TASK 1

Using the code [c(0),c(1),c(2),…,c(n-1)], here n is the participants number and for each i, c(i) is the rank of the i-th participants. (We start the order from 0-th). Then the neighborhood is defined below:

For a code R_1, a permutation operator (i, j) means the 2-permutation of i-th and j-th code bit, for all i, j in [0, code length-1]. (The code length will equal to the participants number). The neighborhood $N = \{R_2\}$ will contain all possible permutation operator bringing to code R_1.

For example, 5 participants with code length = 5, a code R_1 = [c1, c2, c3, c4, c5]. Then we can permute any two code bit to find the neighborhood N. Note that the 2-permutation number for a n-length set will be $\frac{n(n-1)}{2}$. So for 5 participants with code length 5, the 2-permutation number will be 10. The following results show the 10 neighbor R_2 for code R_1= [c1, c2, c3, c4, c5]:

[c2, c1, c3, c4, c5],
[c1, c3, c2, c4, c5],
[c1, c2, c4, c3, c5],
[c1, c2, c3, c5, c4],
[c3, c2, c1, c4, c5],
[c1, c4, c3, c2, c5],
[c1, c2, c5, c4, c3],
[c4, c2, c3, c1, c5],
[c1, c5, c3, c4, c2],
[c1, c2, c3, c4, c5].

For example, the code R_3 = [c2, c1, c3, c4, c5] will not be the neighbor of code R_1.

The cost of R_2 can be generated easily by the following equation: Define the conditional multiplier $d(R, i, j)$ means that R disagree T on $edge(i, j)$, and defined the multiple $d(R, i, j) \times w(i, j)$ as $add(R, i, j)$ means the additive on the result, then we have:

$$
\begin{aligned}
c(R_2,T) = c(R_1,T) \\
&+ \sum_{k=0,k\neq i\neq j}^{k=n-1} \big[add(R_2,i,k) + add(R_2,j,k) + add(R_2,k,i) + add(R_2,k,j) \\
&- \big(add(R_1,i,k) + add(R_1,j,k) + add(R_1,k,i) + add(R_1,k,j)\big)\big] \\
&+ add(R_2,i,j) + add(R_2,j,i) - add(R_1,i,j) - add(R_1,j,i)
\end{aligned}
$$

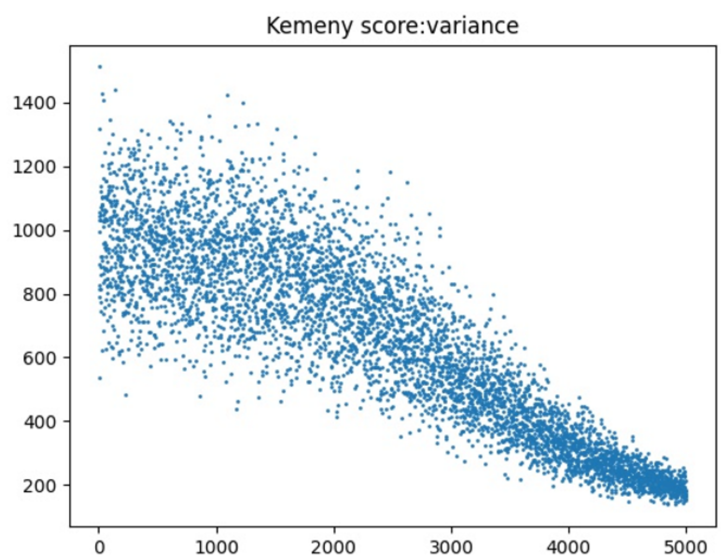
$$where\ R_2\ is\ R_1\ with\ permutation(i,j)$$

TASK 3

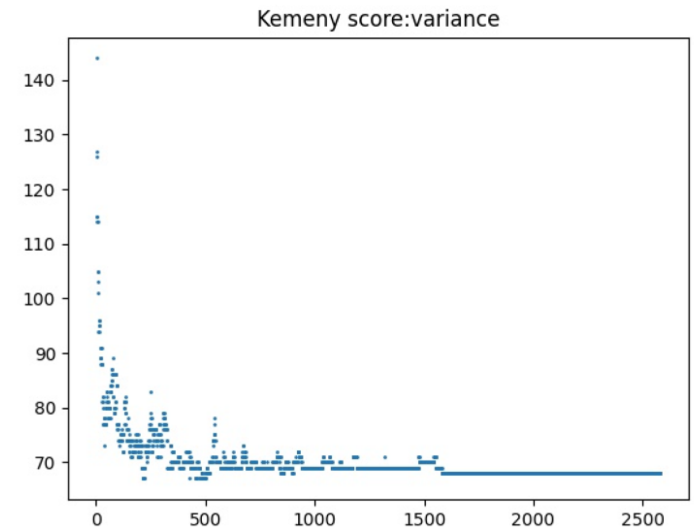
Summary:

The terminate temperature TL markedly determine the result if the SA algorithm have reach its convergency. On the other hands, higher initial temperature provides more possibility for global optimal solution searching. When of convergency, the alpha (temperature multiplier) is a parameter markedly influence time consume.

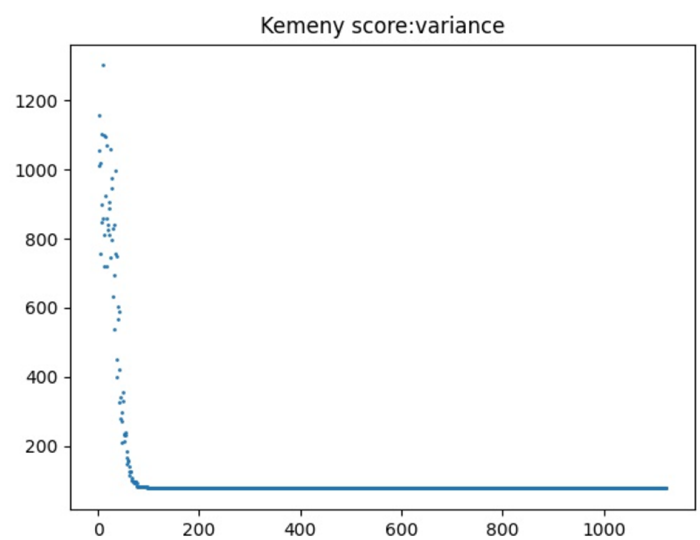
If the TL is not low enough, it may lead to none-convergency, shown below. (TI, T, a, Kemeny score, TL) = (1000, 6.72, 0.999, 138,200)



If the TI is not high enough, it may lead to the incomplete of solution space, shown below: (TI, T, a, Kemeny score, TL) = (1, 0.0756, 0.999, 67, 200)



If the α is too small, it may lead inadequate temperature drop, shown below: (TI, T, a, Kemeny score, TL) = (2000, 8.23e-49, 0.9, 78, 200)



All in all, a proper parameter set should ensure a high enough initially temperature, a low enough terminate temperature, an adequate annealing process (large inner step TL), and a slow enough temperature drop (α close enough to 1).

However, to reach those condition means hard time consume, which may not be accept. Some waste on reaching the convergency can be saved. Please read ‘Extra: time optimization’ to see details on optimizing algorithm implement.

Number_none_impove: 62 (result shown on appendix)

Extra: Time optimization:

There maybe some time consuming optimal. E.g. the inner loop and the outer loop can be deduced when reach adequate ‘waste’ tryout process, that is the tryout(neighbor) do not accepted by the SA energy condition and thus do not update the Kemency score. With this condition, it can be assumed that the SA is reaching convergency. By adding the inner and outer stop condition can we improve the implement on time domain.

Local optima:

Heuristically, for a weight matrix T, it is rational to think that more weight in a row means this participant win more, so he is expected to have greater rank. So we can initially calculate the row summation of weight, and order this weight summation, using the result of sorting (Large first) as heuristic condition for neighbor choice. (That is a permutation change make the participant, who has higher row weight, a greater rank is more likely to be accepted by annealing.)

Appendix: Result (Command Line dump)

TI = 5000, T = 4.98e-3, $\alpha = 0.995$, KS: 62, TL = 200

```
result:
========================================
       rank           participant's name
----------------------------------------
         1     |       Alain Prost
         2     |       Niki Lauda
         3     |       Elio de Angelis
         4     |       Rene Arnoux
         5     |       Corrado Fabi
         6     |       Michele Alboreto
         7     |       Derek Warwick
         8     |       Nelson Piquet
         9     |       Patrick Tambay
        10     |       Andrea de Cesaris
        11     |       Mauro Baldi
        12     |       Thierry Boutsen
        13     |       Teo Fabi
        14     |       Riccardo Patrese
        15     |       Jo Gartner
        16     |       Gerhard Berger
        17     |       Nigel Mansell
        18     |       Keke Rosberg
        19     |       Ayrton Senna
        20     |       Eddie Cheever
        21     |       Marc Surer
        22     |       Jonathan Palmer
        23     |       Martin Brundle
        24     |       Huub Rothengatter
        25     |       Jacques Laffite
        26     |       Stefan Bellof
        27     |       Francois Hesnault
        28     |       Stefan Johansson
        29     |       Piercarlo Ghinzani
        30     |       Manfred Winkelhock
        31     |       Johnny Cecotto
        32     |       Philippe Streiff
        33     |       Philippe Alliot
        34     |       Pierluigi Martini
        35     |       Mike Thackwell
========================================
The Kemeny score of this best ranking is:  62
run time:  52500.06890296936 ms
TI =  5000 , T now =  0.004979705052548735 , alpha =  0.995 , TL =  200 , with outer step:  2758
```

score – step: