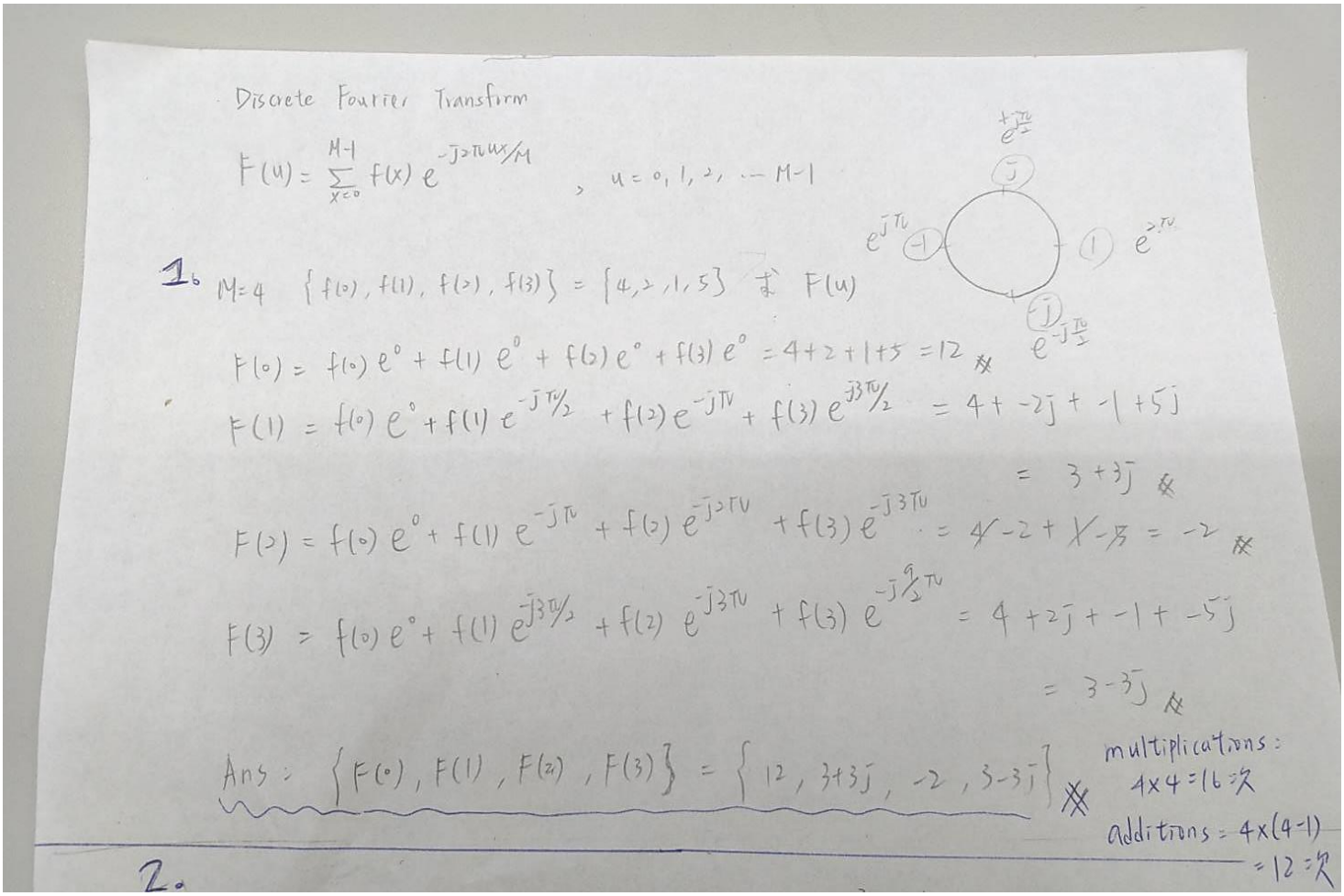
數位影像處理 DIP Homework Chapter 4_1 (100 pts)

1. If a discrete sequence ($M=4$) is $\{f(0), f(1), f(2), f(3)\}=\{4, 2, 1, 5\}$, please find its DFT $F(u)$? and count how many multiplications and additions in the $M=4$ DFT ? (30)

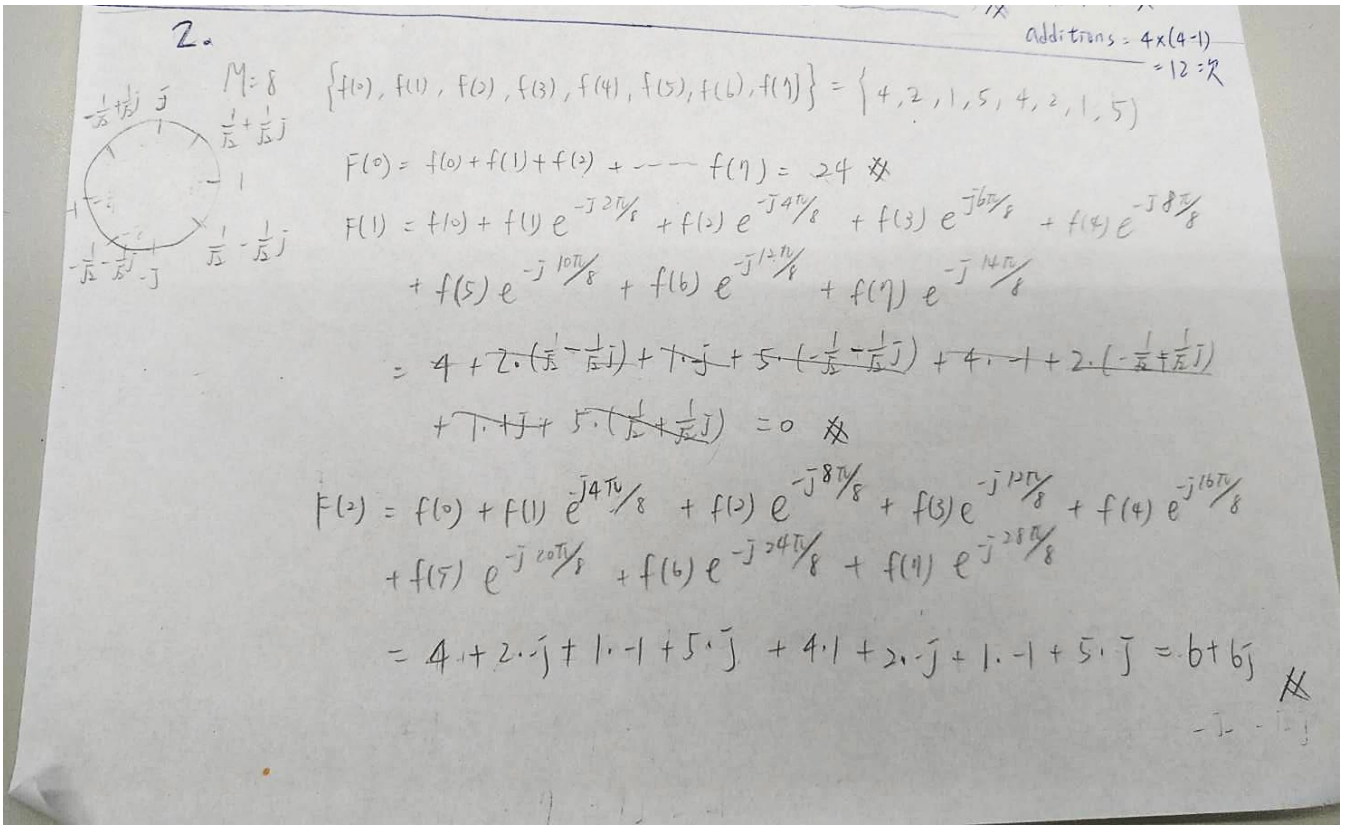


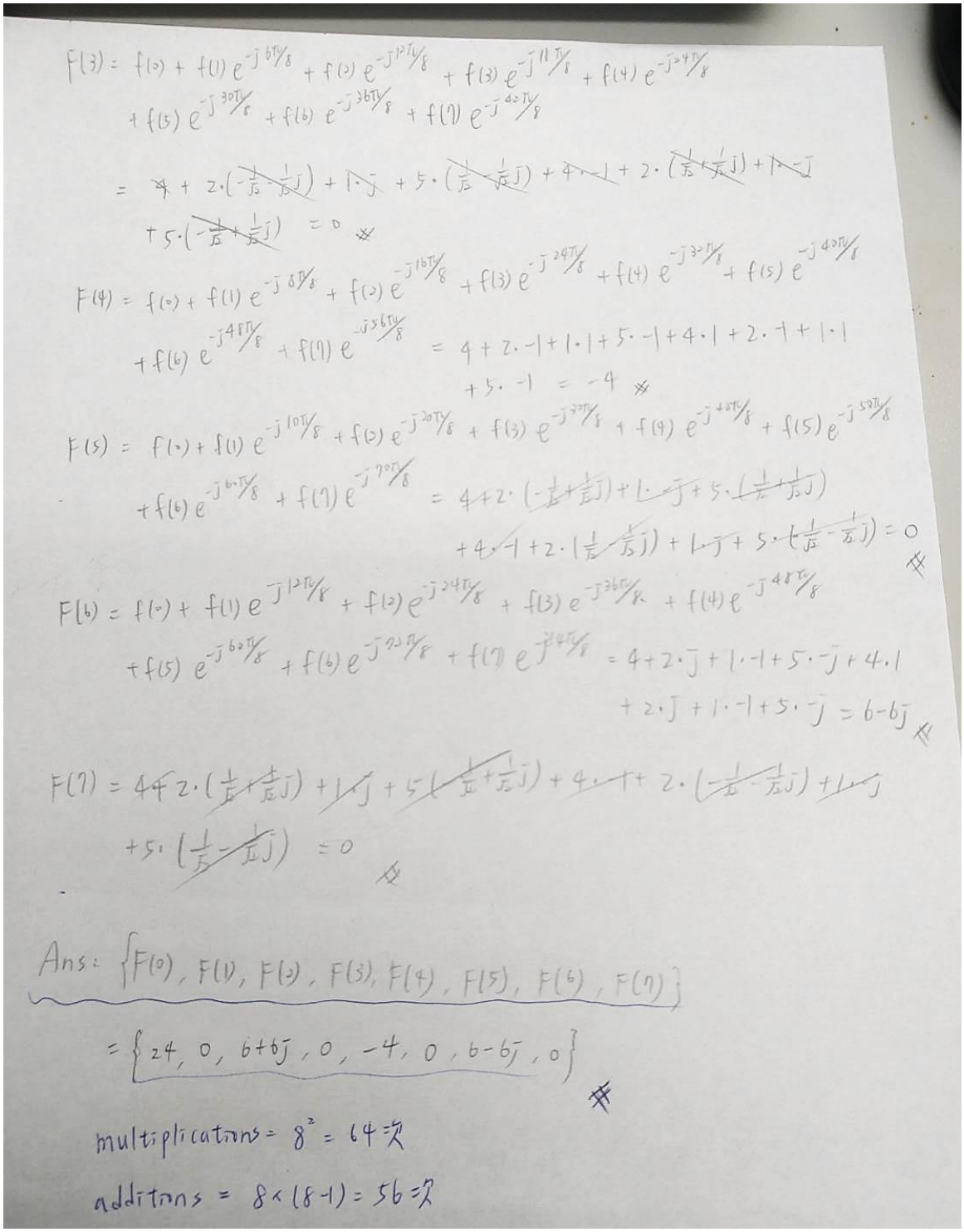
以 code 驗證

```
2   import numpy as np
3
4   x = np.array([4, 2, 1, 5])
5   print(np.fft.fft(x))
```

```
PS C:\Users\USER> & "C:/Program Files/Python37/python.
[12.+0.j  3.+3.j -2.+0.j  3.-3.j]
```

2. If a discrete sequence ($M=8$) is $\{f(0), f(1), f(2), f(3), f(4), f(5), f(6), f(7)\}=\{ 4, 2, 1, 5, 4, 2, 1, 5\}$, please find its DFT $F(u)$? and count how many multiplications and additions in the $M=8$ DFT ? (30)



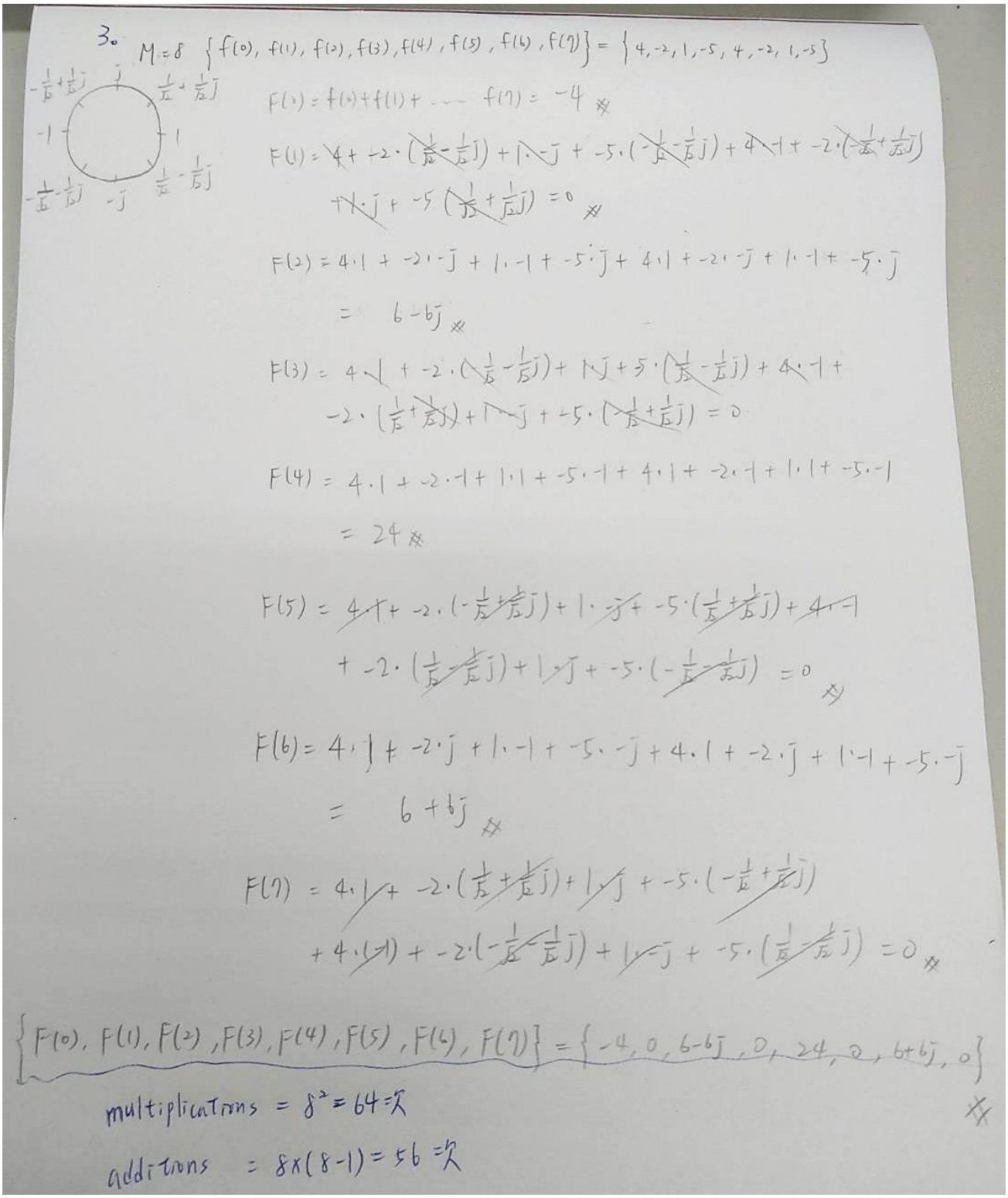


以 code 驗證

```
2   import numpy as np
3
4   x = np.array([4, 2, 1, 5, 4, 2, 1, 5])
5   print(np.fft.fft(x))
```

```
PS C:\Users\USER> & "C:/Program Files/Python37/python.exe" c:/Users
[24.+0.j  0.+0.j  6.+6.j  0.+0.j -4.+0.j  0.+0.j  6.-6.j  0.+0.j]
```

3. If a discrete sequence ($M=8$) is $\{f(0), f(1), f(2), f(3), f(4), f(5), f(6), f(7)\}=\{ 4, -2, 1, -5, 4, -2, 1, -5\}$, please find its DFT $F(u)$? and count how many multiplications and additions in the $M=8$ DFT ? (30)



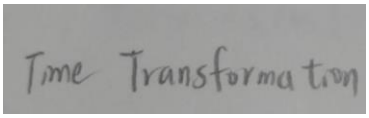
以 code 驗證

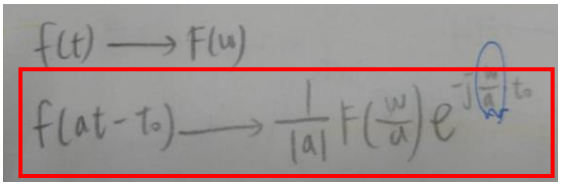
```
2   import numpy as np
3
4   x = np.array([4, -2, 1, -5, 4, -2, 1, -5])
5   print(np.fft.fft(x))
```

```
PS C:\Users\USER> & "C:/Program Files/Python37/python.exe" c:/User
[-4.+0.j  0.+0.j  6.-6.j  0.+0.j 24.+0.j  0.+0.j  6.+6.j  0.+0.j]
```

4. Please state the relation between the question (1), (2), and (3) ? (10)

- (2) 與 (1) 數值一樣，然而 (2) 的 DFT 結果，數值為 (1) 的兩倍。因(2)取樣頻率為(1)的兩倍，因此根據 Fourier Transform 中 Time Transformation 的觀念(如下圖)，當 a 為 1/2，也就是取樣頻率變 2 倍，如同(2)的情況，則做 DFT 的結果，值應為原(1)的兩倍，且在(1)兩倍的位置才出現值。





  - (3) 是 (2)做 Frequency shifting，將(2)的 data 在 time domain *$e^{-jn\pi}$，因此在奇數位的 data 會變為負值，做此處理的結果是 DFT 的結果會往右移動π。(原始 data 的長度是 2π)