1 Problem 1

Because $Tr(AB) = Tr(U^{-1}ABU)$, $Tr(e^{-\beta\mathcal{H}(h)}S_q^z) = -Tr(e^{-\beta\mathcal{H}(-h)}S_q^z)$. Where I use the transform which transforms $S_i^z \to -S_i^z, S_i^x \to -S_i^x$. As a result, it can be shown $\lim_{h\to 0^+} m_q(h) = -\lim_{h\to 0^-} m_q(h)$.

2 Problem 2

Acoording to Mashall's theorem, the ground state of Heisenberg model is not degenate. $[\mathcal{H}_0, P] = 0$, where P is the translation operator and $PS_i^z P^{-1} = S_{i+1}^z$. When $T = 0K$, $< 0|S_i^z|0 > = < 0|S_{i+1}^z|0 >$ and $< 0|\sum_i S_i^z|0 > = 0$. So $m_q(h = 0) == 0$ by deducing.

3 Problem 3

$$< \vec{S}_q \cdot \vec{S}_{-q} > = \sum_n \rho_n < n|\vec{S}_q \vec{S}_{-q}|n > \tag{1}$$

$$N^2|m_q|^2 = |\sum_n \rho_n < n|\vec{S}_q|n > |^2 \tag{2}$$

Because $0 \le \rho_n < 1$, the following equation could come out.

$$\sum_n \rho_n < n|\vec{S}_q|n > = \sum_n (\sqrt{\rho_n})(\sqrt{\rho_n} < n|\vec{S}_q|n >)) = \sum_n a_n b_n, \tag{3}$$

$(\sum_n a_n b_n)^2 <= \sum_n a_n^2 \sum_n b_n^2$, as a result:

$$N^2|m_q|^2 \le (\sum_n \rho_n)(\sum_n \rho_n| < n|\vec{S}_q|n > |^2) = \sum_n \rho_n| < n|\vec{S}_q|n > |^2 \tag{4}$$

At the same time,

$$< \vec{S}_q \cdot \vec{S}_{-q} > = \sum_n \rho_n| < n|\vec{S}_q|n > |^2 + \sum_{n,m\neq n} \rho_n| < n|\vec{S}_q|m > |^2 \ge \sum_n \rho_n| < n|\vec{S}_q|n > |^2 \tag{5}$$

Finally, we get $< \vec{S}_q \cdot \vec{S}_{-q} > \ge N^2|m_q|^2$.

4 Problem 4

It's a mathmatical problem which can be described as below. For a sequence $\{a_n\}$, if $\lim_{n\to\inf} a_n = 0$ and $0 \le a_n < L$ where L is an positive const number. Then whether the sequence $\{\frac{1}{n}S_n\}$ converges where $S_n = \sum_n a_n$. The answer is yes. For $\forall\delta > 0$, exists an N, when $n > N$, $a_n < \delta$. S_N is finite, here exists an $M > N$ satisfying when $m > M$, $\frac{S_N}{m} < \delta$. So $\frac{S_m}{m} = \frac{S_N}{m} + \frac{\sum_{n=N+1}^m a_n < m\delta}{m} < 2\delta$. So sequence $\{\frac{1}{n}S_n\}$ converges to 0.