作业5

7.1

Let $R_1 = (A, B, C)$, $R_2 = (A, D, E)$

$\because R_1 \cap R_2 \to R_1$ or $R_1 \cap R_2 \to R_2$, $\therefore \{R_1, R_2\}$ is lossless decomposition

$\because A \to BC$, $\Rightarrow A \to B, A \to C$; $CD \to E \Rightarrow C \to E, D \to E$

$\therefore A \to C \to E \to A$, A is candidate key

$\because R_1 \cap R_2 = A$, A is candidate key

$\therefore R_1 \cap R_2 \to R_1$

7.2

All nontrivial functional dependencies : $A \to B$, $C \to B$

Logically Imply Dependencies : $AC \to B$

7.6 From 7.1,

$\because A \to A$, $A \to B$, $A \to C$, $A \to B \to D \Rightarrow A \to D$, $A \to E$

$\therefore A \to ABCDE$

$\because E \to A$, $\therefore E \to ABCDE$

$\because CD \to E$, $\therefore C \to ABCDE$, $D \to ABCDE$

$\because B \to D$, $\therefore B \to ABCDE$

$A \to A$, $B \to B$, $C \to C$, $D \to D$, $E \to E$

$\therefore A*\to\alpha$, $BC*\to\alpha$, $CD*\to\alpha$, $E*\to\alpha$, α is subset of $\{A, B, C, D, E\}$

$\therefore A, BC, CD, E$ are candidate keys

7.31

From AB,

$\because AB \rightarrow CD \Rightarrow AB \rightarrow C,\ AB \rightarrow D$

$\therefore AB^+ = \{A, B, C, D\}$

$\because AC$ already exists $\quad AC \rightarrow DE \Rightarrow AC \rightarrow D,\ AC \rightarrow E$

$\therefore AB^+ = \{A, B, C, D, E\}$

$\because$ cannot access G through set F of functional dependencies,

$\therefore AB^+ \neq R$, R is not in BCNF

Let $R_1 = AB \cup CD = \{A, B, C, D\}$, $R_2 = (R - \{C, D\}) \cup AB = \{A, B, E, G\}$

For R_1,

$AB \rightarrow CD$ has candidate key for R1, $AB^+ = \{A, B, C, D\}$, $\therefore R_1$ in BCNF

For R_2,

$\because$ missing C or D attribute, $\therefore R_2$ is not in BCNF

BCNF decomposition is not dependency preserving because not all the FDs are contained in either relation.

7.32

(a): $CD \to AB \Rightarrow CD \to A,\ CD \to B$

$\because A \to BC \Rightarrow A \to B,\ A \to C$

$\because CD \to A \to B \Rightarrow CD \to B,$

$\because C$ or D alone don't determine B

$\therefore CD \to B$ is nontrivial FD

(b)

From A, $A \to BC \Rightarrow A \to B,\ A \to C$, $\therefore A^+ = \{A, B, C\}$

$\because A^+ \neq R$, $\therefore A \to BC$ violates BCNF

Let $R_1 = A \cup BC = \{A, B, C\}$, $R_2 = (R - (BC - A)) = \{A, D, E, G\}$

From R_1,

$A^+ = \{A, B, C\} = R_1$, $\therefore R_1$ is in BCNF

From R_2,

$BD \to E$, B not in R2,

$CD \to AB$, C not in R_2,

$A \to BC$, B and C missing

$\therefore R_2$ is in BCNF

(c) From (b), $R_1 \cap R_2 = A$, $\because A \to BC \Rightarrow A^+ = \{A, B, C\}$

$\therefore A$ is a key, $\therefore R_1 \cap R_2$ is lossless,

(d) $A \to BC$, in R_1; $BD \to E$, not fully contained in either relations;

$CD \to AB$, $CD \to A$, $CD \to B$, not preserved in either relations.

$\therefore$ The BCNF decomposition is not dependency preserving because some of the original FD involve attributes split between the decomposed relations.