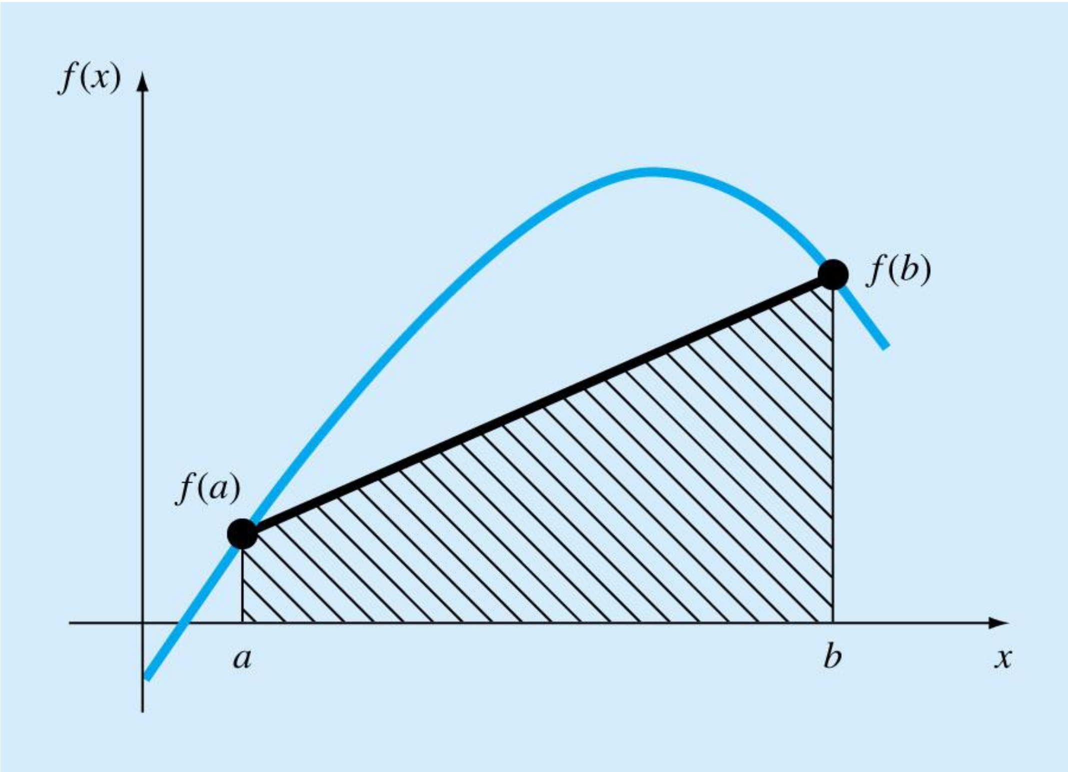
NUMERICAL INTEGRATION

Definite Integral

TRAPEZOIDAL RULE

The trapezoidal rule is the simplest way to approximate an integral

$$I = \int_a^b f(x)dx$$



$$\boxed{I = (b-a)\frac{f(a)+f(b)}{2}}$$

EXAMPLE:

Given the f(x) as $\mathbf{f(x) = 0.2 + 25}\boldsymbol{x}\mathbf{ - 200}\boldsymbol{x}^{\mathbf{2}}\mathbf{ + 675}\boldsymbol{x}^{\mathbf{3}}\mathbf{ - 900}\boldsymbol{x}^{\mathbf{4}}\mathbf{ + 400}\boldsymbol{x}^{\mathbf{5}}$

When we evaluate the definite integral $I = \int_0^{0.8} f(x)dx$ using calculus,

the true answer or the result is 1.640533.

Using Trapezoidal rule, we will have

$$I = (b - a)\frac{f(a) + f(b)}{2}$$

$$= (0.8 - 0)\frac{0.2 + 0.232}{2}$$

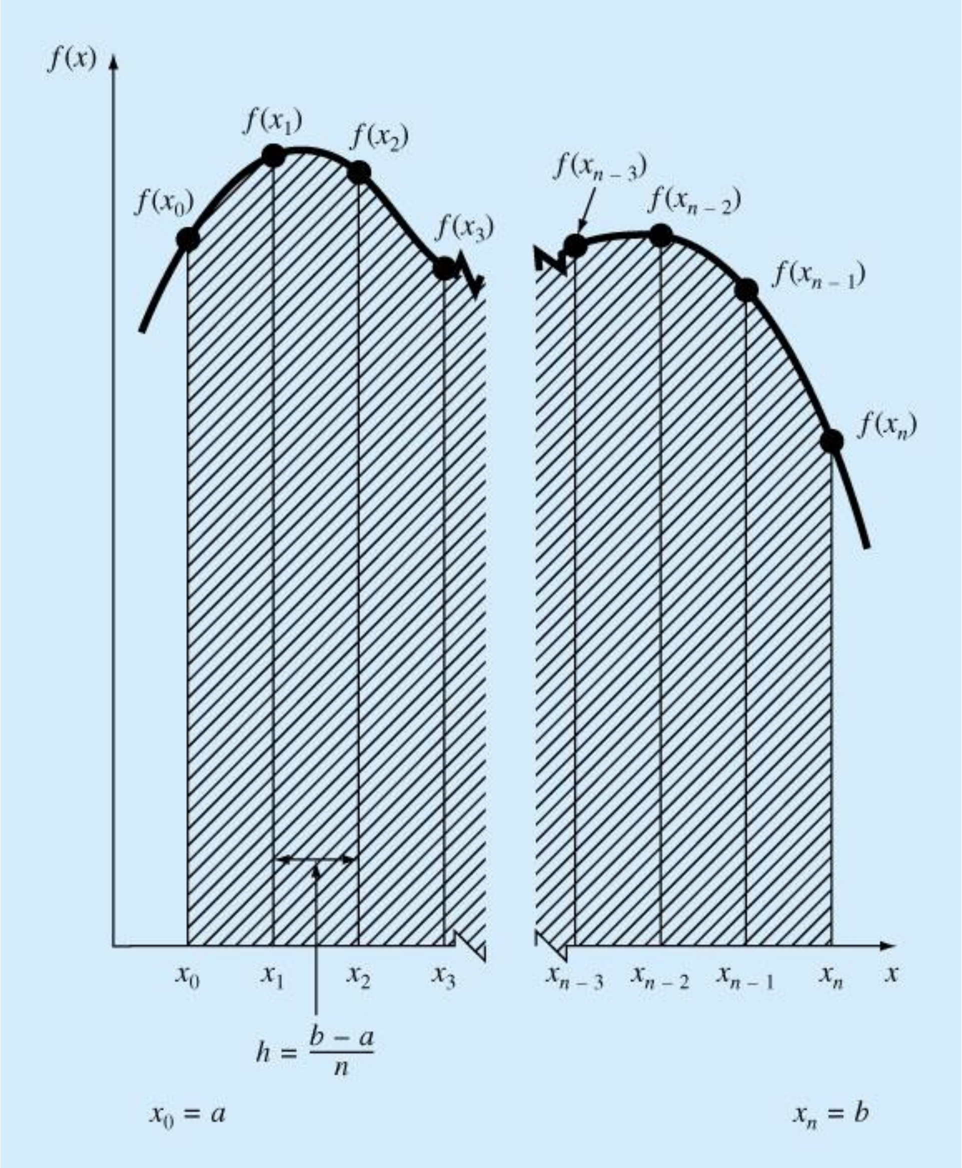
$$\boxed{I = 0.1728}$$

$$True\ Error = \varepsilon_{True} = \left|\frac{1.640533 - 0.1728}{1.640533}\right| x100\% = 89.47\%$$

MULTIPLE APPLICATION OF TRAPEZOIDAL RULE

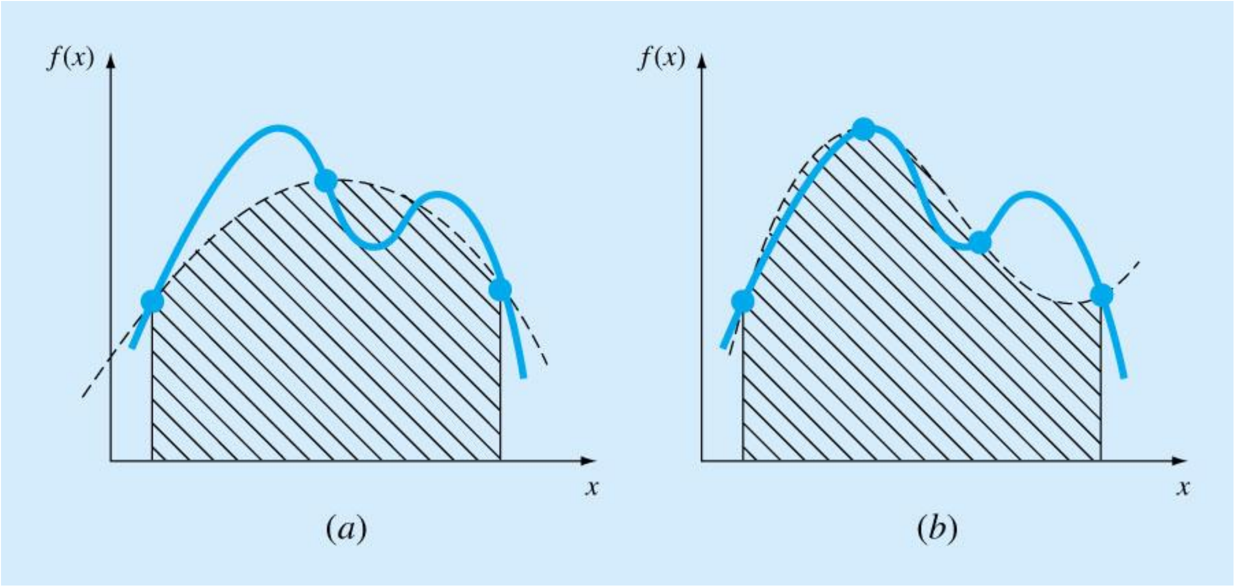
One way to improve the accuracy of the trapezoidal rule is to divide the integration interval from a to b into a number of segments and apply the method to each segment.

$$I = h\frac{f(x_0) + f(x_1)}{2} + h\frac{f(x_1) + f(x_2)}{2} + \cdots + h\frac{f(x_{n-1}) + f(x_n)}{2}$$



SIMPSON'S RULE

A more accurate estimate of Integral is the Simpson's Rule



(a) Simpson's 1/3 Rule

(b) Simpson's 3/8 Rule

SIMPSON'S RULE

Simpson's 1/3 rule or simply Simpson's Rule assumes three equi-spaced data/integration points.

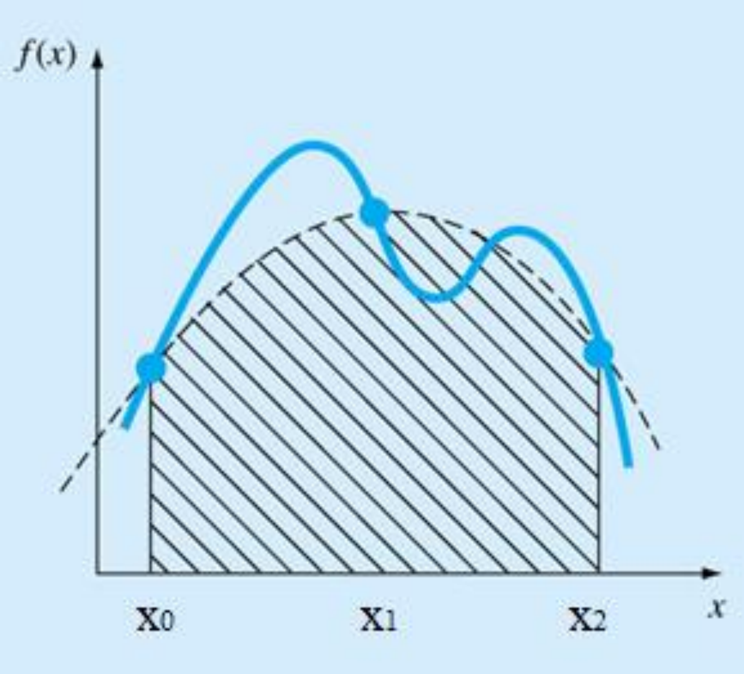
$$I = \int_a^b f(x)dx$$

$$I = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]$$

Where: $h = \frac{b-a}{2}$

$x_0 = a$

$x_2 = b$



SIMPSON'S SECOND RULE

Simpson's Second Rule is also called Simpson's 3/8 rule. It assumes four equi-spaced data/integration points.

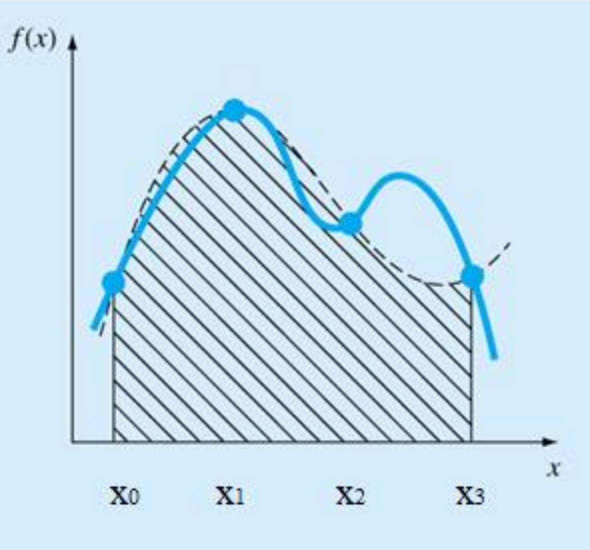
$$I = \int_a^b f(x)dx$$

$$I = \frac{3}{8}h[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]$$

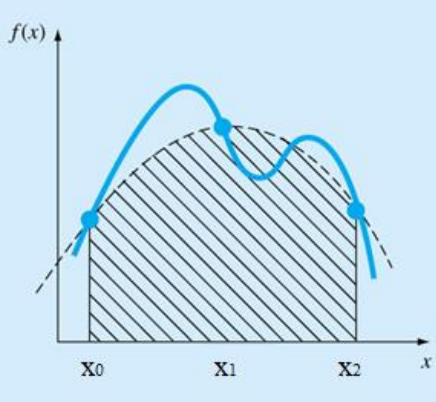
Where: $h = \frac{b-a}{3}$

$$x_0 = a$$

$$x_3 = b$$



EXAMPLE 1 – SIMPSON'S 1/3 RULE



Given the f(x) as $\mathbf{f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5}$

Shown below is the calculation when we evaluate $I = \int_0^{0.8} f(x)dx$ using

Simpson's 1/3 rule

$$\boldsymbol{I = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]}$$

$$h = x_1 = \frac{b-a}{2} = \frac{0.8-0}{2} = 0.4$$

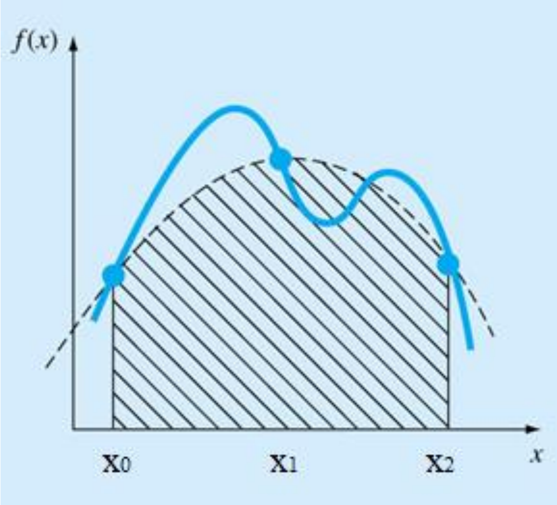
$$x_0 = a = 0$$

$$x_2 = b = 0.8$$

$$I = \frac{0.4}{3}[f(0) + 4f(0.4) + f(0.8)]$$

$$I = \frac{0.4}{3}[0.2 + 4(2.456) + 0.232] = 1.3675$$

$$True\ Error = \varepsilon_{True} = \left|\frac{1.640533 - 1.3675}{1.640533}\right| x100\% = 16.64\%$$



EXAMPLE 2 – SIMPSON'S 1/3 RULE

Evaluate $I = \int_2^7 e^{-2.4x} dx$ using Simpson's 1/3 rule

$$\boldsymbol{I = \frac{h}{3}[f(x_0) + 4f(x_1) + f(x_2)]}$$

$$h = \frac{b-a}{2} = \frac{7-2}{2} = 2.5$$

$$x_0 = a = 2$$

$$x_1 = 2 + 2.5 = 4.5$$

$$x_2 = b = 7$$

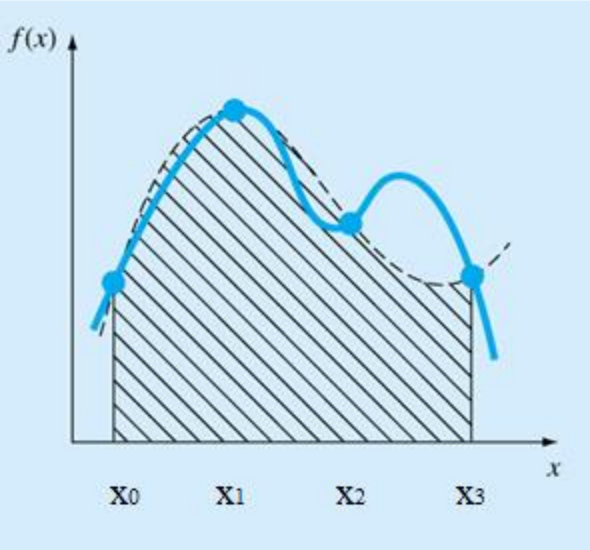
$$I = \frac{2.5}{3}[f(2) + 4f(4.5) + f(7)]$$

$$I = \frac{2.5}{3}\left[e^{-2.4(2)} + 4\left(e^{-2.4(4.5)}\right) + e^{-2.4(7)}\right] = 0.006926$$

SIMPSON'S SECOND RULE

Simpson's Second Rule is also called Simpson's 3/8 rule. It assumes four equi-spaced data/integration points.

$$I = \int_a^b f(x)dx$$



$$I = \frac{3}{8}h[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]$$

Where: $h = \frac{b-a}{3}$

$$x_0 = a$$

$$x_3 = b$$

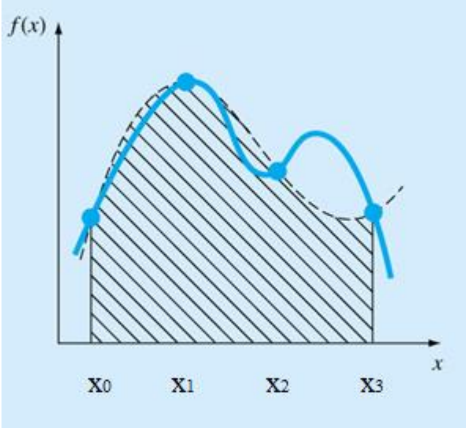
EXAMPLE – SIMPSON'S 3/8 RULE

Given the f(x) as $\mathbf{f(x) = 0.2 + 25}x\mathbf{ - 200}x^{\mathbf{2}}\mathbf{ + 675}x^{\mathbf{3}}\mathbf{ - 900}x^{\mathbf{4}}\mathbf{ + 400}x^{\mathbf{5}}$

Shown on the next slide is the calculation when we evaluate $I = \int_0^{0.8} f(x)dx$ using

Simpson's 3/8 rule

EXAMPLE – SIMPSON'S 3/8 RULE (SOLUTION)



$I = \int_0^{0.8} f(x)dx$; $\mathbf{f(x) = 0.2 + 25x - 200x^2 + 675x^3 - 900x^4 + 400x^5}$

$$\boldsymbol{I = \frac{3h}{8}[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]}$$

$$h = \frac{0.8 - 0}{3} = \frac{4}{15}$$

$x_0 = a = 0$ $\qquad f(0) = 0.2$

$x_1 = h = \frac{4}{15}$ $\qquad f\left(\frac{4}{15}\right) = 1.4327$

$x_2 = 2h = 2\left(\frac{4}{15}\right) = \frac{8}{15}$ $\qquad f\left(\frac{8}{15}\right) = 3.4872$

$x_3 = b = 0.8$ $\qquad f(0.8) = 0.232$

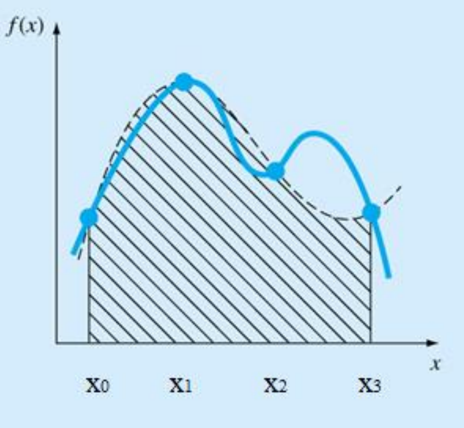
$$I = \frac{3}{8}h[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]$$

$$I = \frac{3}{8}\left(\frac{4}{15}\right)[0.2 + 3(1.4327) + 3(3.4872) + 0.232]$$

$$I = 1.51917$$

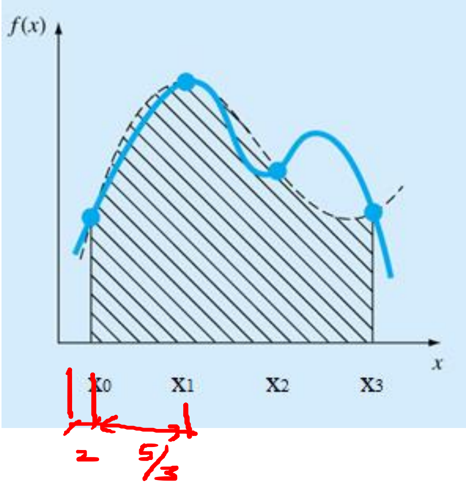
$$True\ Error = \varepsilon_{True} = \left|\frac{1.640533. -1.51917}{1.640533}\right| x100\% = 7.4\%$$

PRACTICE – SIMPSON'S 3/8 RULE



Evaluate $I = \int_2^7 e^{-2.4x} dx$ using Simpson's 3/8 rule

PRACTICE – SIMPSON'S 3/8 RULE (SOLUTION)



Evaluate $I = \int_2^7 e^{-2.4x} dx$ using Simpson's 3/8 rule

$h = \frac{7-2}{3} = \frac{5}{3}$

$x_0 = a = 2$

$x_1 = 2 + \frac{5}{3} = \frac{11}{3}$

$x_2 = \frac{11}{3} + \frac{5}{3} = \frac{16}{3}$

$x_3 = b = 7$

$$I = \int_2^7 e^{-2.4x} dx$$

$$= \frac{3}{8}(1.66)\left[e^{-2.4(2)} + 3e^{-2.4\left(\frac{11}{3}\right)} + 3e^{-2.4\left(\frac{16}{3}\right)} + e^{-2.4(7)}\right]$$

$$I = 0.005431$$