CS 4602

Introduction to Machine Learning

Convolutional Neural Network

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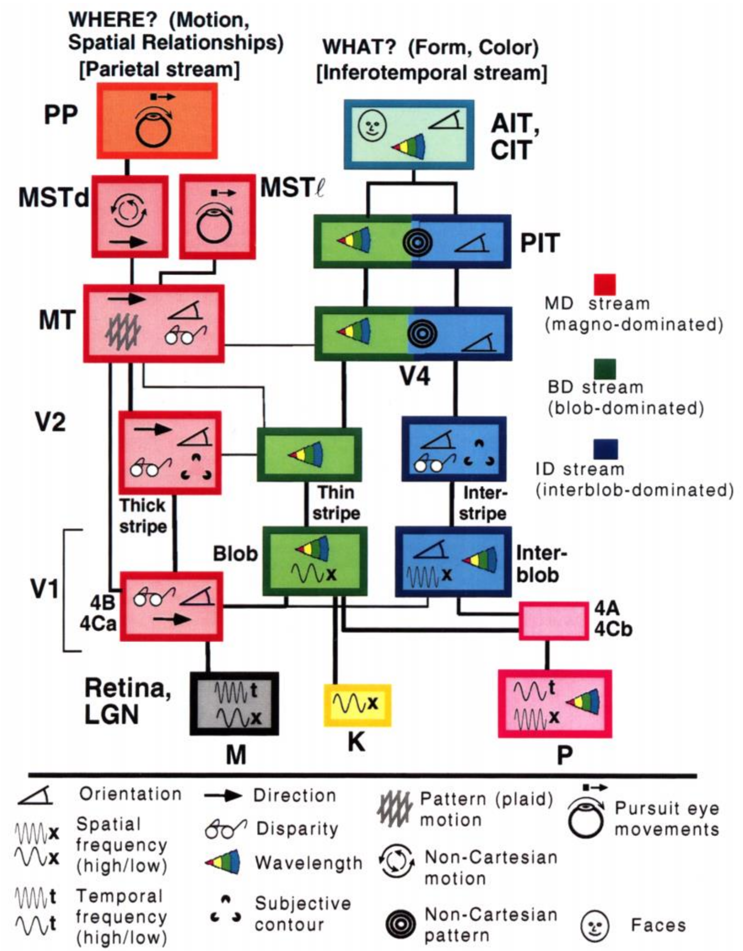
Roadmap

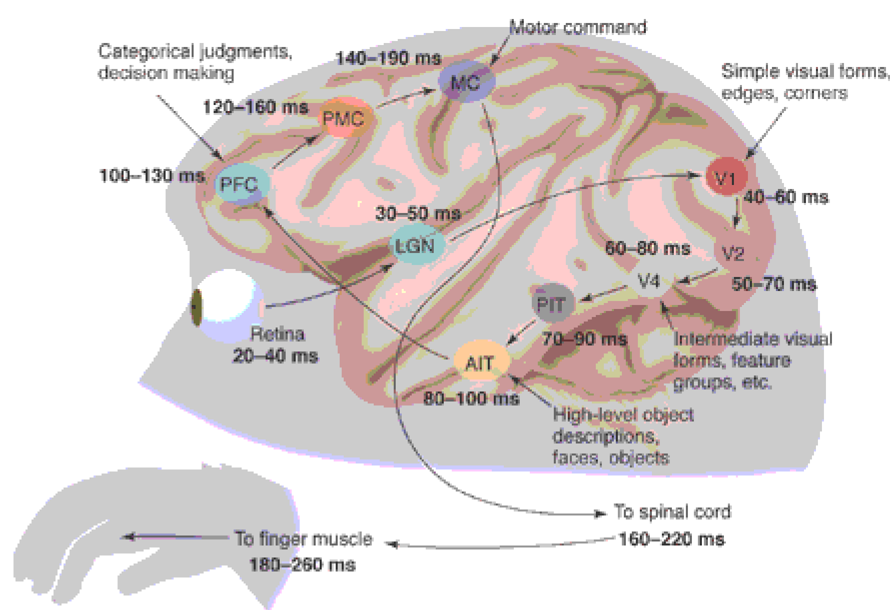
- Introduction and Basic Concepts
- Regression
- Bayesian Classifiers
- Decision Trees
- Linear Classifier
- Neural Networks
- Deep learning
- Convolutional Neural Networks
- ~~Reinforcement Learning~~
- ~~KNN~~
- Clustering
- Dimensionality reduction
- Model Selection and Evaluation

How does the brain interpret images?

- The ventral (recognition) pathway in the visual cortex has multiple stages
- Retina - LGN - V1 - V2 - V4 - PIT - AIT ....

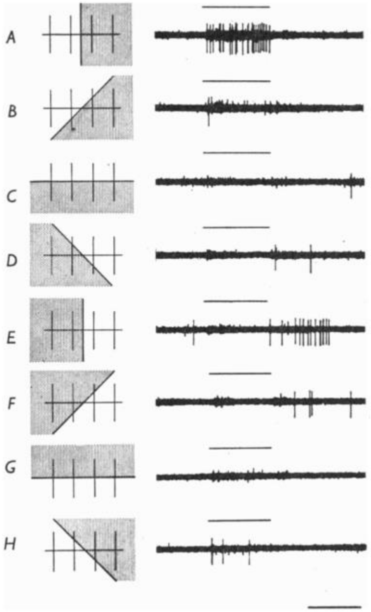


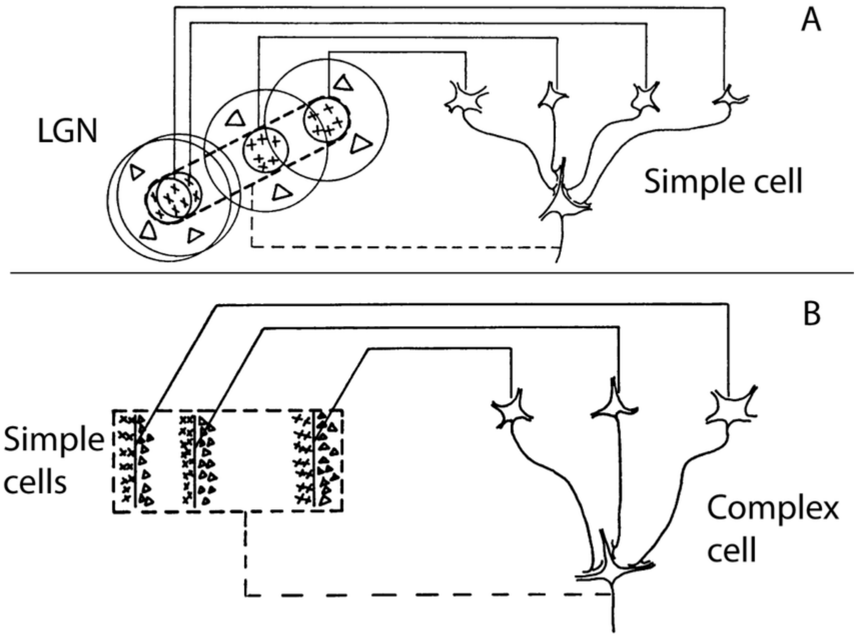
Gallant & Van Essen, 1994



Visual Cortex

- Hubel & Wiesel 1962
- Simple cells detect local features
- Complex cells “pool” the outputs of simple cells



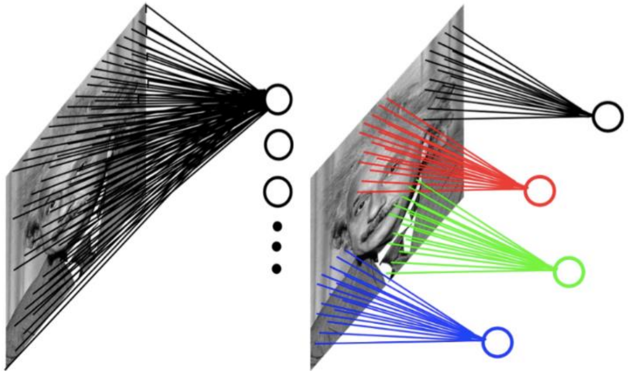


Convolutional Neural Network (CNN)

- Using a fully-connected neural network would need a large amount of parameters.
- CNNs are a special type of neural network whose hidden units are only connected to **local receptive field**
- The number of parameters needed by CNNs is much smaller.

Example: 200x200 image

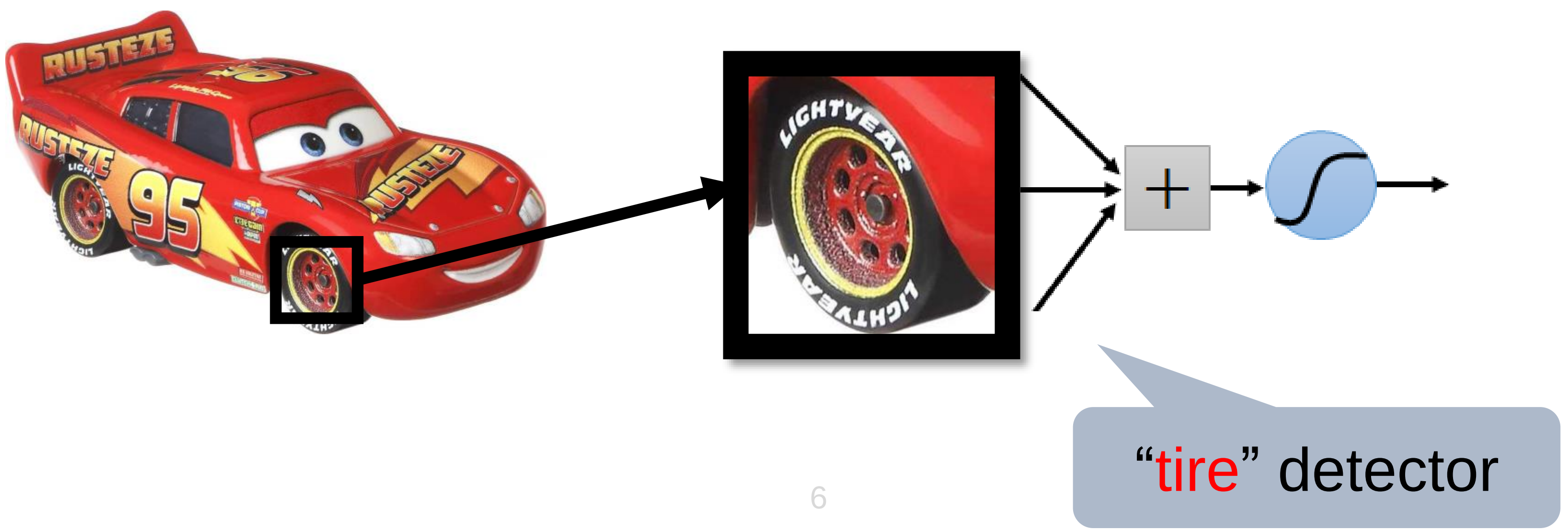
fully connected: 40,000 hidden units
=> 1.6 billion parameters



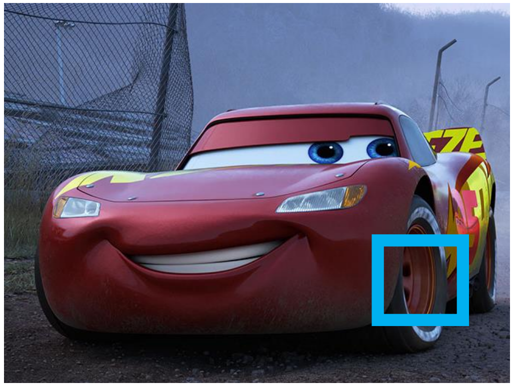
CNN: 5x5 kernel, 100 feature maps
=> 2,500 parameters

Learning a pattern

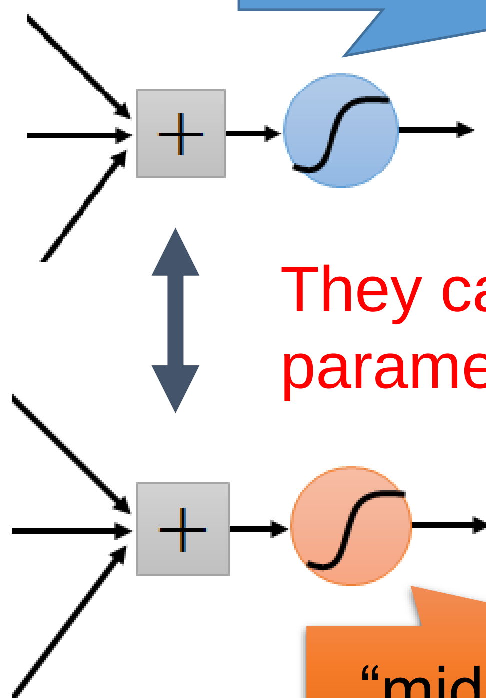
- Some patterns are much smaller than the whole image
- Can represent a small region with fewer parameters



- Same pattern appears in different places



"lower-right tire" detector



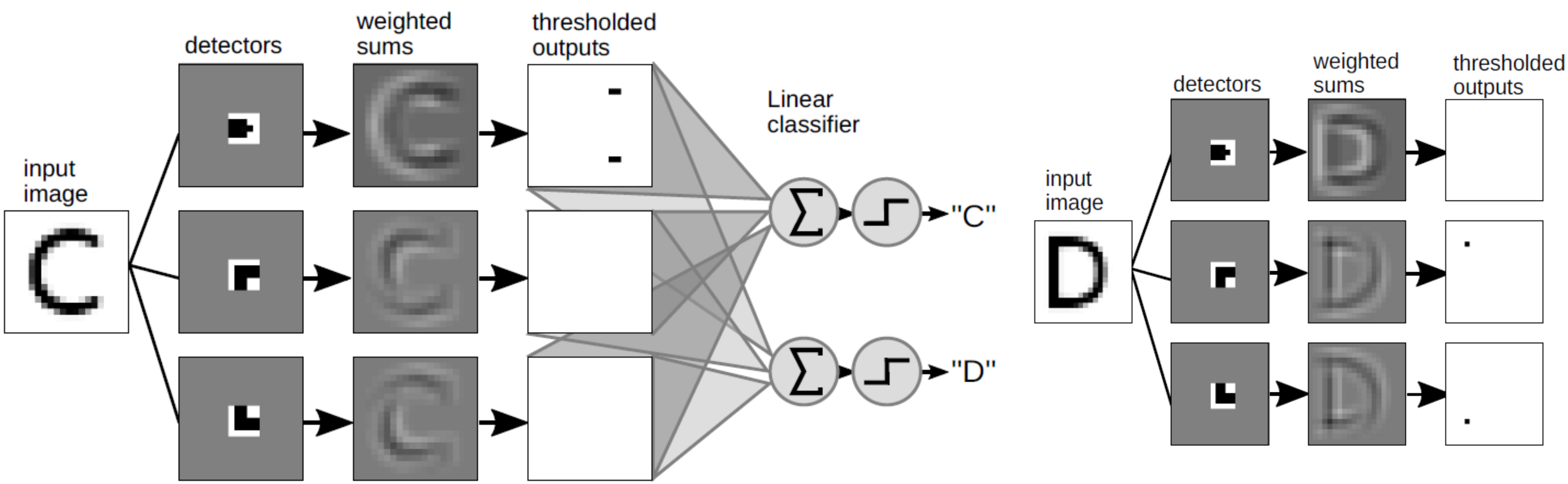
They can use same parameters.

"middle tire" detector

What about training a lot of such small detectors and each detector must move around".

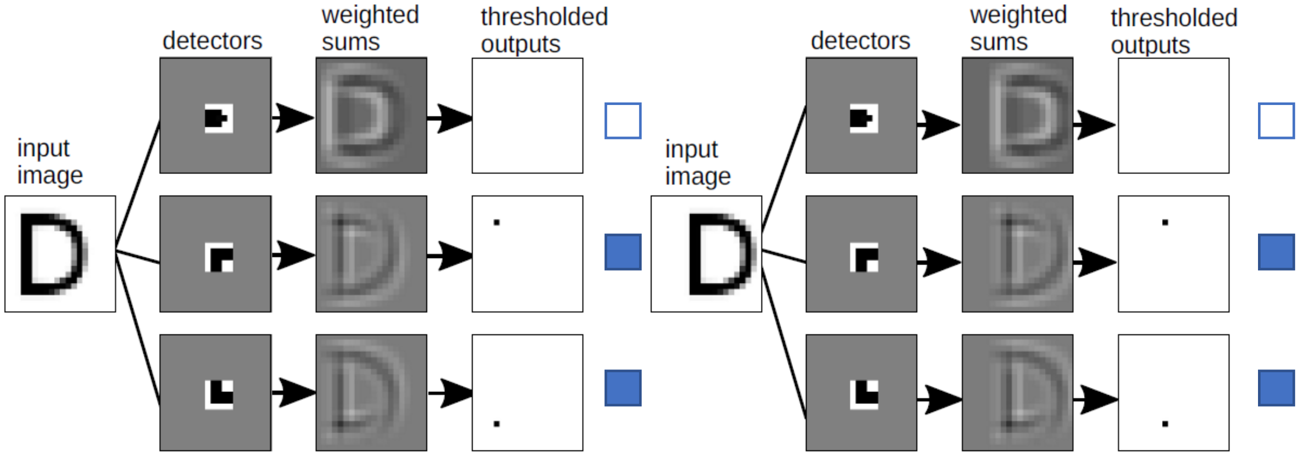
Detecting Motifs in Images

- Swipe “templates” over the image to detect motifs



Detecting Motifs in Images

- Shift invariance

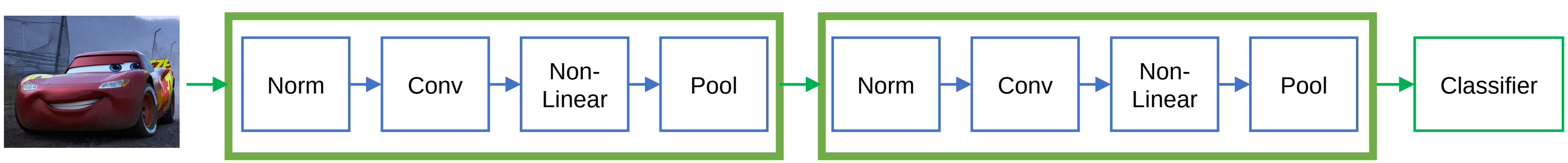


How can we have similar outputs from the model?

To combine outputs in a common pool!

Overall Architecture

- Multiple stages: Normalization→ Convolution→Non-Linearity→Pooling
  - Normalization: average removal, variance normalization...
  - Convolution: dimension expansion, projection on basis…
  - Non-Linearity: Rectification (ReLU), tanh…
  - Pooling: Max, average…



Normalization

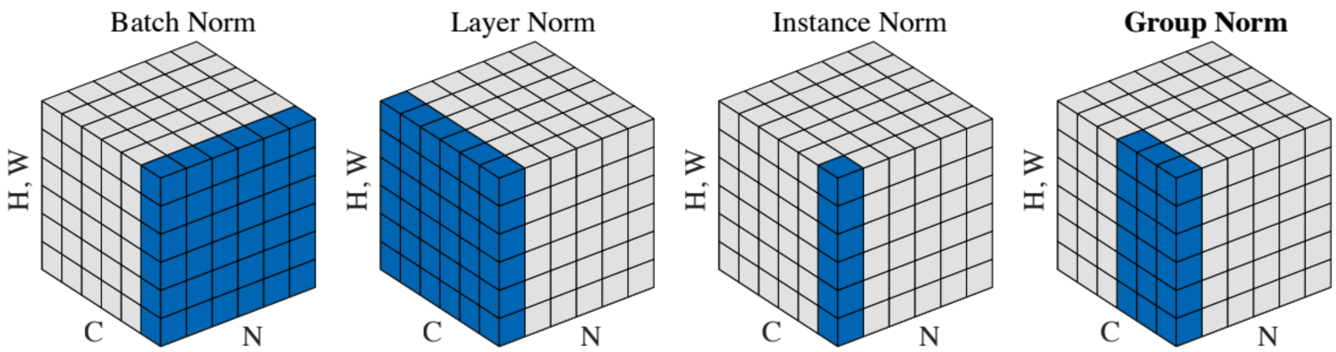


Figure 2. **Normalization methods**. Each subplot shows a feature map tensor, with N as the batch axis, C as the channel axis, and (H, W) as the spatial axes. The pixels in blue are normalized by the same mean and variance, computed by aggregating the values of these pixels.

(Wu, Y. and He, K., 2018. Group normalization. arXiv preprint arXiv: 1803.08494.)

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

Dot product → 3 -1

Convolution

If stride=2

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

3 -3

Convolution

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

stride=1

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image

3	-1	-3	-1
-3	1	0	-3
-3	-3	0	1
3	-2	-2	-1

Convolution

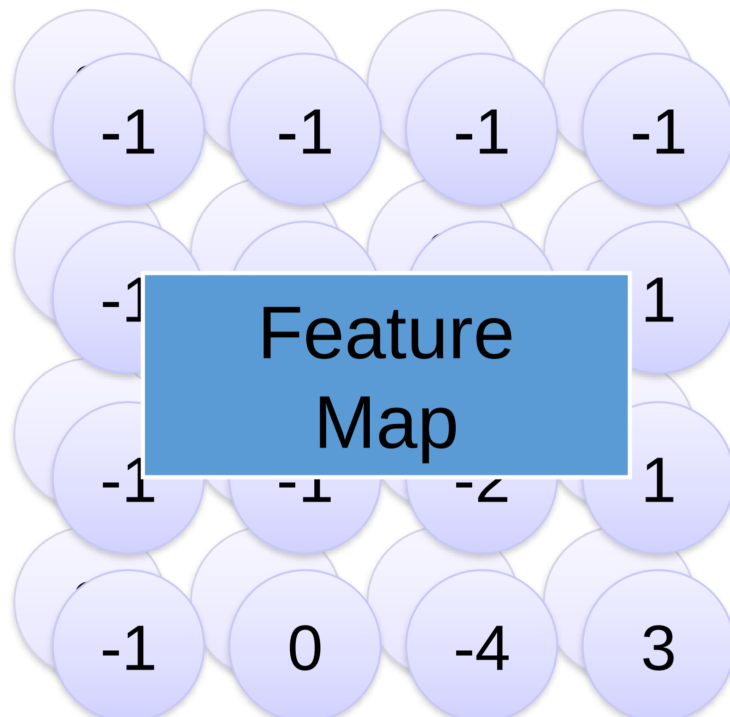
-1	1	-1
-1	1	-1
-1	1	-1

Filter 2

stride=1

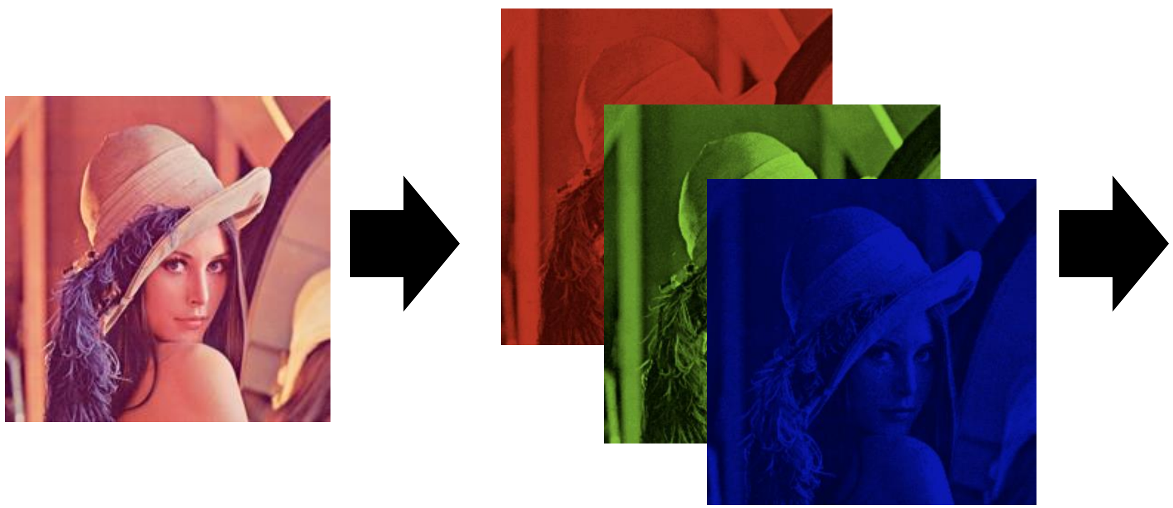
1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

6 x 6 image



Two 4 x 4 images
Forming 2 x 4 x 4 matrix

Color image (3 channels)



1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

1	-1	-1
-1	1	-1
-1	-1	1

Filter 1

-1	1	-1
-1	1	-1
-1	1	-1

Filter 2

Padding

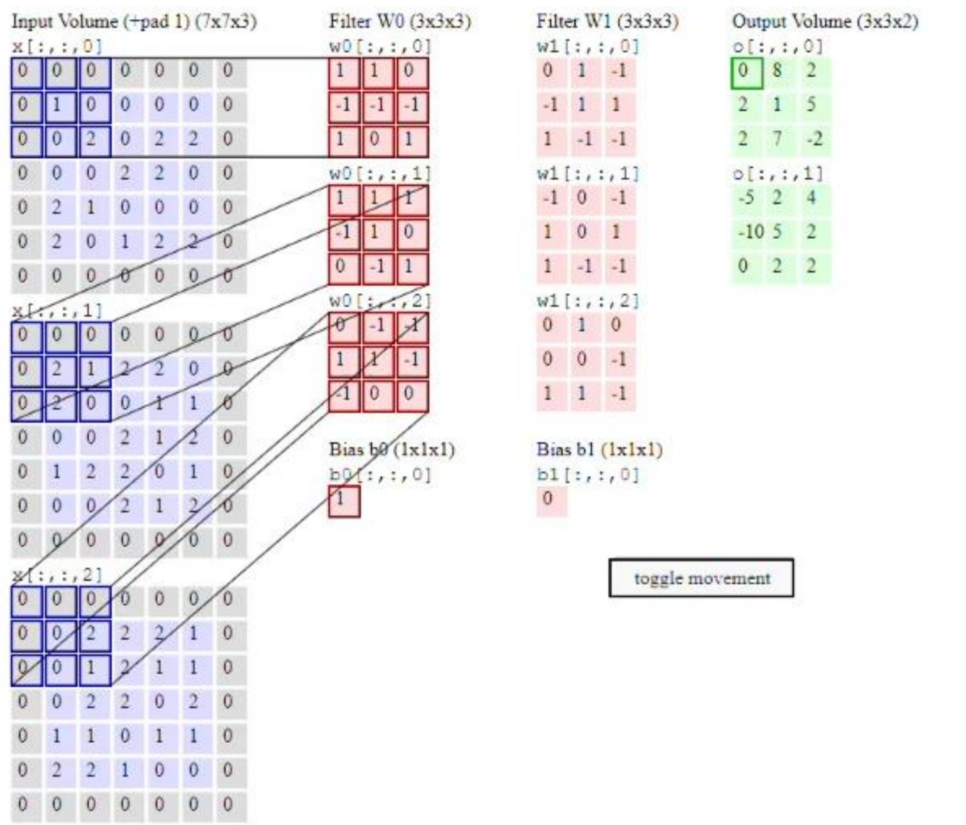
- Conv 3x3 with stride=1, padding=1

0	0	0	0	0	0
0	1	5	3	9	0
0	4	4	3	5	0
0	6	4	2	6	0
0	6	5	2	1	0
0	0	0	0	0	0

4 x 4 image

14	24	33	24
27	41	32	25
33	34	32	26
26	32	27	16

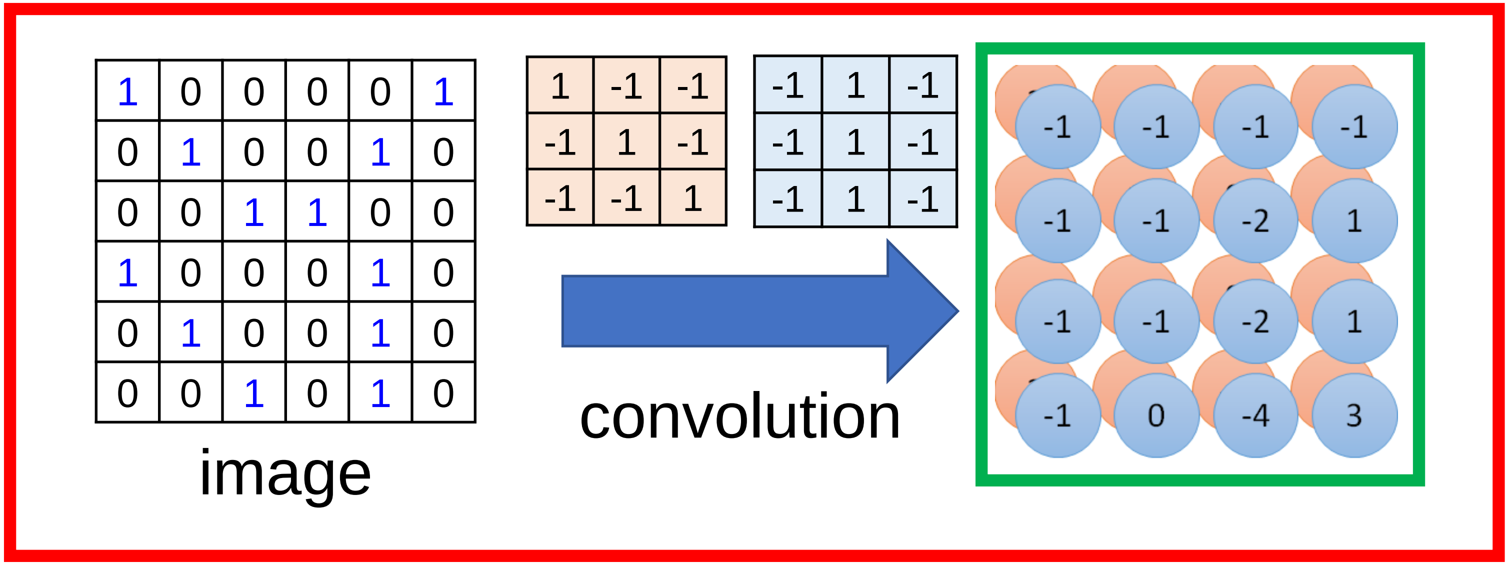
4 x 4 image



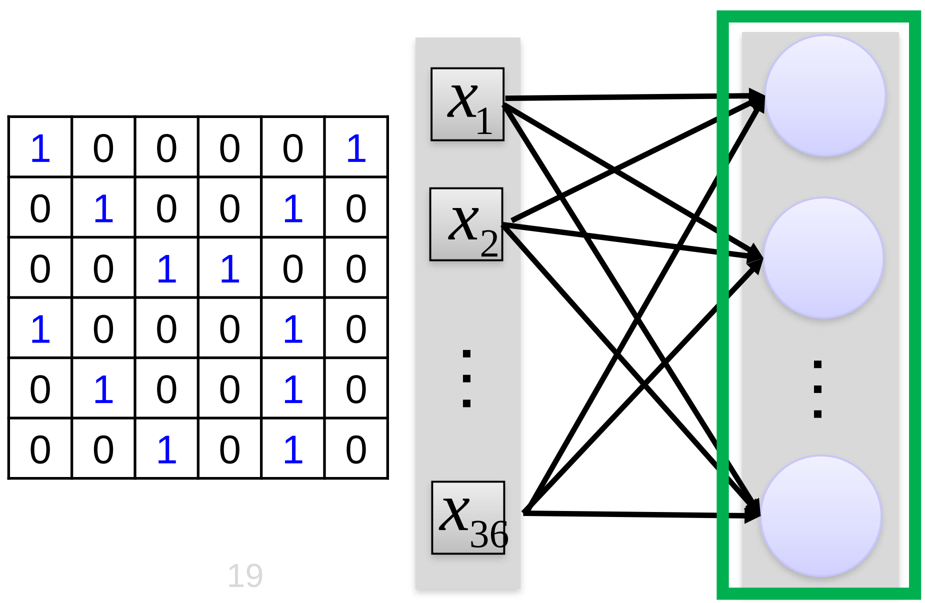
Implementation as Matrix Multiplication. Note that the convolution operation essentially performs dot products between the filters and local regions of the input. A common implementation pattern of the CONV layer is to take advantage of this fact and formulate the forward pass of a convolutional layer as one big matrix multiply as follows:

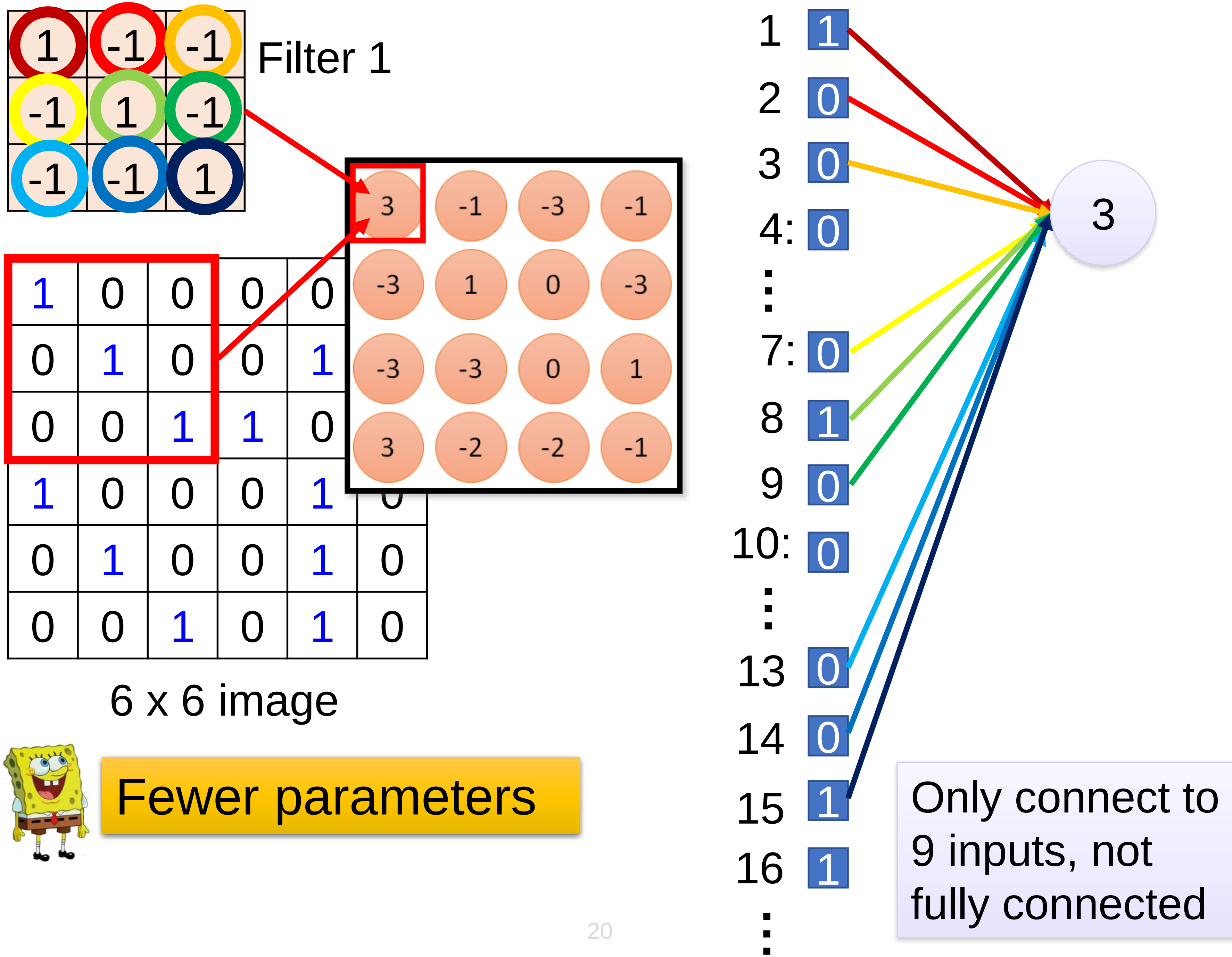
https://cs231n.github.io/convolutional-networks/

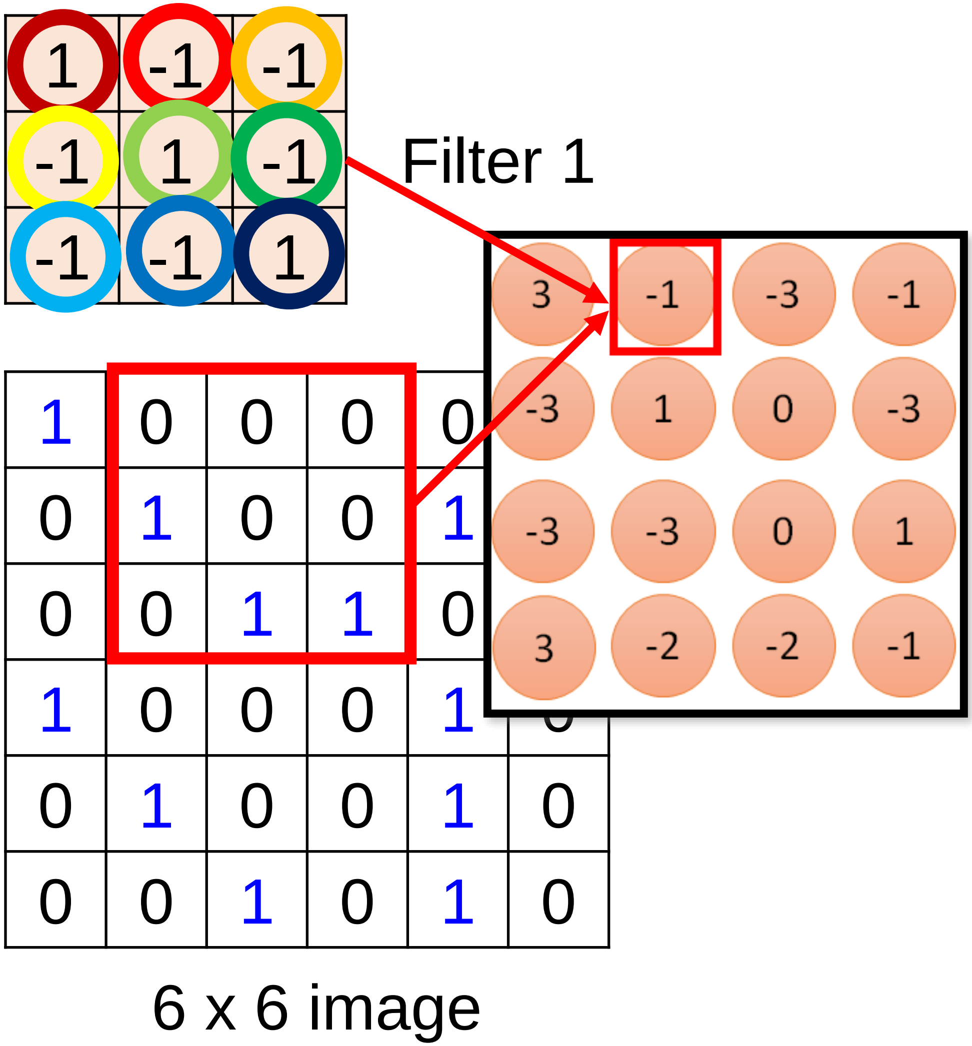
Convolution v.s. Fully Connected

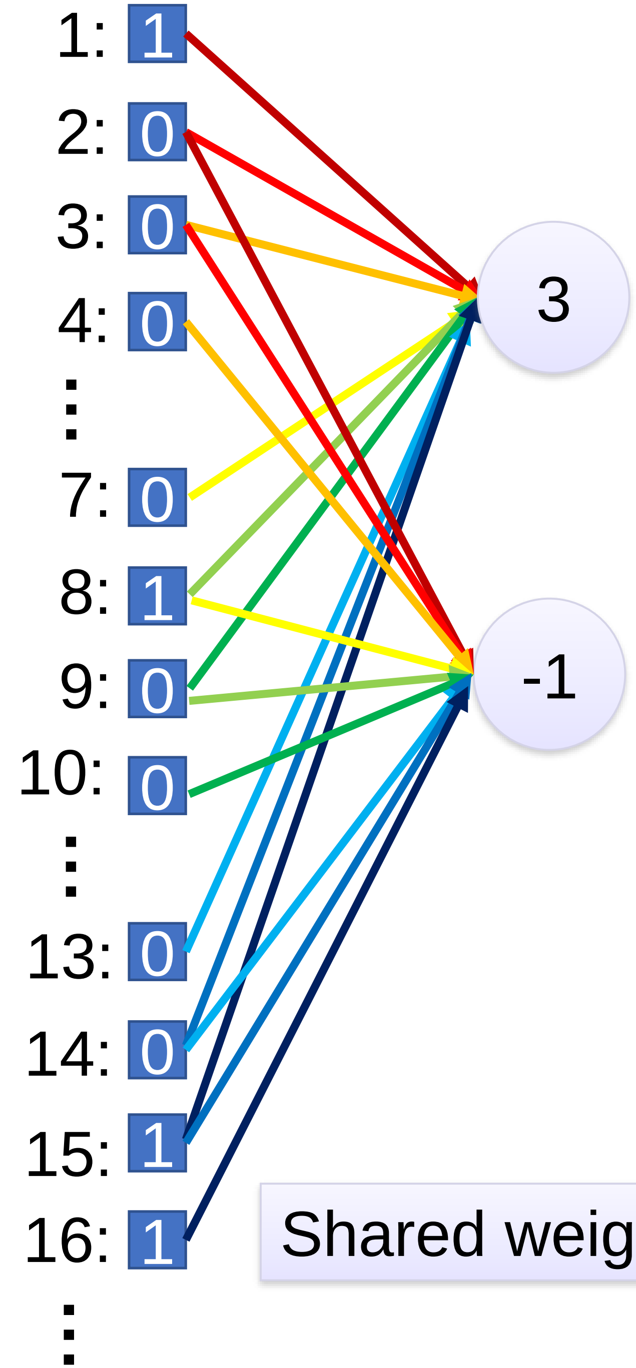


Fully-connected




Filter 1
6 x 6 image
3
Fewer parameters
Only connect to 9 inputs, not fully connected



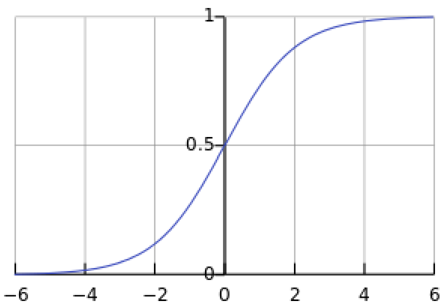


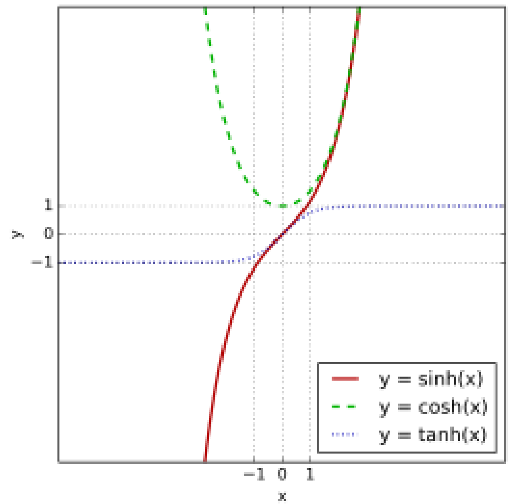
Shared weights

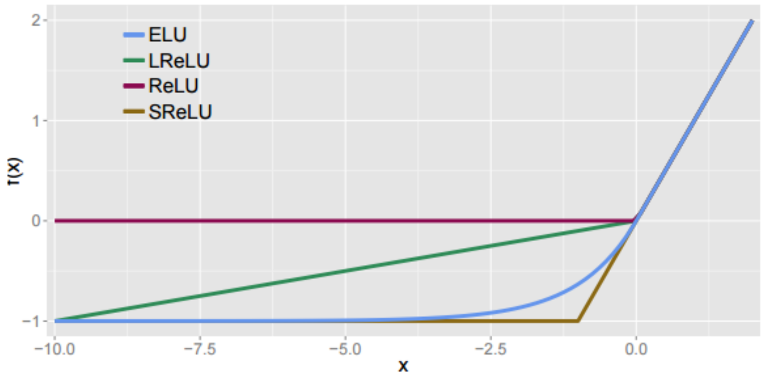
Even fewer parameters

Nonlinear Activations

- Why activation? Nonlinearity
  - Sigmoid
  - tanh
  - ReLU family

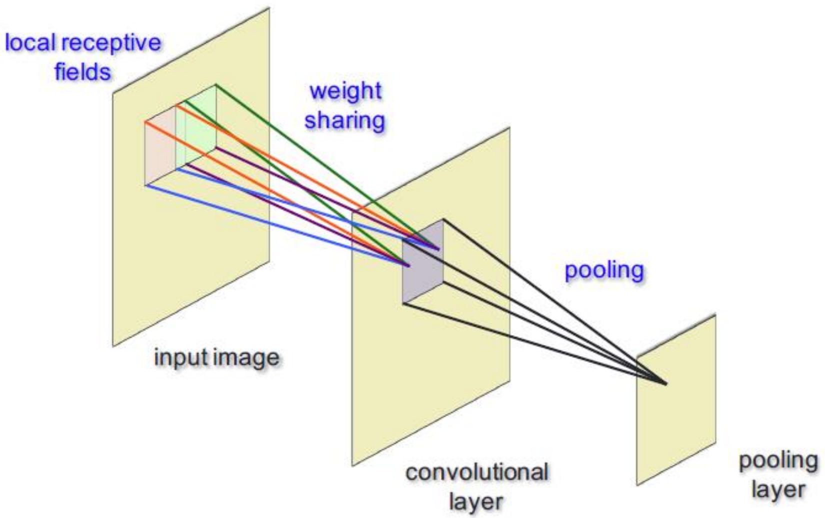






Pooling

- Common pooling operations:
  - Max pooling: reports the maximum output within a rectangular neighborhood.
  - Average pooling: reports the average output of a rectangular neighborhood (possibly weighted by the distance from the central pixel).



Pooling Example (Summing or averaging)

1	0	0	0	0	1
0	1	0	0	1	0
0	0	1	1	0	0
1	0	0	0	1	0
0	1	0	0	1	0
0	0	1	0	1	0

Convolved feature

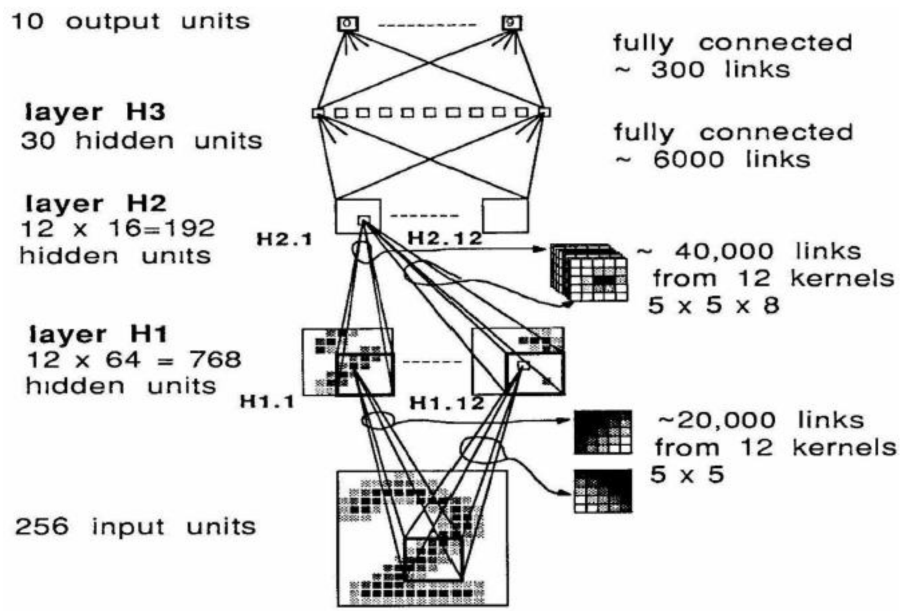
1	

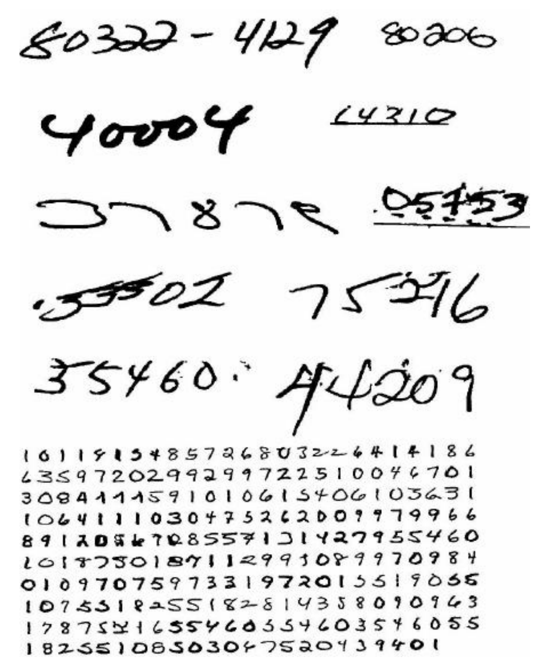
Pooled feature

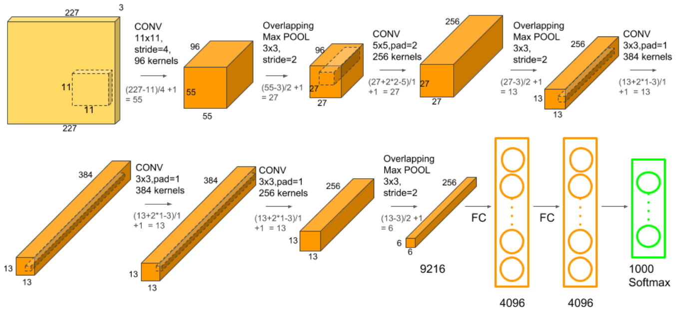
Fewer parameters to characterize the image

First ConvNets [LeCun et al 89]

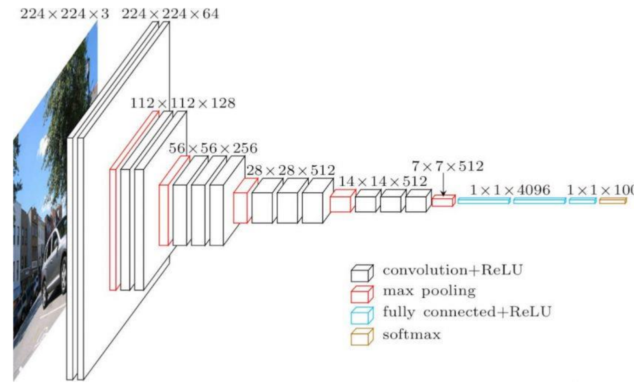
- Trained with Backprop
- USPS Zipcode digits: 7300 training, 2000 test
- Convolution with stride. No separate pooling



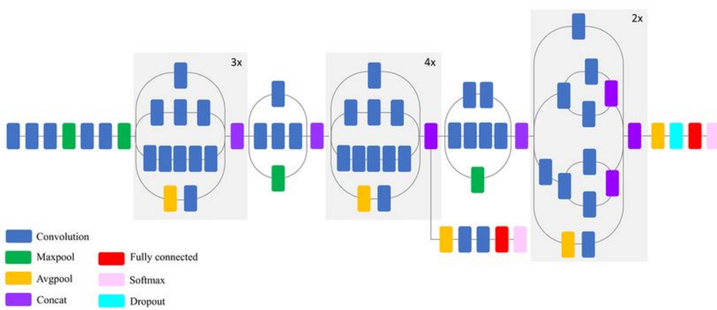




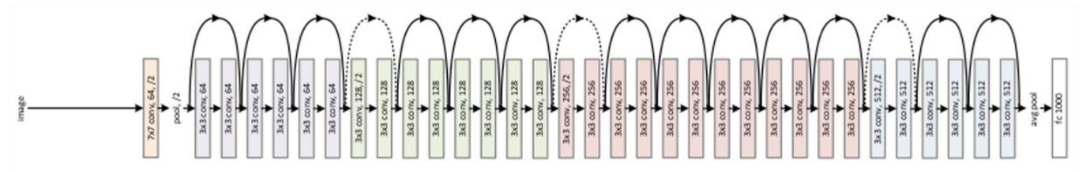
AlexNet (NIPS 2012)



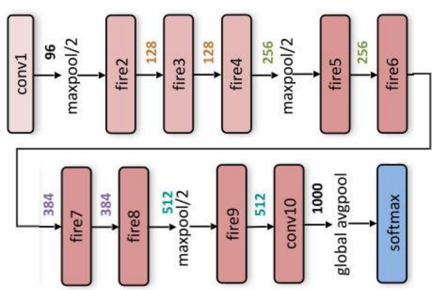
VGG-16 (ICLR 2015)



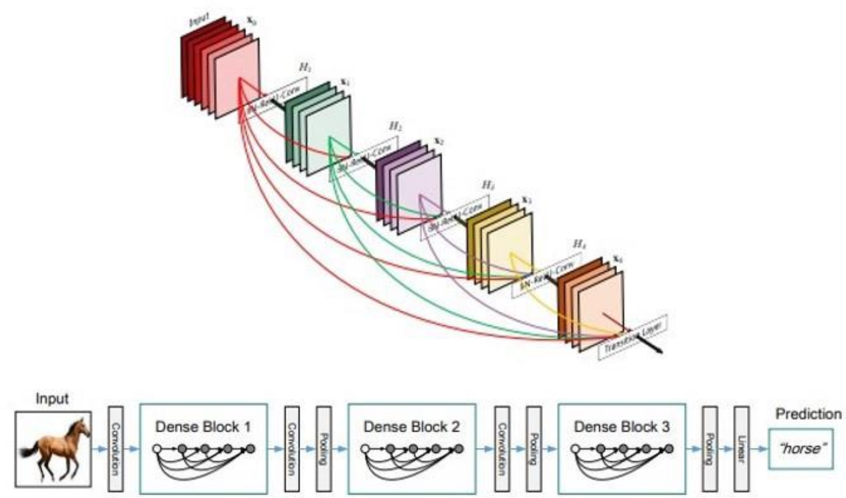
Inception v3 (CVPR 2016)



ResNet50 (CVPR 2015)



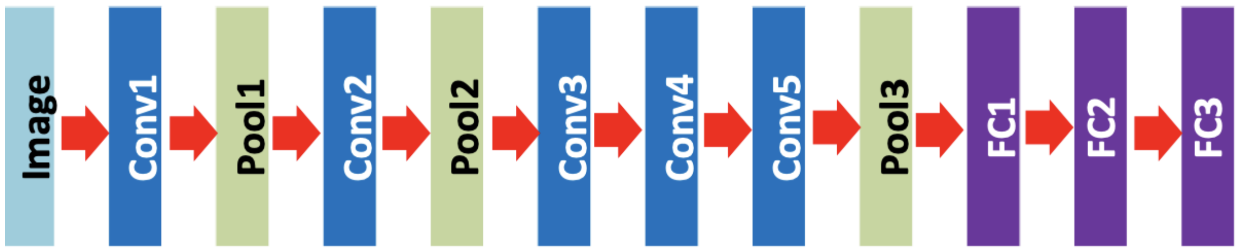
SqeezeNet (2016)



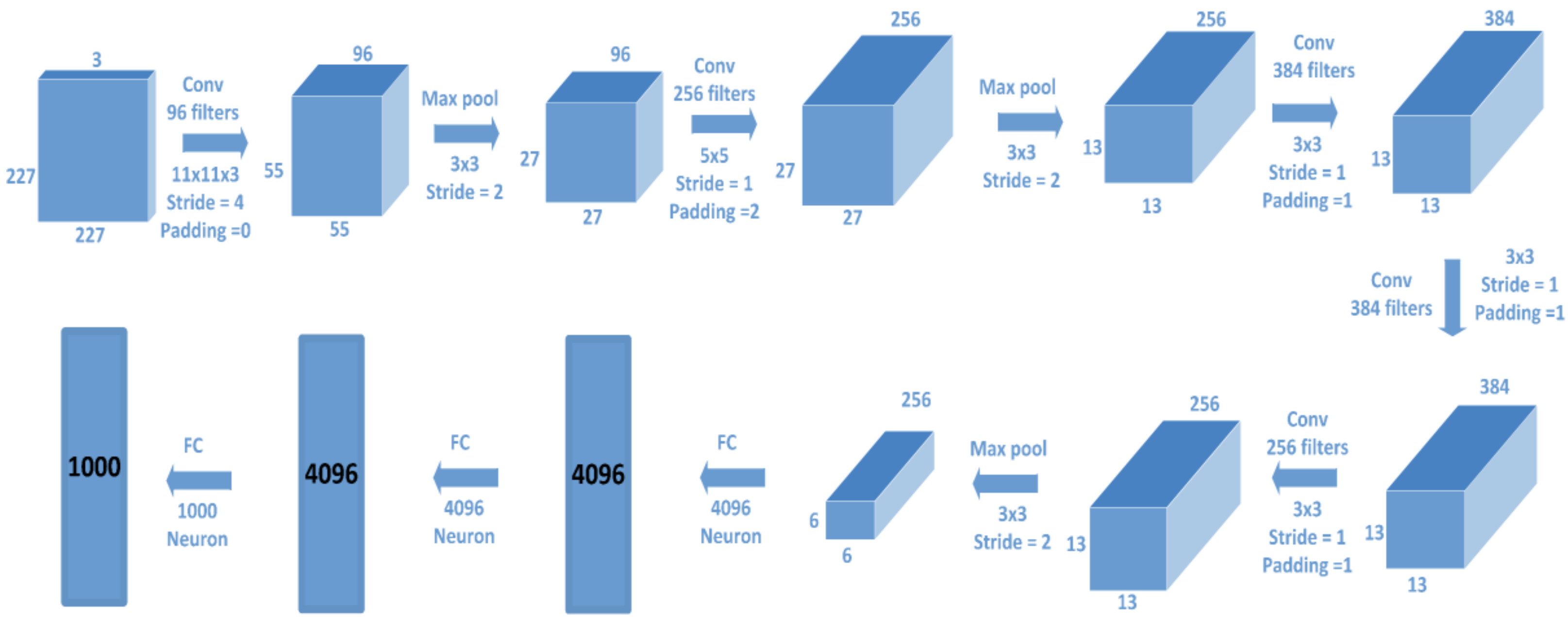
Densenet121 (CVPR 2017)

AlexNet (2012)

- AlexNet achieves on ILSVRC 2012 competition 15.3% Top-5 error rate compare to 26.2% achieved by the second best entry.
- AlexNet has 8 layers without counting pooling layers.
- AlexNet trained on two GTX 580 GPUs for five to six days



AlexNet (2012)



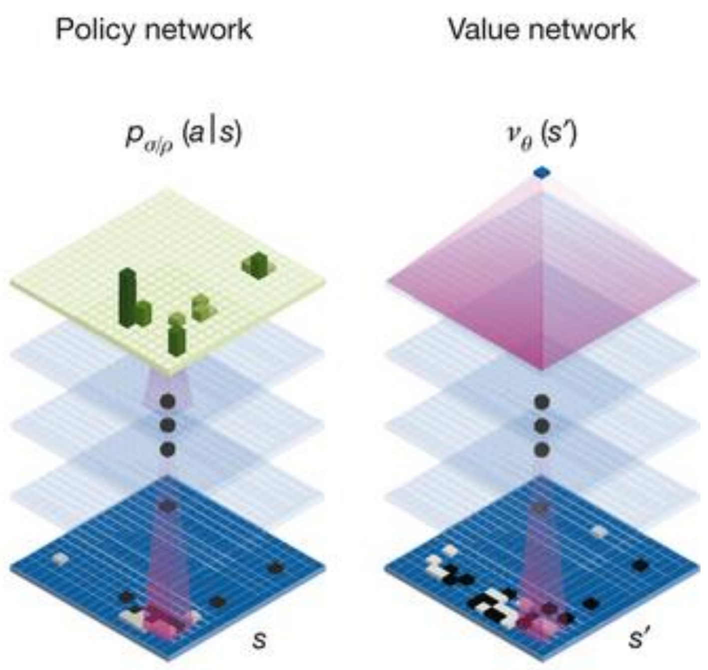
Total (label and softmax not included)
Memory: 2.24 million
Weights: 62.37 million

(Figure from Dr. Mohamed Loey)

AlexNet (2012)

- ReLU
- Norm layers
- Data augmentation
- Dropout 0.5
- Batch size is 128
- SGD Momentum 0.9
- Learning rate 1e-2

Deep CNN in AlphaGO



(Silver et al, 2016)

Policy network:

- Input: 19x19, 48 input channels
- Layer 1: 5x5 kernel, 192 filters
- Layer 2 to 12: 3x3 kernel, 192 filters
- Layer 13: 1x1 kernel, 1 filter

Value network has similar architecture to policy network

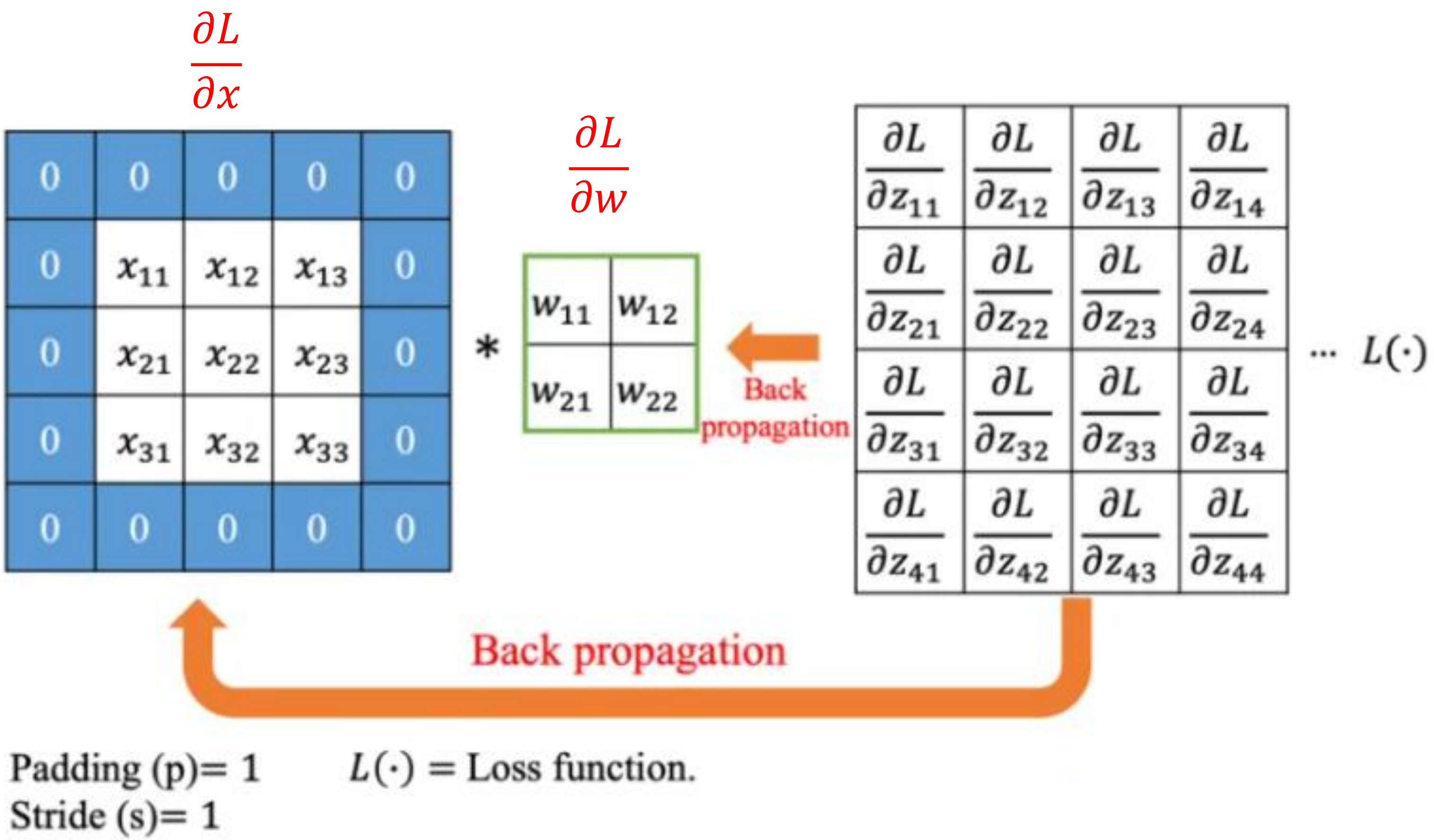
How to backpropagate with convolution?

$$\begin{bmatrix} x_{11} & x_{12} & x_{13} \\ x_{21} & x_{22} & x_{23} \\ x_{31} & x_{32} & x_{33} \end{bmatrix} * \begin{bmatrix} w_{11} & w_{12} \\ w_{21} & w_{22} \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} & z_{13} & z_{14} \\ z_{21} & z_{22} & z_{23} & z_{24} \\ z_{31} & z_{32} & z_{33} & z_{34} \\ z_{41} & z_{42} & z_{43} & z_{44} \end{bmatrix} \cdots \; L(\cdot)$$

Padding (p)= 1
Stride (s)= 1

$L(\cdot)$ = Loss function.

Ref: https://www.brilliantcode.net/1670/convolutional-neural-networks-4-backpropagation-in-kernels-of-cnns/?cli_action=1604504837.339



Ref: https://www.brilliantcode.net/1670/convolutional-neural-networks-4-backpropagation-in-kernels-of-cnns/?cli_action=1604504837.339

$$z_{11} = 0w_{11} + 0w_{12} + 0w_{21} + x_{11}w_{22}$$
$$z_{12} = 0w_{11} + 0w_{12} + x_{11}w_{21} + x_{12}w_{22}$$
$$z_{13} = 0w_{11} + 0w_{12} + x_{12}w_{21} + x_{13}w_{22}$$
$$z_{14} = 0w_{11} + 0w_{12} + x_{13}w_{21} + 0w_{22}$$

$$z_{21} = 0w_{11} + x_{11}w_{12} + 0w_{21} + x_{21}w_{22}$$
$$z_{22} = x_{11}w_{11} + x_{12}w_{12} + x_{21}w_{21} + x_{22}w_{22}$$
$$z_{23} = x_{12}w_{11} + x_{13}w_{12} + x_{22}w_{21} + x_{23}w_{22}$$
$$z_{24} = x_{13}w_{11} + 0w_{12} + x_{23}w_{21} + 0w_{22}$$

$$z_{31} = 0w_{11} + x_{21}w_{12} + 0w_{21} + x_{31}w_{22}$$
$$z_{32} = x_{21}w_{11} + x_{22}w_{12} + x_{31}w_{21} + x_{32}w_{22}$$
$$z_{33} = x_{22}w_{11} + x_{23}w_{12} + x_{32}w_{21} + x_{33}w_{22}$$
$$z_{34} = x_{23}w_{11} + 0w_{12} + x_{33}w_{21} + 0w_{22}$$

$$z_{41} = 0w_{11} + x_{31}w_{12} + 0w_{21} + 0w_{22}$$
$$z_{42} = x_{31}w_{11} + x_{32}w_{12} + 0w_{21} + 0w_{22}$$
$$z_{43} = x_{32}w_{11} + x_{33}w_{12} + 0w_{21} + 0w_{22}$$
$$z_{44} = x_{33}w_{11} + 0w_{12} + 0w_{21} + 0w_{22}$$

Ref: https://www.brilliantcode.net/1670/convolutional-neural-networks-4-backpropagation-in-kernels-of-cnns/?cli_action=1604504837.339

$$
\begin{aligned}
z_{11} &= 0w_{11} + 0w_{12} + 0w_{21} + x_{11}w_{22} \\
z_{12} &= 0w_{11} + 0w_{12} + x_{11}w_{21} + x_{12}w_{22} \\
z_{13} &= 0w_{11} + 0w_{12} + x_{12}w_{21} + x_{13}w_{22} \\
z_{14} &= 0w_{11} + 0w_{12} + x_{13}w_{21} + 0w_{22}
\end{aligned}
$$

$$
\begin{aligned}
z_{21} &= 0w_{11} + x_{11}w_{12} + 0w_{21} + x_{21}w_{22} \\
z_{22} &= x_{11}w_{11} + x_{12}w_{12} + x_{21}w_{21} + x_{22}w_{22} \\
z_{23} &= x_{12}w_{11} + x_{13}w_{12} + x_{22}w_{21} + x_{23}w_{22} \\
z_{24} &= x_{13}w_{11} + 0w_{12} + x_{23}w_{21} + 0w_{22}
\end{aligned}
$$

$\frac{\partial L}{\partial w_{11}}$?

$$
\begin{aligned}
z_{31} &= 0w_{11} + x_{21}w_{12} + 0w_{21} + x_{31}w_{22} \\
z_{32} &= x_{21}w_{11} + x_{22}w_{12} + x_{31}w_{21} + x_{32}w_{22} \\
z_{33} &= x_{22}w_{11} + x_{23}w_{12} + x_{32}w_{21} + x_{33}w_{22} \\
z_{34} &= x_{23}w_{11} + 0w_{12} + x_{33}w_{21} + 0w_{22}
\end{aligned}
$$

$$
\begin{aligned}
z_{41} &= 0w_{11} + x_{31}w_{12} + 0w_{21} + 0w_{22} \\
z_{42} &= x_{31}w_{11} + x_{32}w_{12} + 0w_{21} + 0w_{22} \\
z_{43} &= x_{32}w_{11} + x_{33}w_{12} + 0w_{21} + 0w_{22} \\
z_{44} &= x_{33}w_{11} + 0w_{12} + 0w_{21} + 0w_{22}
\end{aligned}
$$

Ref: https://www.brilliantcode.net/1670/convolutional-neural-networks-4-backpropagation-in-kernels-of-cnns/?cli_action=1604504837.339

$$\frac{\partial L}{\partial w} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial w}$$

$$\begin{aligned}
\frac{\partial L}{\partial w_{11}} &= \frac{\partial L}{\partial z_{22}}\boxed{\frac{\partial z_{22}}{\partial w_{11}}} + \frac{\partial L}{\partial z_{23}}\frac{\partial z_{23}}{\partial w_{11}} + \frac{\partial L}{\partial z_{24}}\frac{\partial z_{24}}{\partial w_{11}} \\
&+ \frac{\partial L}{\partial z_{32}}\frac{\partial z_{32}}{\partial w_{11}} + \frac{\partial L}{\partial z_{33}}\frac{\partial z_{33}}{\partial w_{11}} + \frac{\partial L}{\partial z_{34}}\frac{\partial z_{34}}{\partial w_{11}} \\
&+ \frac{\partial L}{\partial z_{42}}\frac{\partial z_{42}}{\partial w_{11}} + \frac{\partial L}{\partial z_{43}}\frac{\partial z_{43}}{\partial w_{11}} + \frac{\partial L}{\partial z_{44}}\frac{\partial z_{44}}{\partial w_{11}} \\
&= \frac{\partial L}{\partial z_{22}}\boxed{x_{11}} + \frac{\partial L}{\partial z_{23}}x_{12} + \frac{\partial L}{\partial z_{24}}x_{13} \\
&+ \frac{\partial L}{\partial z_{32}}x_{21} + \frac{\partial L}{\partial z_{33}}x_{22} + \frac{\partial L}{\partial z_{34}}x_{23} \\
&+ \frac{\partial L}{\partial z_{42}}x_{31} + \frac{\partial L}{\partial z_{43}}x_{32} + \frac{\partial L}{\partial z_{44}}x_{33}
\end{aligned}$$

$$\frac{\partial L}{\partial w_{11}} = \frac{\partial L}{\partial z_{22}}x_{11} + \frac{\partial L}{\partial z_{23}}x_{12} + \frac{\partial L}{\partial z_{24}}x_{13}$$
$$+\frac{\partial L}{\partial z_{32}}x_{12} + \frac{\partial L}{\partial z_{33}}x_{22} + \frac{\partial L}{\partial z_{34}}x_{23}$$
$$+\frac{\partial L}{\partial z_{42}}x_{31} + \frac{\partial L}{\partial z_{43}}x_{32} + \frac{\partial L}{\partial z_{44}}x_{33}$$

$$\frac{\partial L}{\partial w_{12}} = \frac{\partial L}{\partial z_{21}}x_{11} + \frac{\partial L}{\partial z_{22}}x_{12} + \frac{\partial L}{\partial z_{23}}x_{13}+$$
$$+\frac{\partial L}{\partial z_{31}}x_{21} + \frac{\partial L}{\partial z_{32}}x_{22} + \frac{\partial L}{\partial z_{33}}x_{23}$$
$$+\frac{\partial L}{\partial z_{41}}x_{31} + \frac{\partial L}{\partial z_{42}}x_{32} + \frac{\partial L}{\partial z_{43}}x_{33}$$

$$\frac{\partial L}{\partial w_{21}} = \frac{\partial L}{\partial z_{12}}x_{11} + \frac{\partial L}{\partial z_{13}}x_{12} + \frac{\partial L}{\partial z_{14}}x_{13}$$
$$+\frac{\partial L}{\partial z_{22}}x_{21} + \frac{\partial L}{\partial z_{23}}x_{22} + \frac{\partial L}{\partial z_{24}}x_{23}$$
$$+\frac{\partial L}{\partial z_{32}}x_{31} + \frac{\partial L}{\partial z_{33}}x_{32} + \frac{\partial L}{\partial z_{34}}x_{33}$$

$$\frac{\partial L}{\partial w_{22}} = \frac{\partial L}{\partial z_{11}}x_{11} + \frac{\partial L}{\partial z_{12}}x_{12} + \frac{\partial L}{\partial z_{13}}x_{13}$$
$$+\frac{\partial L}{\partial z_{21}}x_{21} + \frac{\partial L}{\partial z_{22}}x_{22} + \frac{\partial L}{\partial z_{23}}x_{23}$$
$$+\frac{\partial L}{\partial z_{31}}x_{31} + \frac{\partial L}{\partial z_{32}}x_{32} + \frac{\partial L}{\partial z_{33}}x_{33}$$

$$
\begin{aligned}
z_{11} &= 0w_{11} + 0w_{12} + 0w_{21} + \boxed{x_{11}}w_{22} \\
z_{12} &= 0w_{11} + 0w_{12} + \boxed{x_{11}}w_{21} + x_{12}w_{22} \\
z_{13} &= 0w_{11} + 0w_{12} + x_{12}w_{21} + x_{13}w_{22} \\
z_{14} &= 0w_{11} + 0w_{12} + x_{13}w_{21} + 0w_{22}
\end{aligned}
$$

$$
\begin{aligned}
z_{21} &= 0w_{11} + \boxed{x_{11}}w_{12} + 0w_{21} + x_{21}w_{22} \\
z_{22} &= \boxed{x_{11}}w_{11} + x_{12}w_{12} + x_{21}w_{21} + x_{22}w_{22} \\
z_{23} &= x_{12}w_{11} + x_{13}w_{12} + x_{22}w_{21} + x_{23}w_{22} \\
z_{24} &= x_{13}w_{11} + 0w_{12} + x_{23}w_{21} + 0w_{22}
\end{aligned}
$$

$\frac{\partial L}{\partial x_{11}}$?

$$
\begin{aligned}
z_{31} &= 0w_{11} + x_{21}w_{12} + 0w_{21} + x_{31}w_{22} \\
z_{32} &= x_{21}w_{11} + x_{22}w_{12} + x_{31}w_{21} + x_{32}w_{22} \\
z_{33} &= x_{22}w_{11} + x_{23}w_{12} + x_{32}w_{21} + x_{33}w_{22} \\
z_{34} &= x_{23}w_{11} + 0w_{12} + x_{33}w_{21} + 0w_{22}
\end{aligned}
$$

$$
\begin{aligned}
z_{41} &= 0w_{11} + x_{31}w_{12} + 0w_{21} + 0w_{22} \\
z_{42} &= x_{31}w_{11} + x_{32}w_{12} + 0w_{21} + 0w_{22} \\
z_{43} &= x_{32}w_{11} + x_{33}w_{12} + 0w_{21} + 0w_{22} \\
z_{44} &= x_{33}w_{11} + 0w_{12} + 0w_{21} + 0w_{22}
\end{aligned}
$$

Ref: https://www.brilliantcode.net/1670/convolutional-neural-networks-4-backpropagation-in-kernels-of-cnns/?cli_action=1604504837.339

$$\boxed{\frac{\partial L}{\partial x} = \frac{\partial L}{\partial z}\frac{\partial z}{\partial x}}$$

$$\frac{\partial L}{\partial x_{11}} = \frac{\partial L}{\partial z_{11}}\frac{\partial z_{11}}{\partial x_{11}} + \frac{\partial L}{\partial z_{12}}\frac{\partial z_{12}}{\partial x_{11}} + \frac{\partial L}{\partial z_{21}}\frac{\partial z_{21}}{\partial x_{11}} + \frac{\partial L}{\partial z_{22}}\frac{\partial z_{22}}{\partial x_{11}}$$

$$= \frac{\partial L}{\partial z_{11}} w_{22} + \frac{\partial L}{\partial z_{12}} w_{21} + \frac{\partial L}{\partial z_{21}} w_{12} + \frac{\partial L}{\partial z_{22}} w_{11}$$

$$\frac{\partial L}{\partial x_{22}} = \frac{\partial L}{\partial z_{22}}\frac{\partial z_{22}}{\partial x_{22}} + \frac{\partial L}{\partial z_{23}}\frac{\partial z_{23}}{\partial x_{22}} + \frac{\partial L}{\partial z_{32}}\frac{\partial z_{32}}{\partial x_{22}} + \frac{\partial L}{\partial z_{33}}\frac{\partial z_{33}}{\partial x_{22}}$$

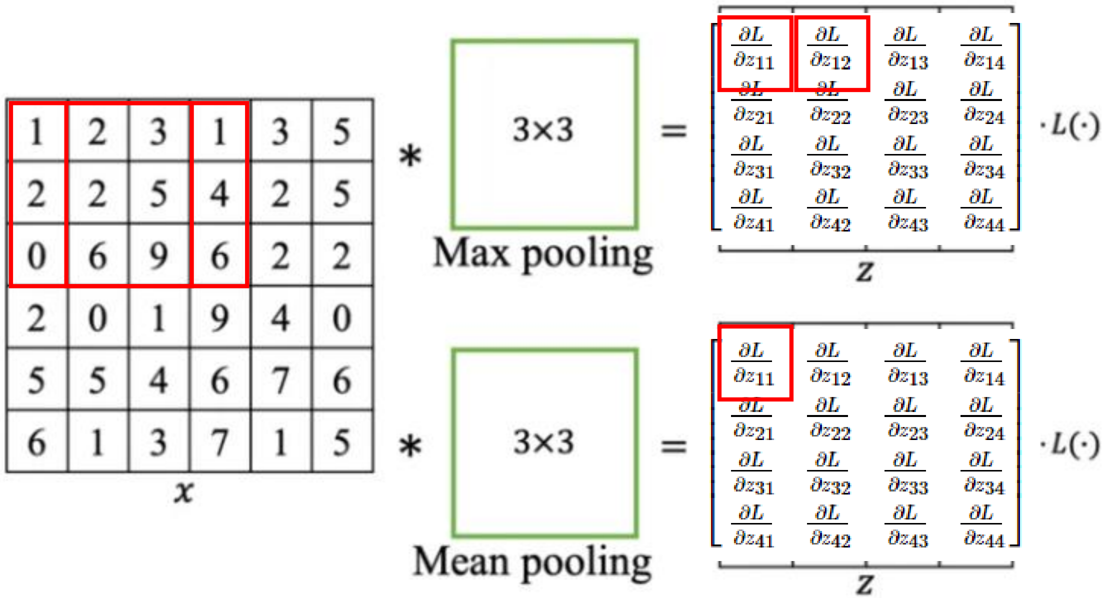
$$= \frac{\partial L}{\partial z_{22}} w_{22} + \frac{\partial L}{\partial z_{23}} w_{21} + \frac{\partial L}{\partial z_{32}} w_{12} + \frac{\partial L}{\partial z_{33}} w_{11}$$

$$\vdots$$

$$
\frac{\partial L}{\partial w} =
\left[\begin{array}{c|c}
\begin{array}{l}
= \frac{\partial L}{\partial z_{22}} x_{11} + \frac{\partial L}{\partial z_{23}} x_{12} + \frac{\partial L}{\partial z_{24}} x_{13} \\
+ \frac{\partial L}{\partial z_{32}} x_{12} + \frac{\partial L}{\partial z_{33}} x_{22} + \frac{\partial L}{\partial z_{34}} x_{23} \\
+ \frac{\partial L}{\partial z_{42}} x_{31} + \frac{\partial L}{\partial z_{43}} x_{32} + \frac{\partial L}{\partial z_{44}} x_{33}
\end{array}
&
\begin{array}{l}
= \frac{\partial L}{\partial z_{21}} x_{11} + \frac{\partial L}{\partial z_{22}} x_{12} + \frac{\partial L}{\partial z_{23}} x_{13} + \\
+ \frac{\partial L}{\partial z_{31}} x_{21} + \frac{\partial L}{\partial z_{32}} x_{22} + \frac{\partial L}{\partial z_{33}} x_{23} \\
+ \frac{\partial L}{\partial z_{41}} x_{31} + \frac{\partial L}{\partial z_{42}} x_{32} + \frac{\partial L}{\partial z_{43}} x_{33}
\end{array}
\\ \hline
\begin{array}{l}
= \frac{\partial L}{\partial z_{12}} x_{11} + \frac{\partial L}{\partial z_{13}} x_{12} + \frac{\partial L}{\partial z_{14}} x_{13} \\
+ \frac{\partial L}{\partial z_{22}} x_{21} + \frac{\partial L}{\partial z_{23}} x_{22} + \frac{\partial L}{\partial z_{24}} x_{23} \\
+ \frac{\partial L}{\partial z_{32}} x_{31} + \frac{\partial L}{\partial z_{33}} x_{32} + \frac{\partial L}{\partial z_{34}} x_{33}
\end{array}
&
\begin{array}{l}
= \frac{\partial L}{\partial z_{11}} x_{11} + \frac{\partial L}{\partial z_{12}} x_{12} + \frac{\partial L}{\partial z_{13}} x_{13} \\
+ \frac{\partial L}{\partial z_{21}} x_{21} + \frac{\partial L}{\partial z_{22}} x_{22} + \frac{\partial L}{\partial z_{23}} x_{23} \\
+ \frac{\partial L}{\partial z_{31}} x_{31} + \frac{\partial L}{\partial z_{32}} x_{32} + \frac{\partial L}{\partial z_{33}} x_{33}
\end{array}
\end{array}\right]
$$

$$
\frac{\partial L}{\partial x} =
\left[\begin{array}{c|c|c}
\begin{array}{c}
\frac{\partial L}{\partial z_{11}} w_{22} + \frac{\partial L}{\partial z_{12}} w_{21} + \\
\frac{\partial L}{\partial z_{21}} w_{12} + \frac{\partial L}{\partial z_{22}} w_{11}
\end{array}
&
\begin{array}{c}
\frac{\partial L}{\partial z_{12}} w_{22} + \frac{\partial L}{\partial z_{13}} w_{21} + \\
\frac{\partial L}{\partial z_{22}} w_{12} + \frac{\partial L}{\partial z_{23}} w_{11}
\end{array}
&
\begin{array}{c}
\frac{\partial L}{\partial z_{13}} w_{22} + \frac{\partial L}{\partial z_{14}} w_{21} + \\
\frac{\partial L}{\partial z_{23}} w_{12} + \frac{\partial L}{\partial z_{24}} w_{11}
\end{array}
\\ \hline
\begin{array}{c}
\frac{\partial L}{\partial z_{21}} w_{22} + \frac{\partial L}{\partial z_{22}} w_{21} + \\
\frac{\partial L}{\partial z_{31}} w_{12} + \frac{\partial L}{\partial z_{32}} w_{11}
\end{array}
&
\begin{array}{c}
\frac{\partial L}{\partial z_{22}} w_{22} + \frac{\partial L}{\partial z_{23}} w_{21} + \\
\frac{\partial L}{\partial z_{32}} w_{12} + \frac{\partial L}{\partial z_{33}} w_{11}
\end{array}
&
\begin{array}{c}
\frac{\partial L}{\partial z_{23}} w_{22} + \frac{\partial L}{\partial z_{24}} w_{21} + \\
\frac{\partial L}{\partial z_{33}} w_{12} + \frac{\partial L}{\partial z_{34}} w_{11}
\end{array}
\\ \hline
\begin{array}{c}
\frac{\partial L}{\partial z_{31}} w_{22} + \frac{\partial L}{\partial z_{32}} w_{21} + \\
\frac{\partial L}{\partial z_{41}} w_{12} + \frac{\partial L}{\partial z_{42}} w_{11}
\end{array}
&
\begin{array}{c}
\frac{\partial L}{\partial z_{32}} w_{22} + \frac{\partial L}{\partial z_{33}} w_{21} + \\
\frac{\partial L}{\partial z_{42}} w_{12} + \frac{\partial L}{\partial z_{43}} w_{11}
\end{array}
&
\begin{array}{c}
\frac{\partial L}{\partial z_{33}} w_{22} + \frac{\partial L}{\partial z_{34}} w_{21} + \\
\frac{\partial L}{\partial z_{43}} w_{12} + \frac{\partial L}{\partial z_{44}} w_{11}
\end{array}
\end{array}\right]
$$

How about Pooling layers?



Pooling kernel size= (3×3), Stride (s)= 1, $L(\cdot)$ = Loss function.

Max: $$\begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{\partial L}{\partial z_{11}} + \frac{\partial L}{\partial z_{12}} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

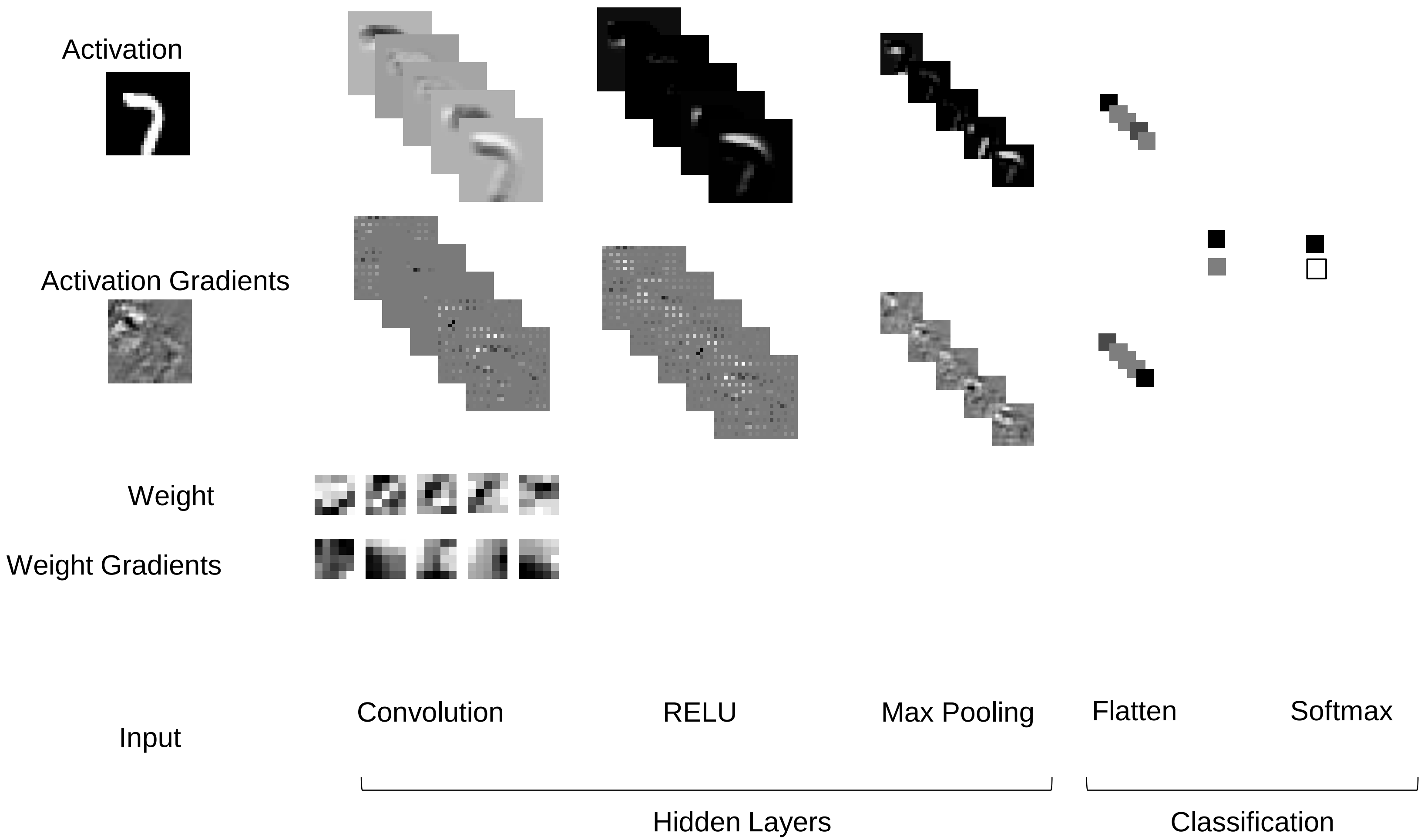
Mean: $$\begin{bmatrix} (\frac{\partial L}{\partial z_{11}})/9 & (\frac{\partial L}{\partial z_{11}})/9 & (\frac{\partial L}{\partial z_{11}})/9 & 0 & 0 & 0 \\ (\frac{\partial L}{\partial z_{11}})/9 & (\frac{\partial L}{\partial z_{11}})/9 & (\frac{\partial L}{\partial z_{11}})/9 & 0 & 0 & 0 \\ (\frac{\partial L}{\partial z_{11}})/9 & (\frac{\partial L}{\partial z_{11}})/9 & (\frac{\partial L}{\partial z_{11}})/9 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

ConvNets are good for

- Signals that comes to you in the form of (multidimensional) arrays.
- Signals that have strong local correlations
- Signals where features can appear anywhere
- Signals in which objects are invariant to translations.
- 1D ConvNets: sequential signals, text
  - Text, music, audio, speech, time series.
- 2D ConvNets: images, time-frequency representations (speech and audio)
  - Object detection, localization, recognition
- 3D ConvNets: video, volumetric images, tomography images
  - Video recognition / understanding
  - Biomedical image analysis
  - Hyperspectral image analysis

Model Visualization

http://cs.stanford.edu/people/karpathy/convnetjs/



Questions?

How to confuse your ConvNets?