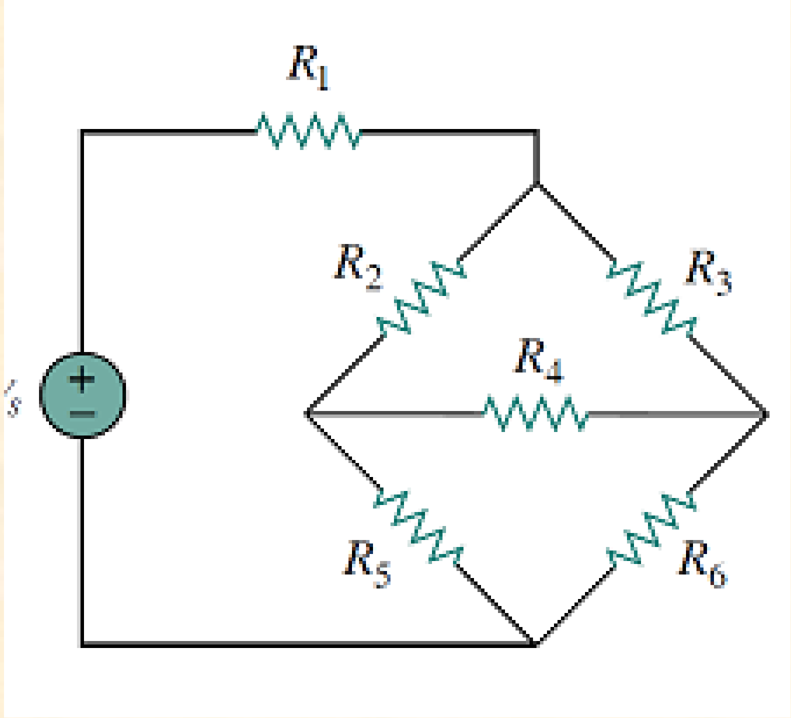
Lecture 2

Electrical Basics

Resistors in Series and in Parallel

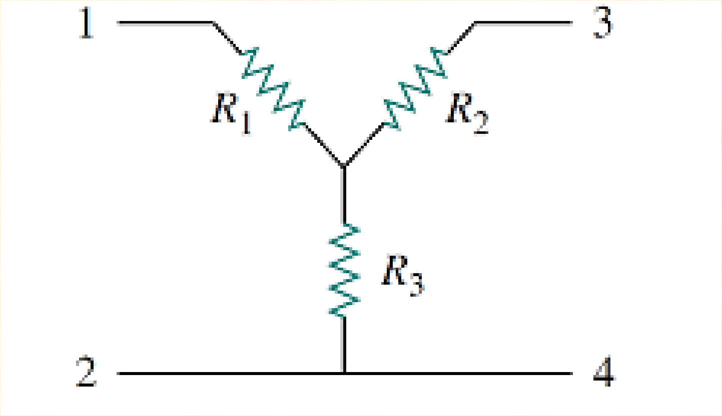
Wye and Delta (Y – Δ) circuit:

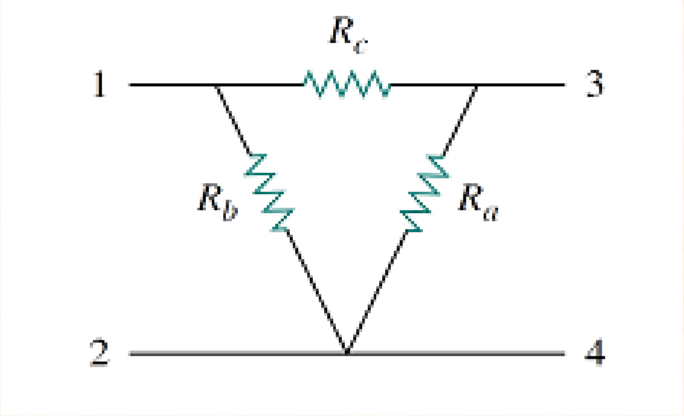


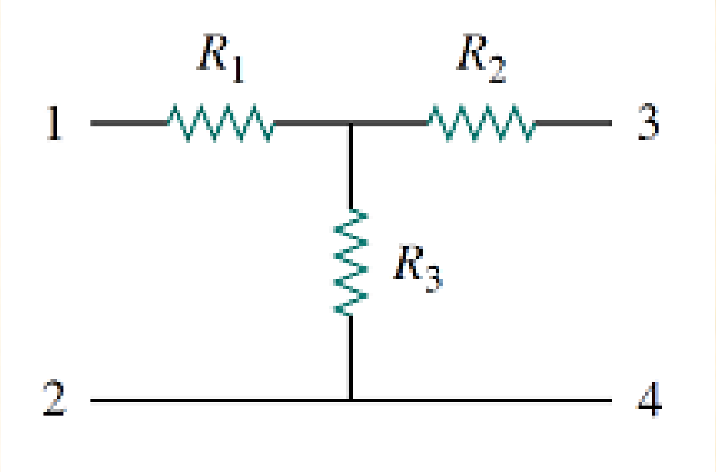
This circuit has no resistor series or parallel

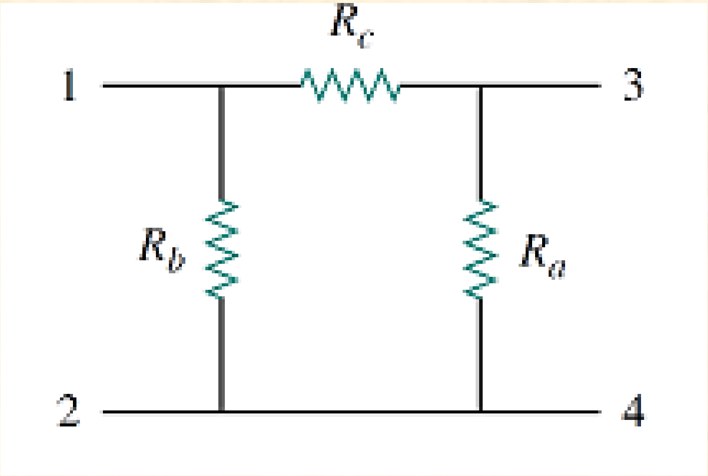
Resistors in Series and in Parallel

Wye and Delta (Y – Δ) circuitos:









Wye circuit

Delta circuit

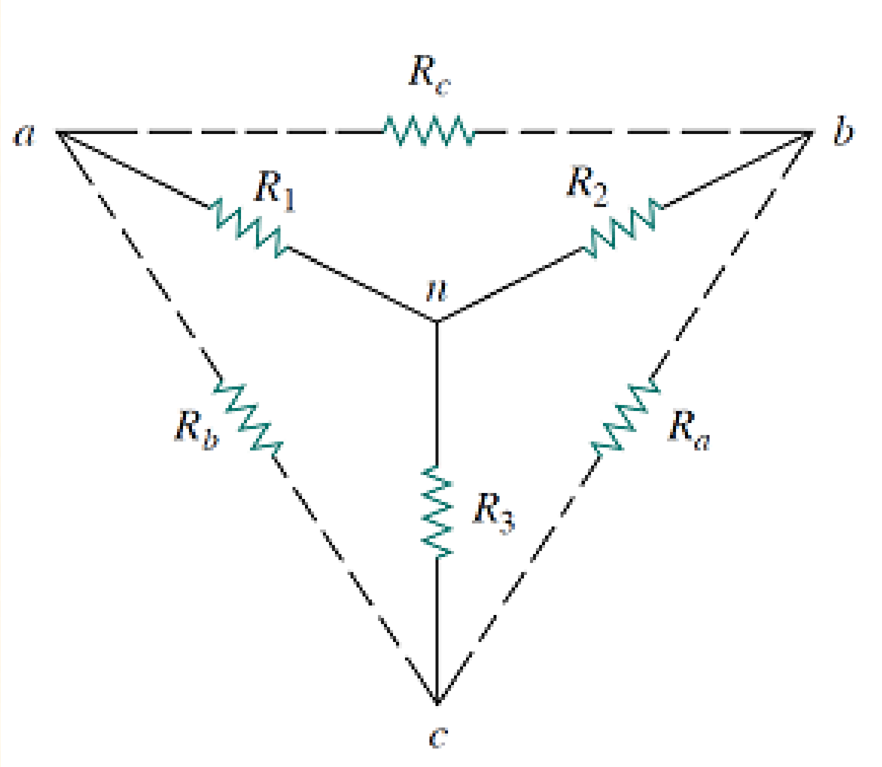
Resistors in Series and in Parallel

Wye to Delta Transformation:

$$R_a = R_2 + R_3 + \frac{R_2 R_3}{R_1}$$

$$R_b = R_1 + R_3 + \frac{R_1 R_3}{R_2}$$

$$R_c = R_1 + R_2 + \frac{R_1 R_2}{R_3}$$



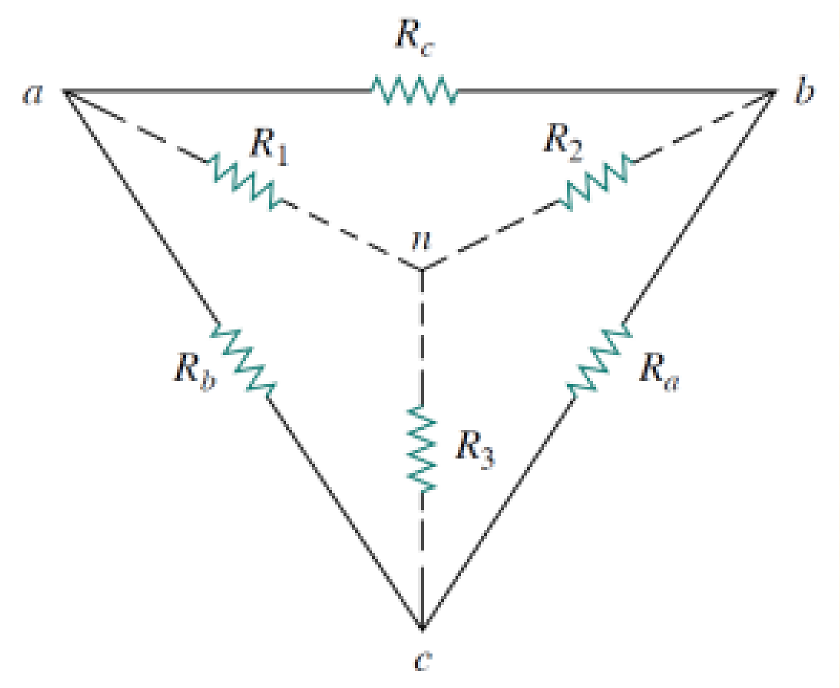
Resistors in Series and in Parallel

Delta to Wye Transformation:

$$R_1 = \frac{R_b R_c}{R_a + R_b + R_c}$$

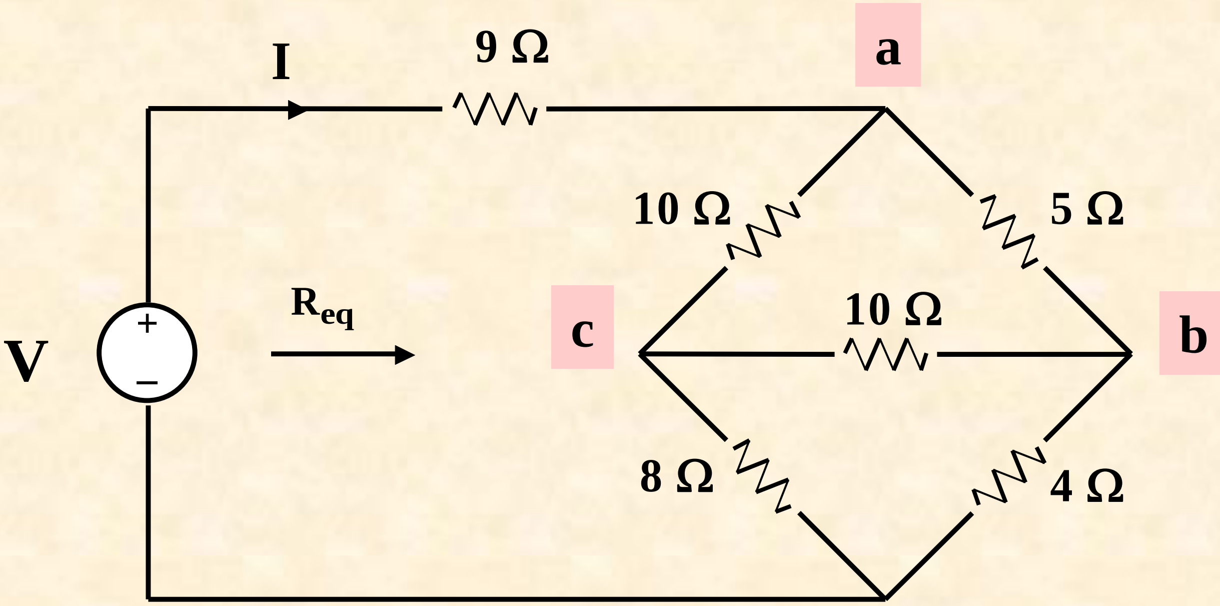
$$R_2 = \frac{R_c R_a}{R_a + R_b + R_c}$$

$$R_3 = \frac{R_a R_b}{R_a + R_b + R_c}$$



Resistors in Series and in Parallel

Example : Given the circuit below. Find R_{eq}.



Convert the delta around a – b – c to a wye.

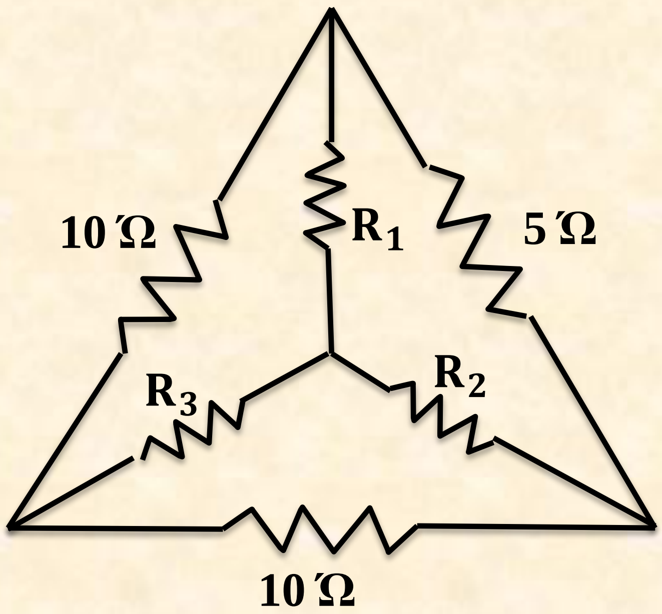
Resistors in Series and in Parallel

Delta to Wye Transformation:

$$\mathbf{R_1} = \frac{\mathbf{5 * 10}}{\mathbf{10 + 10 + 5}} = \mathbf{2}\ \Omega$$

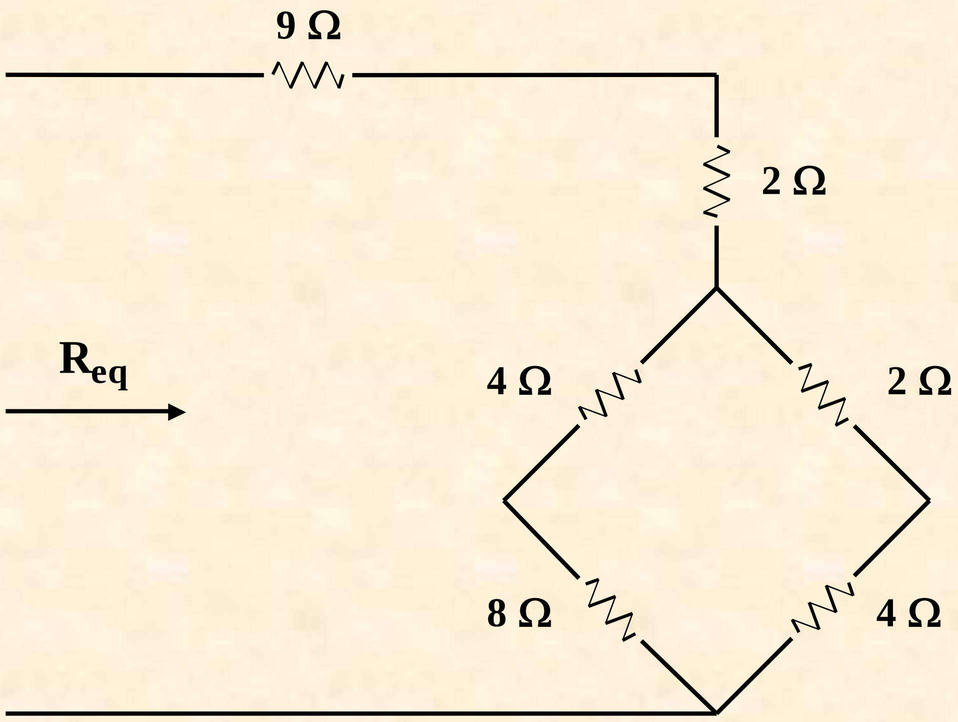
$$\mathbf{R_2} = \frac{\mathbf{5 * 10}}{\mathbf{10 + 10 + 5}} = \mathbf{2}\ \Omega$$

$$\mathbf{R_3} = \frac{\mathbf{10 * 10}}{\mathbf{10 + 10 + 5}} = \mathbf{4}\ \Omega$$



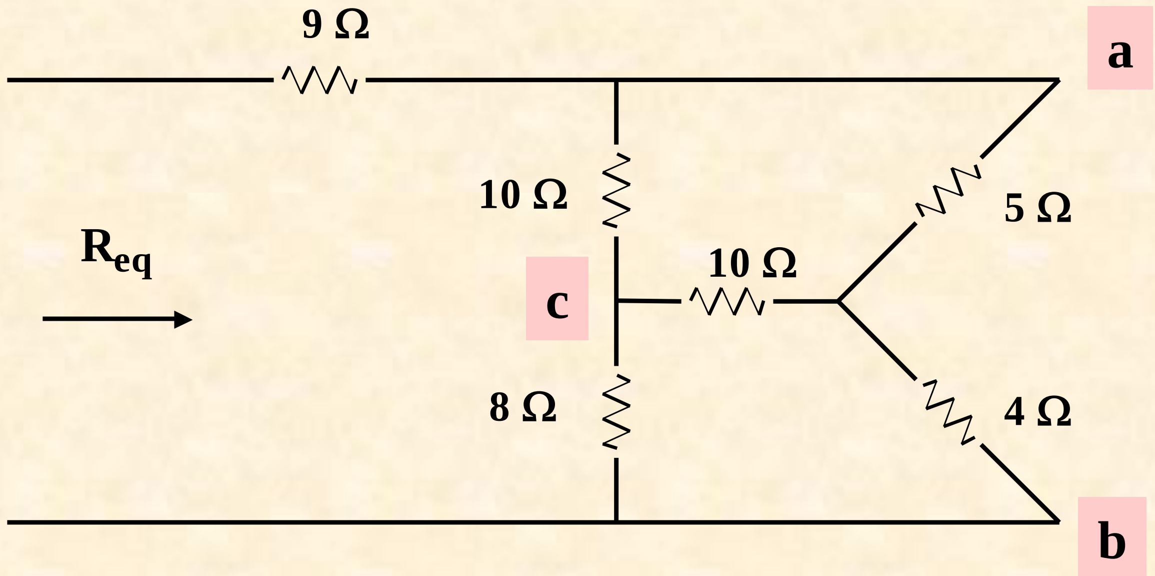
Resistors in Series and in Parallel

$R_{eq} = 15\ \Omega$



Resistors in Series and in Parallel

Example : Given the circuit below. Find R_{eq}.



Convert the wye around a – b – c to delta.

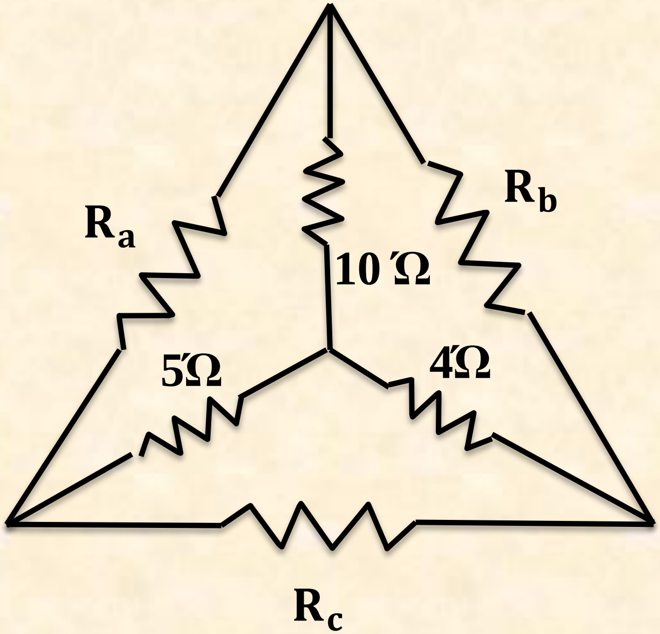
Resistors in Series and in Parallel

Wye to Delta Transformation:

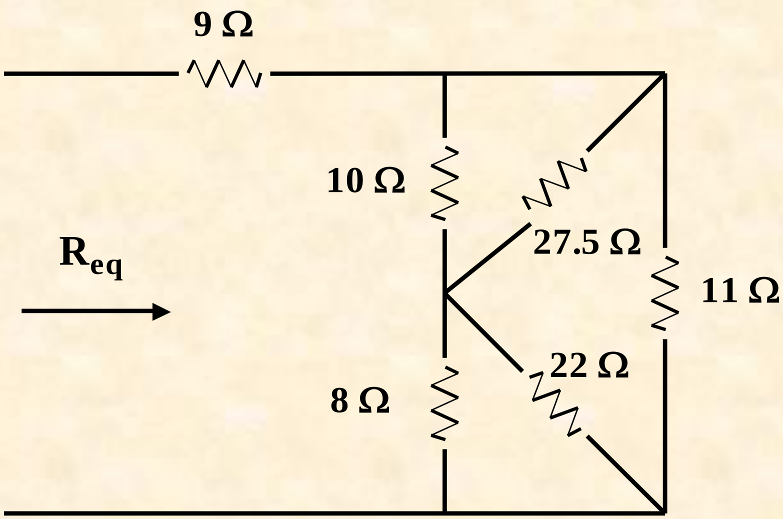
$$\mathbf{R_a = 5 + 10 + \frac{5 * 10}{4} = 27.5\ \Omega}$$

$$\mathbf{R_b = 10 + 4 + \frac{4 * 10}{5} = 22\ \Omega}$$

$$\mathbf{R_c = 5 + 4 + \frac{5 * 4}{10} = 11\ \Omega}$$

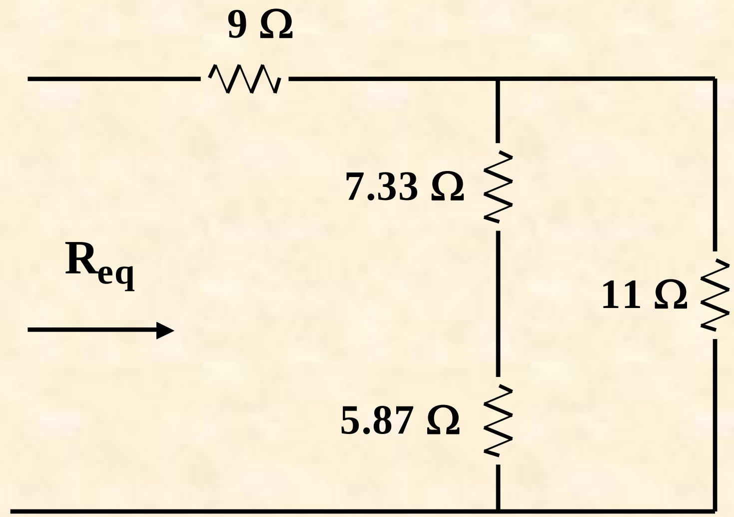


Resistors in Series and in Parallel



$10\ \Omega \; // \; 27.5\ \Omega$

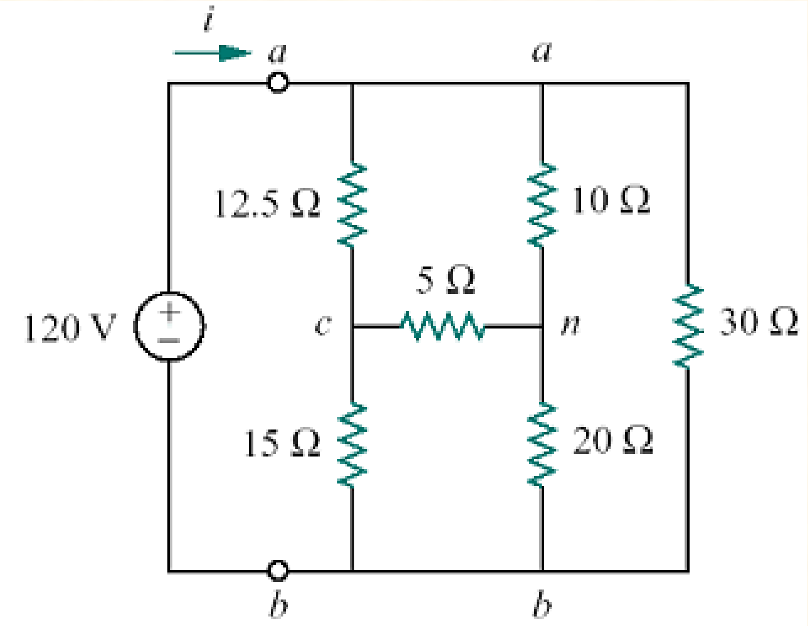
$8\ \Omega \; // \; 22\ \Omega$



$R_{eq} = 15\ \Omega$

Resistors in Series and in Parallel

Example : Obtain the equivalent resistance R_{ab} for the given circuit and find current i.



Convert the delta around a – b – c to a wye.

Resistors in Series and in Parallel

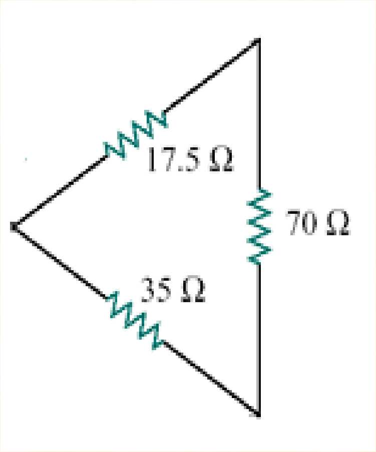
Convert the wye around a – b – c to delta.

$\mathbf{R_1 = 10\,\Omega}$ $\mathbf{R_2 = 20\,\Omega}$ $\mathbf{R_2 = 5\,\Omega}$

$$\mathbf{R_a = 5 + 10 + \frac{5 * 10}{20} = 17.5\,\Omega}$$

$$\mathbf{R_b = 5 + 20 + \frac{5 * 20}{10} = 35\,\Omega}$$

$$\mathbf{R_c = 10 + 20 + \frac{10 * 20}{5} = 70\,\Omega}$$



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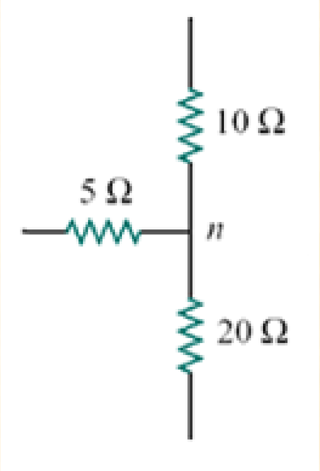
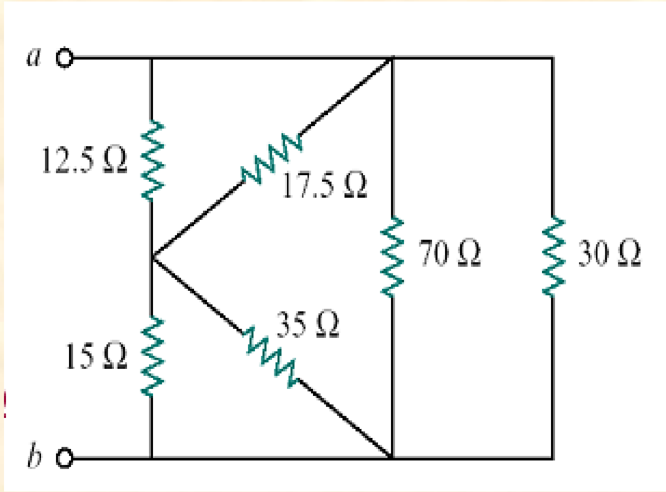


Resistors in Series and in Parallel

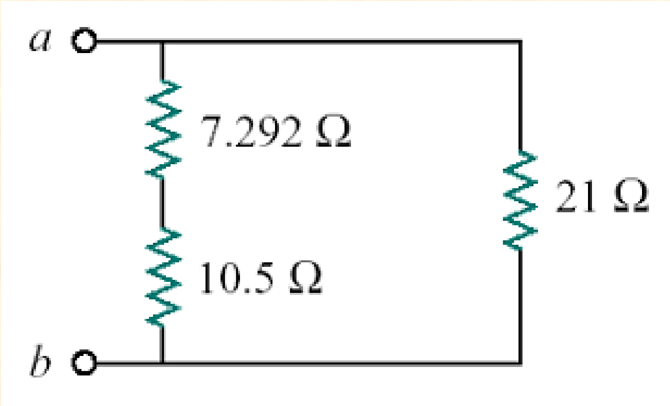
Combining the three pairs of resistors in parallel, we obtain



$$30\ \Omega \,//\, 70\ \Omega = \frac{70 * 30}{70 + 30} = 21\ \Omega$$

$$12.5\ \Omega \,//\, 17.5\ \Omega = \frac{12.5 * 17.5}{12.5 + 17.5} = 7.29\ \Omega$$

$$15\ \Omega \,//\, 35\ \Omega = \frac{15 * 35}{15 + 35} = 10.5\ \Omega$$



$$\mathrm{R_{eq}} = \frac{17.79 * 21}{17.79 + 21} = 9.6\ \Omega$$

$$\mathrm{I} = \frac{V}{\mathrm{R_{eq}}} = \frac{120}{9.6} = 12.4\ A$$

Kirchhoff's Laws

Kirchhoff's Current Law (KCL)

Sum of all currents entering a node is zero

$$\sum_n I_n = 0$$

Sum of currents entering node is equal to sum of currents leaving node

$$\sum I_{in} = \sum I_{out}$$

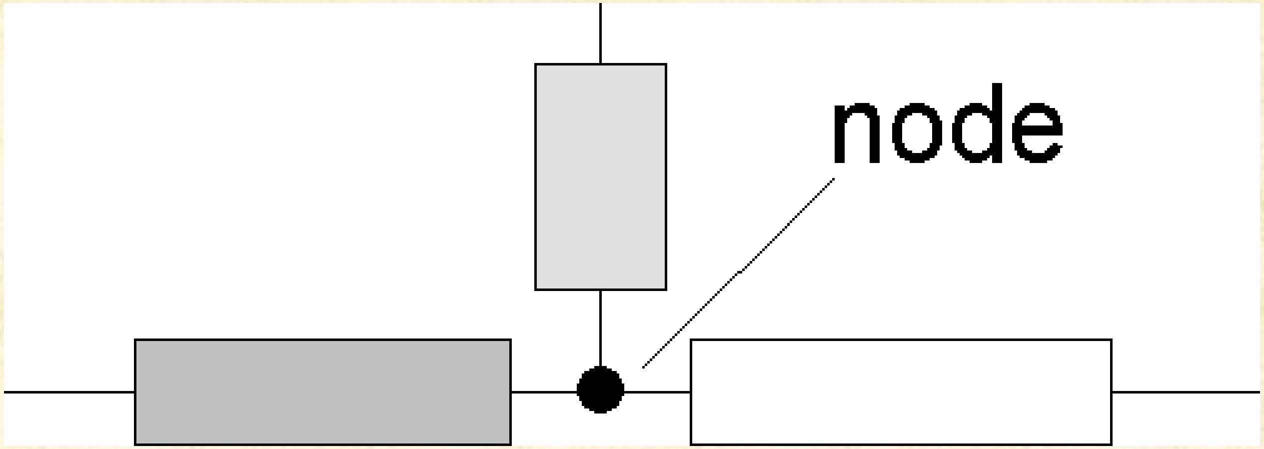
Kirchhoff's Voltage Law (KVL)

Sum of voltages around any loop in a circuit is zero

$$\sum_n V_n = 0$$

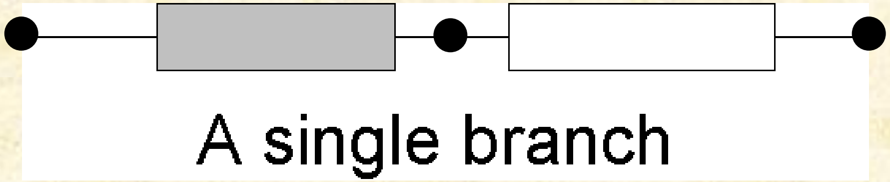
Circuit Topology

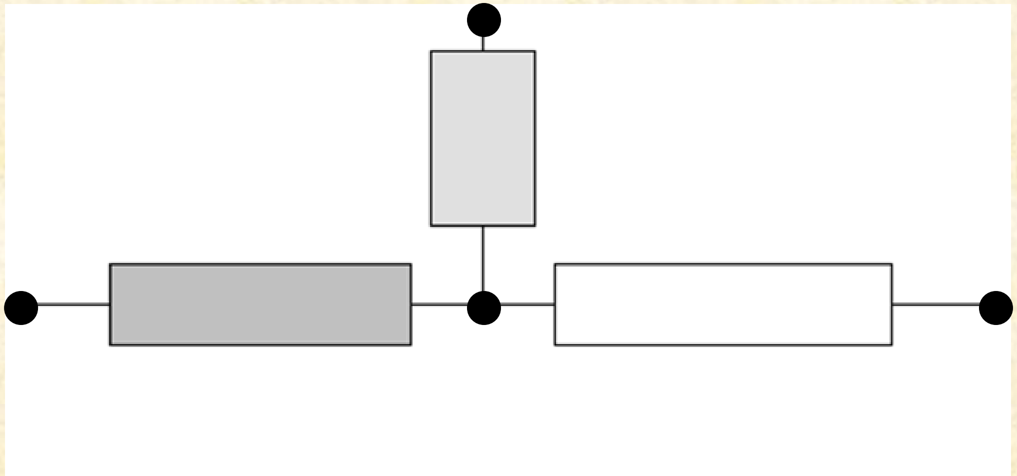
A **node** is the point of connection between two or more circuit elements



Circuit Topology

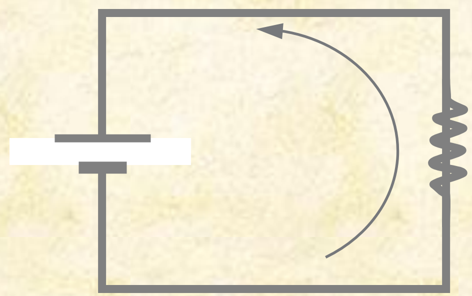
A **branch** represents a single circuit element; between any two nods.





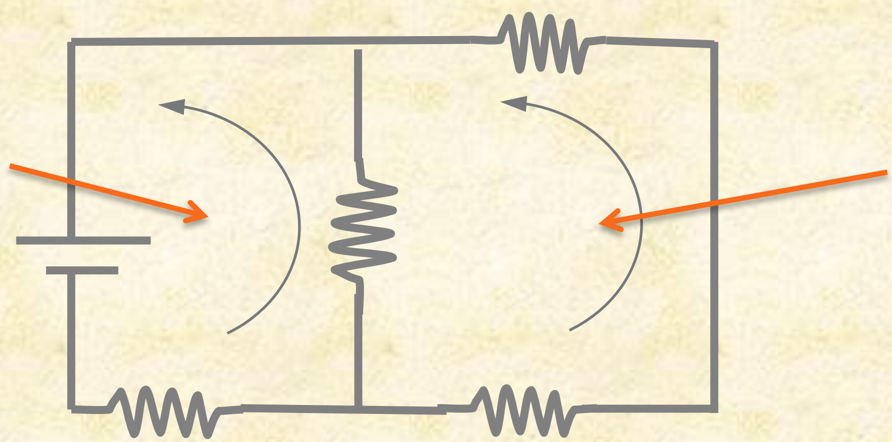
Circuit Topology

A **loop** is any closed path in a circuit (network).



A **loop** is said to be **independent** if it contains at least one branch which is not a part of any other **independent loop**

Independent loop

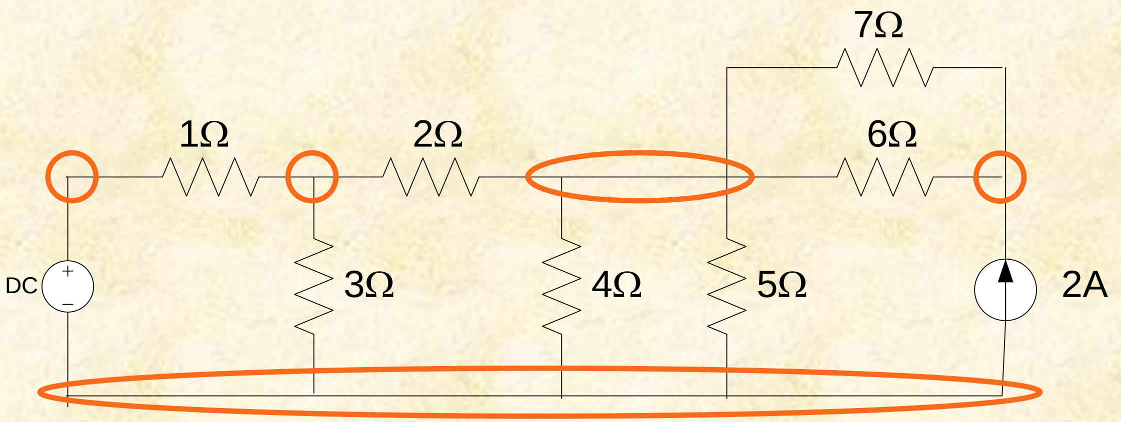


Independent loop

For a network with ***b*** branches, ***n*** nodes and ***l*** loops:

$$b = l + n - 1$$

Example



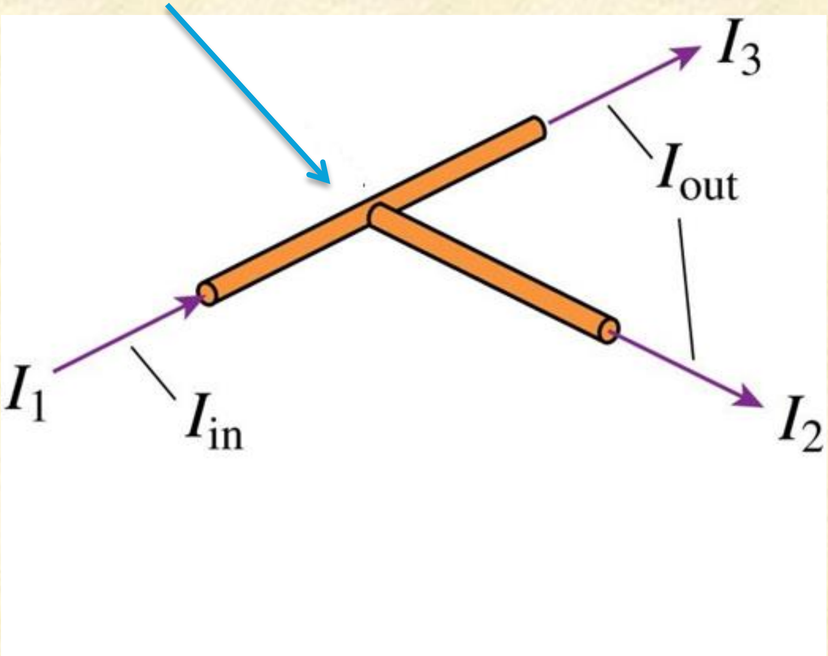
$n =$ **5**

$l =$ **5**

$b =$ **9**

Kirchhoff's Current Law (KCL)

Node



Sum of all currents entering a node is zero

$$I_1 + I_2 + I_3 = 0$$

Sum of currents entering node is equal to sum of currents leaving node

$$I_{in} = I_1$$

$$I_{out} = I_2 + I_3$$

$$I_1 = I_2 + I_3$$

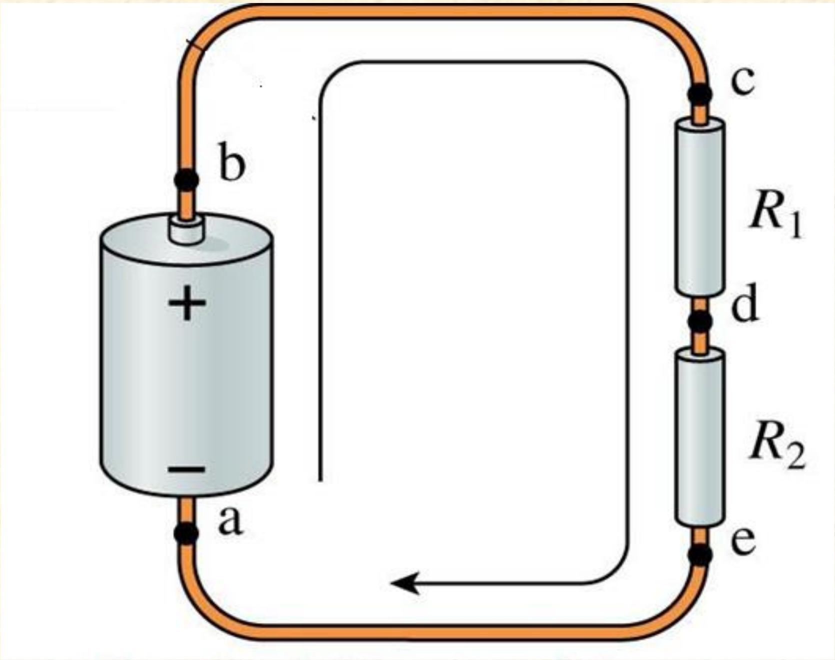
Kirchhoff's Voltage Law (KVL)

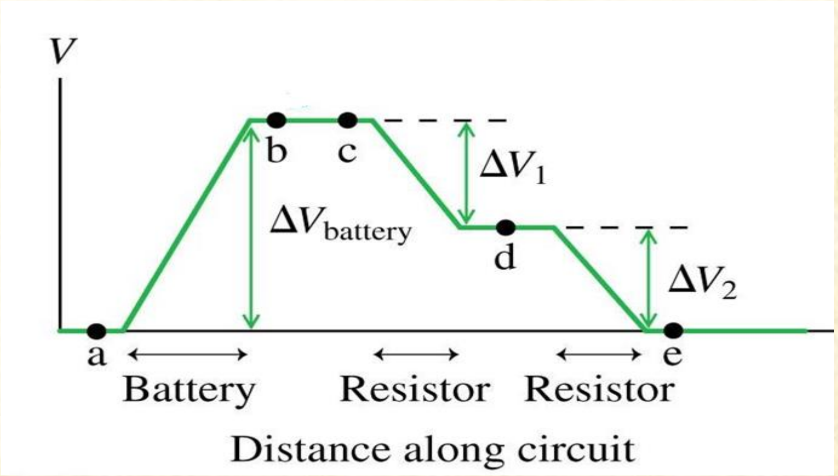
Sum of voltages around any loop in a circuit is zero

$$\Delta V_{bettary} = \Delta V_{R1} + \Delta V_{R2}$$

$$\Delta V_{bettary} + \Delta V_{R1} + \Delta V_{R2} = 0$$

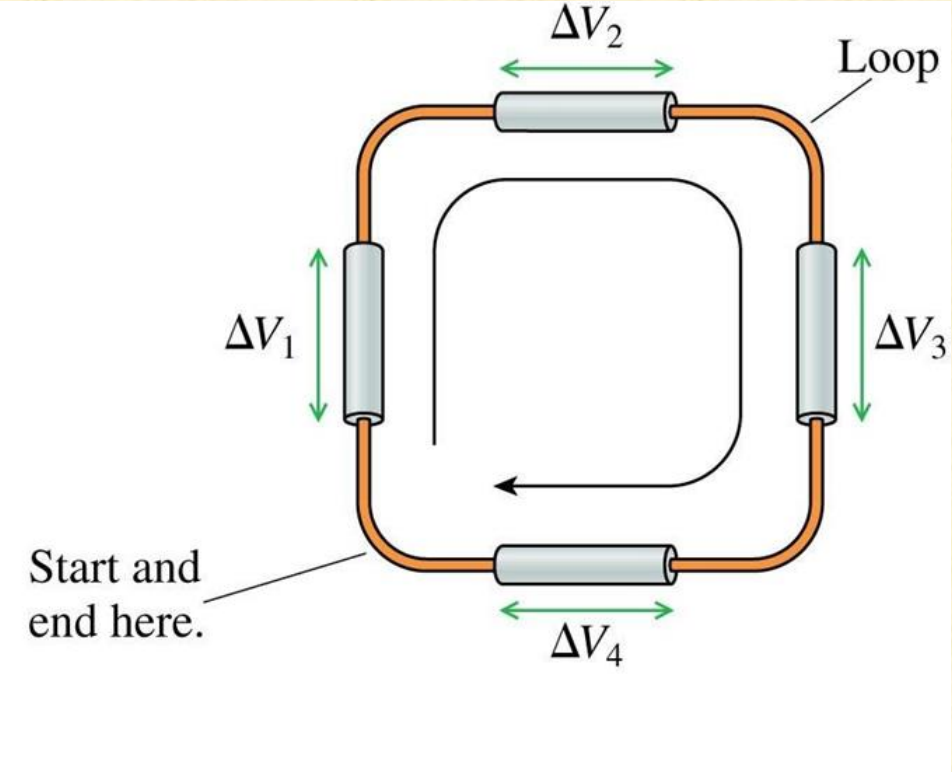
$$\sum_{n} V_n = 0$$





Kirchhoff's Voltage Law (KVL)

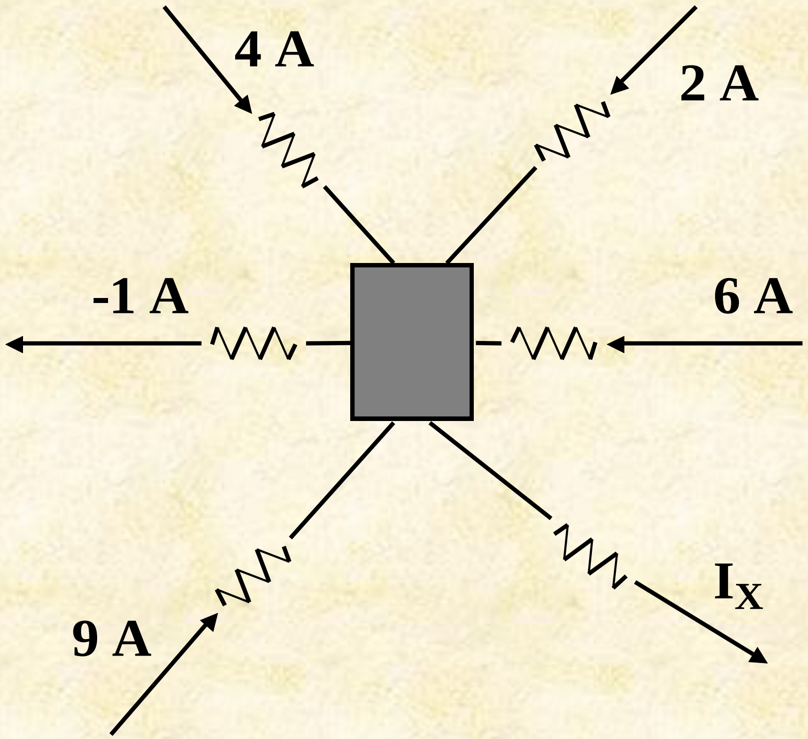
For any circuit, if we add all of the potential differences around the loop formed by the circuit, the sum must be zero.



$$\Delta V_1 + \Delta V_2 + \Delta V_3 + \Delta V_4 = 0$$

Example

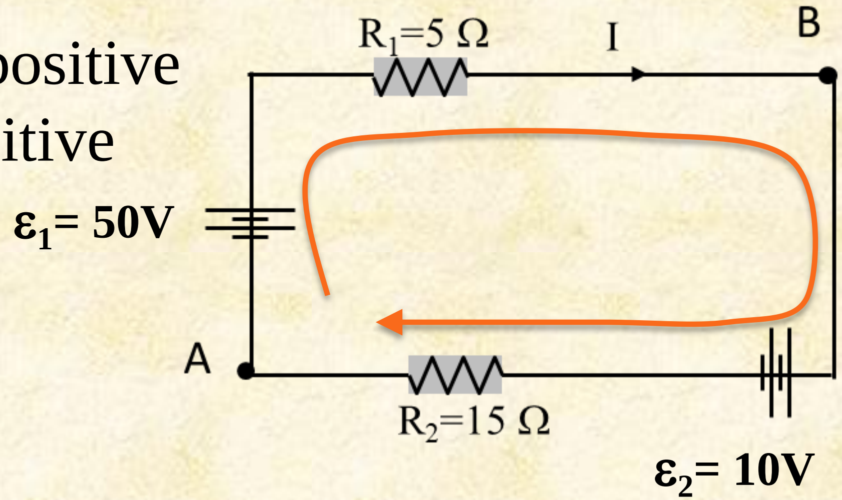
Find the current I_x.



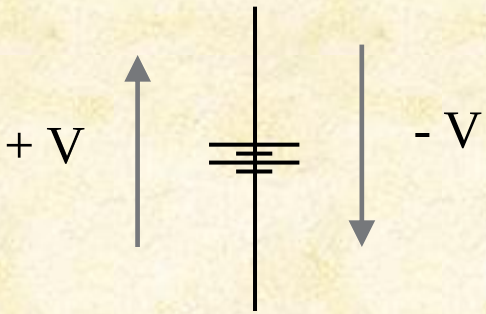
$I_X =$ 22 A

Example

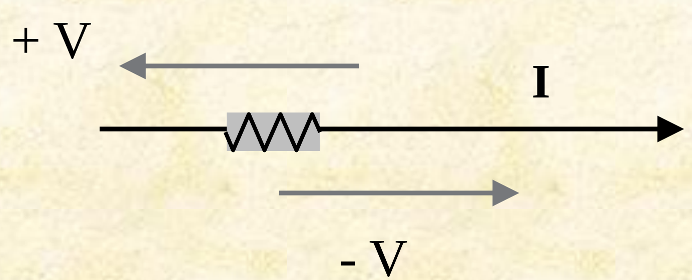
In the shown circuit . Find I .



For **batteries** – voltage change is positive when summing from negative to positive



For **resistors** – voltage change is negative when summing in the direction of the current

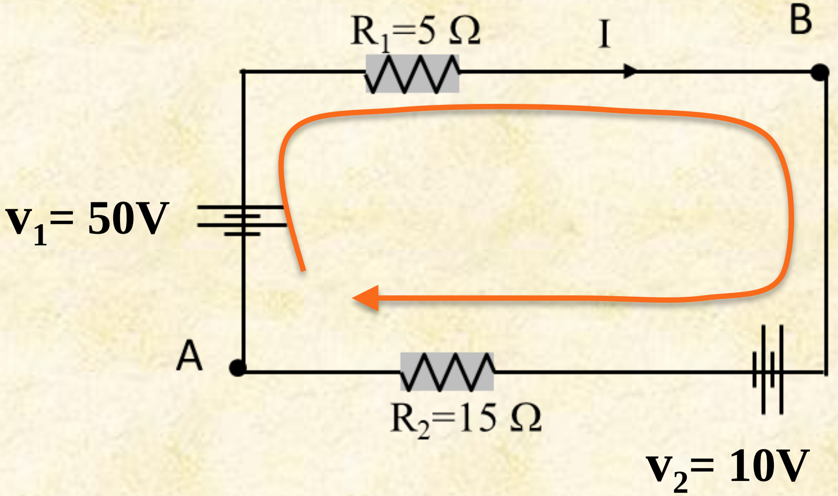


Electrical Basics

$$+V_1 - IR_1 - V_2 - IR_2 = 0$$

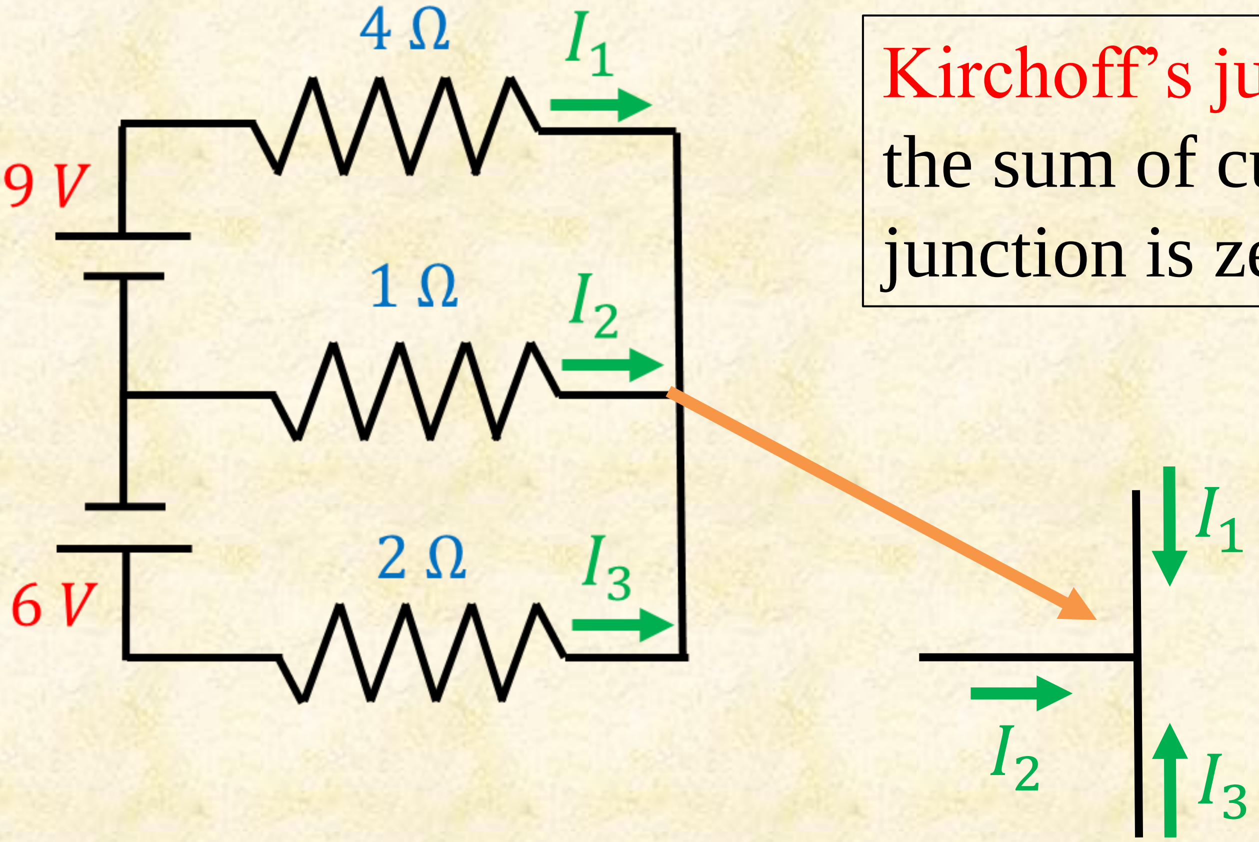
+50 - 5 I - 10 - 15 I = 0

I = +2 Amps



Example

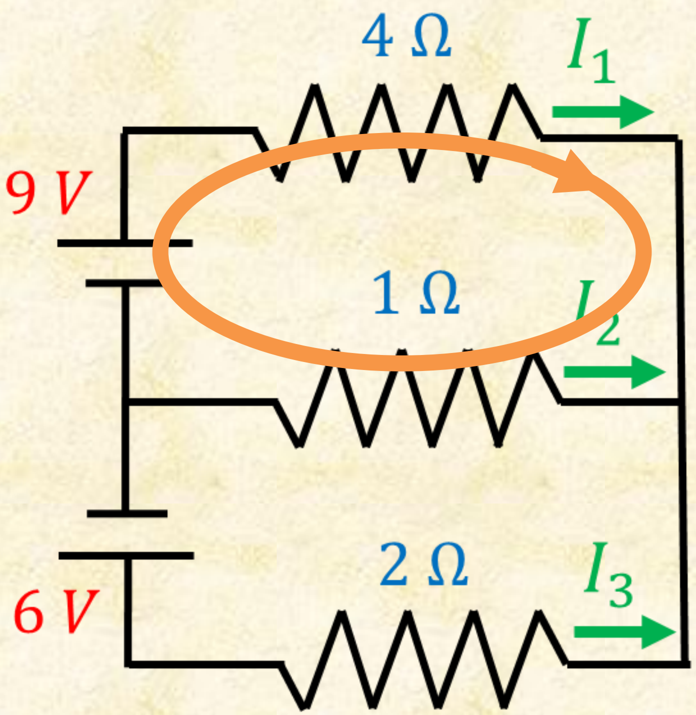
What are the currents flowing in the 3 resistors?

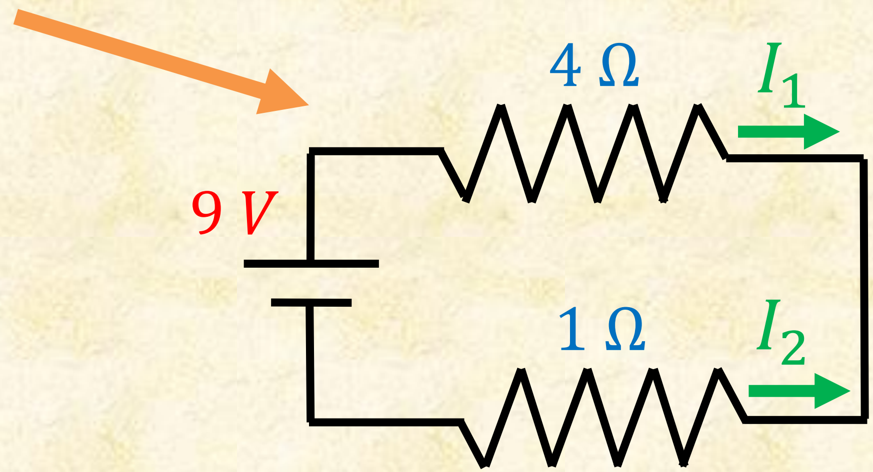


Kirchoff's junction rule :
the sum of currents at any junction is zero

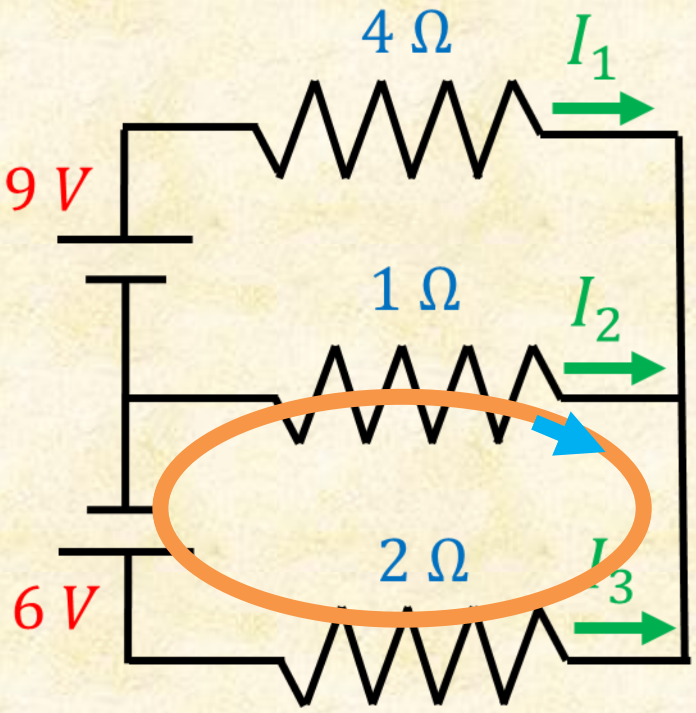
$$I_1 + I_2 + I_3 = 0$$

Kirchoff's loop rule : the sum of voltage changes around a closed loop is zero

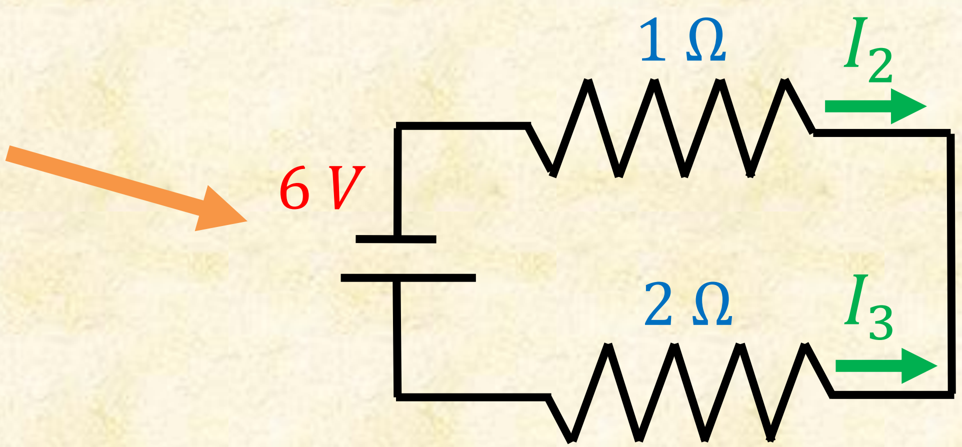




$$9 - 4\,I_1 + 1\,I_2 = 0$$



Kirchoff's loop rule : the sum of voltage changes around a closed loop is zero



$$-6 - 1\, I_2 + 2\, I_3 = 0$$

We now have 3 equations:

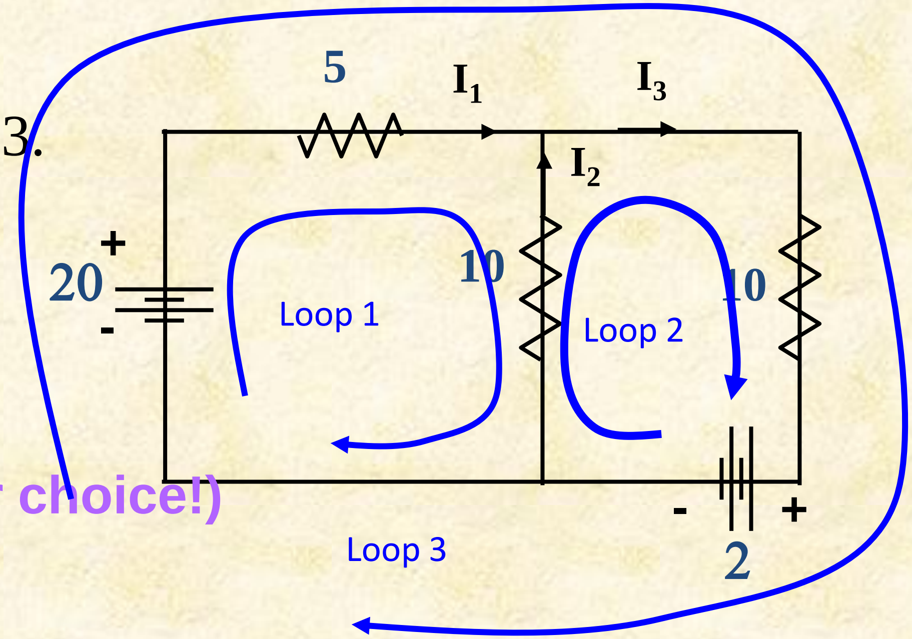
$$I_1 + I_2 + I_3 = 0 \quad (1)$$
$$9 - 4I_1 + I_2 = 0 \quad (2)$$
$$-6 - I_2 + 2I_3 = 0 \quad (3)$$

To solve for I_1 we can use algebra to eliminate I_2 and I_3 :

$$(1) \rightarrow \quad I_3 = -I_1 - I_2$$
$$Sub.in\ (3) \rightarrow I_2 = -2 - \tfrac{2}{3} I_1$$
$$Sub.in\ (2) \rightarrow I_1 = 1.5\ A$$

Example

In the shown circuit . Find I1, I2 and I3.



1. **Label all currents**
2. **Write down node equation**

   Node: $I_1 + I_2 = I_3$
3. **Choose loop and direction** **(Your choice!)**
4. **Write down voltage changes**

Loop 1: $+V_1 - I_1R_1 + I_2R_2 = 0$

$20 - 5I_1 + 10I_2 = 0$

Loop 2: $- I_2R_2 - I_3R_3 - V_2 = 0$

$-2 - 10I_2 - 10I_3 = 0$

Loop 3: $+V_1 - I_1R_1 - I_3R_3 - V_2 = 0$

$20 - 5I_1 - 10I_3 - 2 = 0$

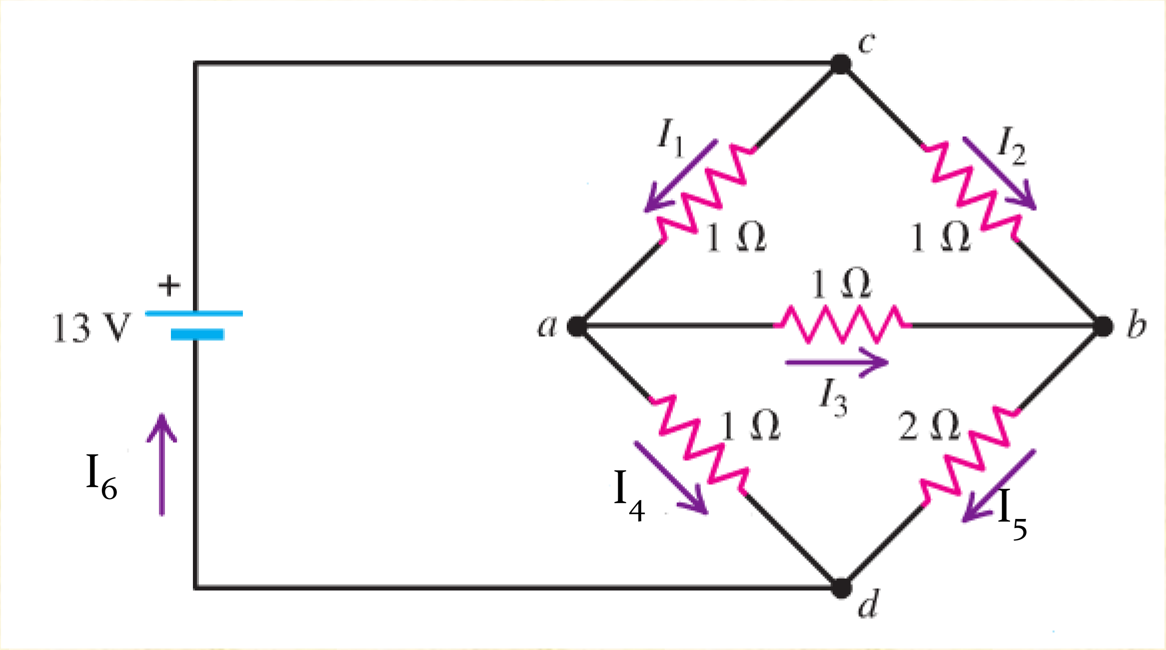
$I_1 = 1.90$ A

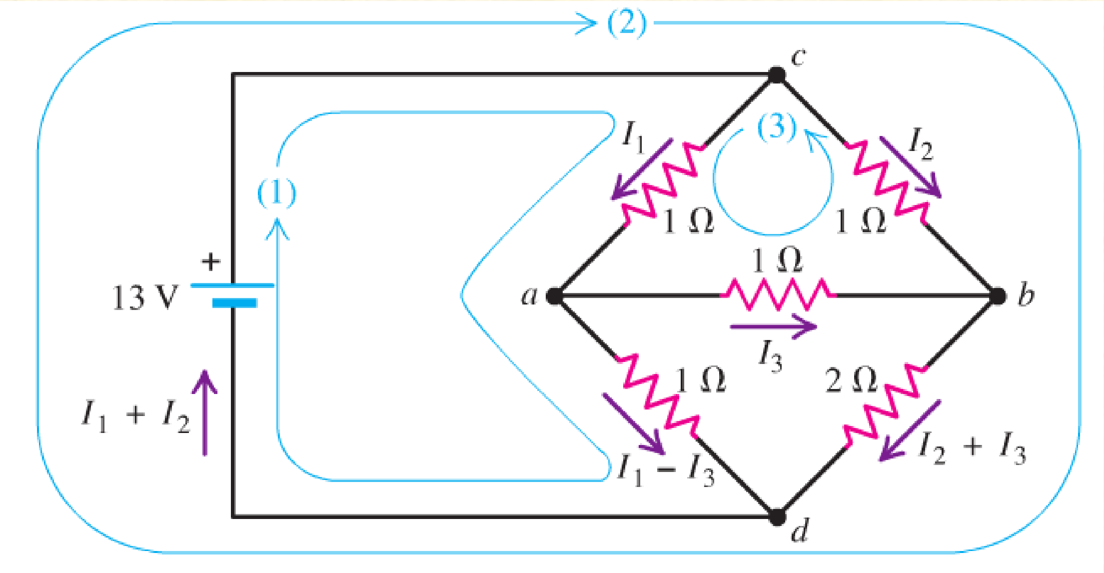
$I_2 = -1.05$ A

$I_3 = 0.85$ A

Example

Find the equivalent resistance for the following circuit





loop1: $13V - I_1 \times 1\Omega - \left(I_1 - I_3\right) \times 1\Omega = 0$

loop2: $13V - I_2 \times 1\Omega - \left(I_2 + I_3\right) \times 2\Omega = 0$

loop3: $-I_1 \times 1\Omega - I_3 \times 1\Omega + I_2 \times 1\Omega = 0$

From 3: $I_1 = I_2 - I_3$ **into 1:**

$$13V - I_2 \times 1\Omega + I_3 \times 1\Omega - \left(I_2 - 2I_3\right) \times 1\Omega = 0$$

$$13V - 2I_2 \times 1\Omega + 3I_3 \times 1\Omega = 0$$

Together with eq. 2 we get

$$\left.\begin{aligned} &13V - 2I_2 \times 1\Omega + 3I_3 \times 1\Omega = 0 \\ &13V - 3I_2 \times 1\Omega - 2I_3 \times 1\Omega = 0 \end{aligned}\right\} \quad \left.\begin{aligned} &39V - 6I_2 \times 1\Omega + 9I_3 \times 1\Omega = 0 \\ &-26V + 6I_2 \times 1\Omega + 4I_3 \times 1\Omega = 0 \end{aligned}\right\} \Rightarrow$$

$$13V + 13I_3 \times 1\Omega = 0$$

$\Rightarrow$ $I_3 = -1A$

$\Rightarrow$ $I_2 = 5A$

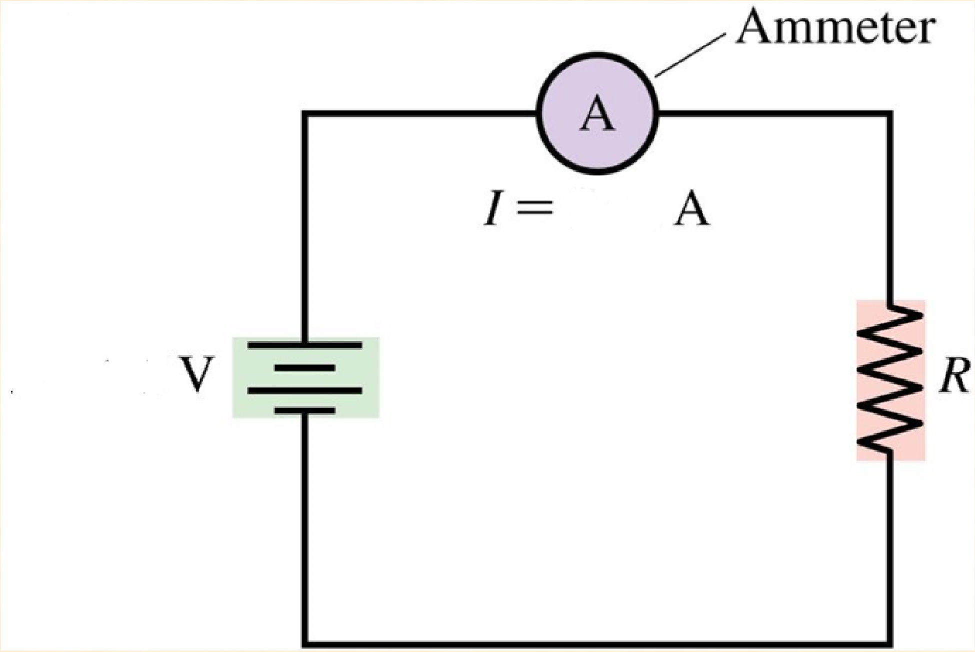
$\Rightarrow$ $I_1 = 6A$

$\Rightarrow$ **Total current through the network** $I_6 = I_1 + I_2 = 11A$

$\Rightarrow$ **Equivalent resistance** $R_{eq} = \dfrac{13V}{11A} = 1.18\Omega$

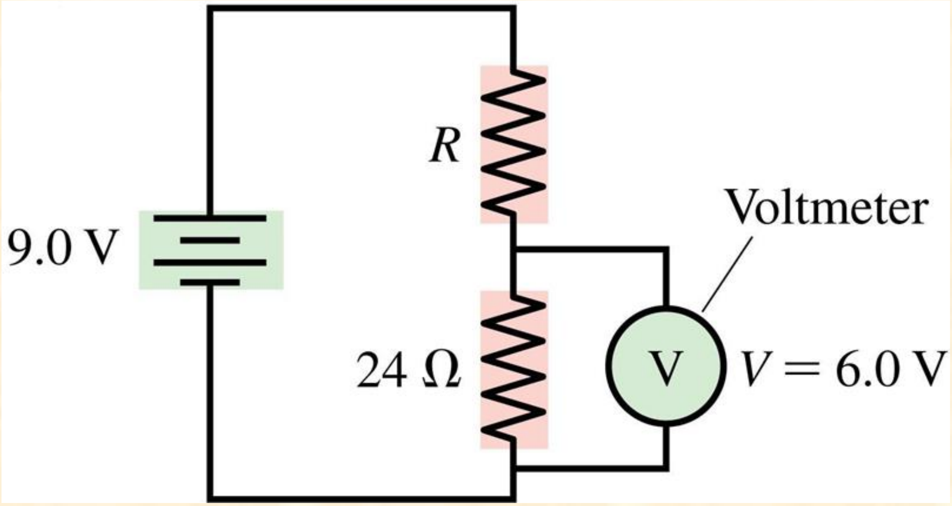
Measuring Voltage and Current

- An **ammeter** is a device that measures the current in a circuit element.
- Because charge flows *through* circuit elements, an ammeter must be placed *in series* with the circuit element whose current is to be measured.



Measuring Voltage and Current

- A **voltmeter** is used to measure the potential differences in a circuit.
- Because the potential difference is measured *across* a circuit element, a voltmeter is placed in *parallel* with the circuit element whose potential difference is to be measured.



Variable Resistor Construction

The variable resistor is variable resistances used to vary the amount of current or voltage in a circuit

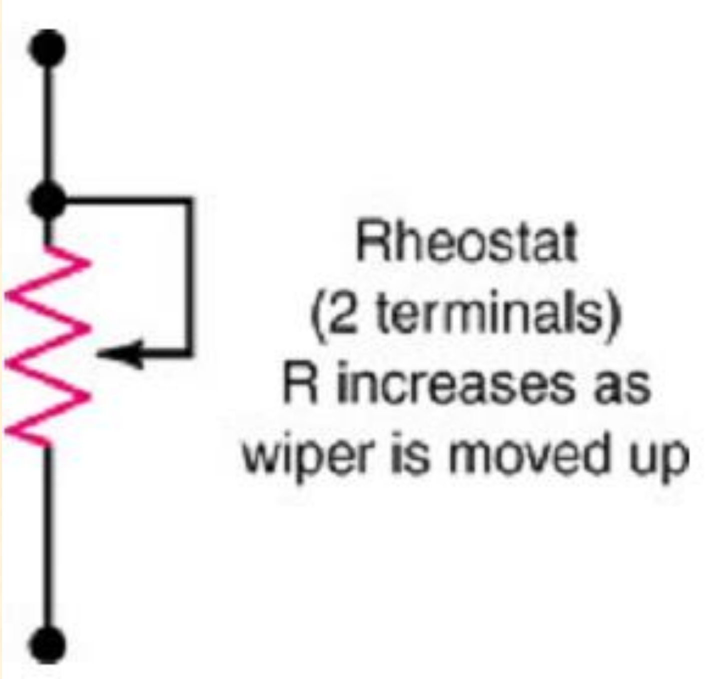
Type of variable Resistor

Rheostats

Two terminals.

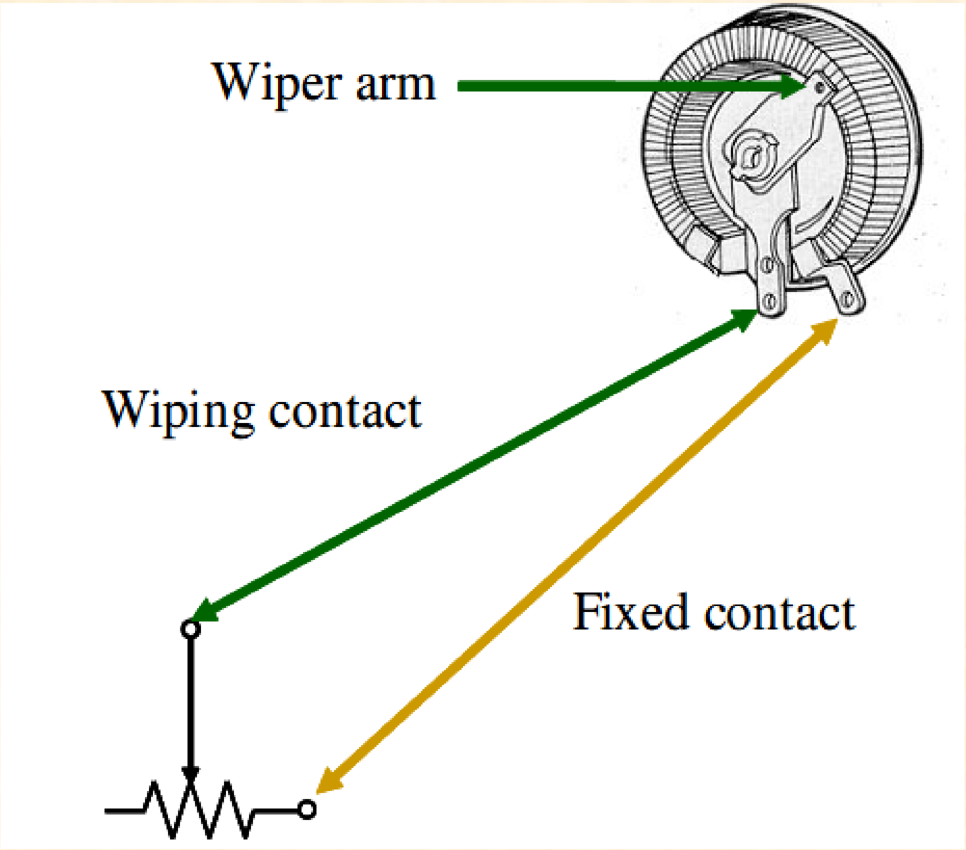
Connected in series with the load and the voltage source.

Varies the current.

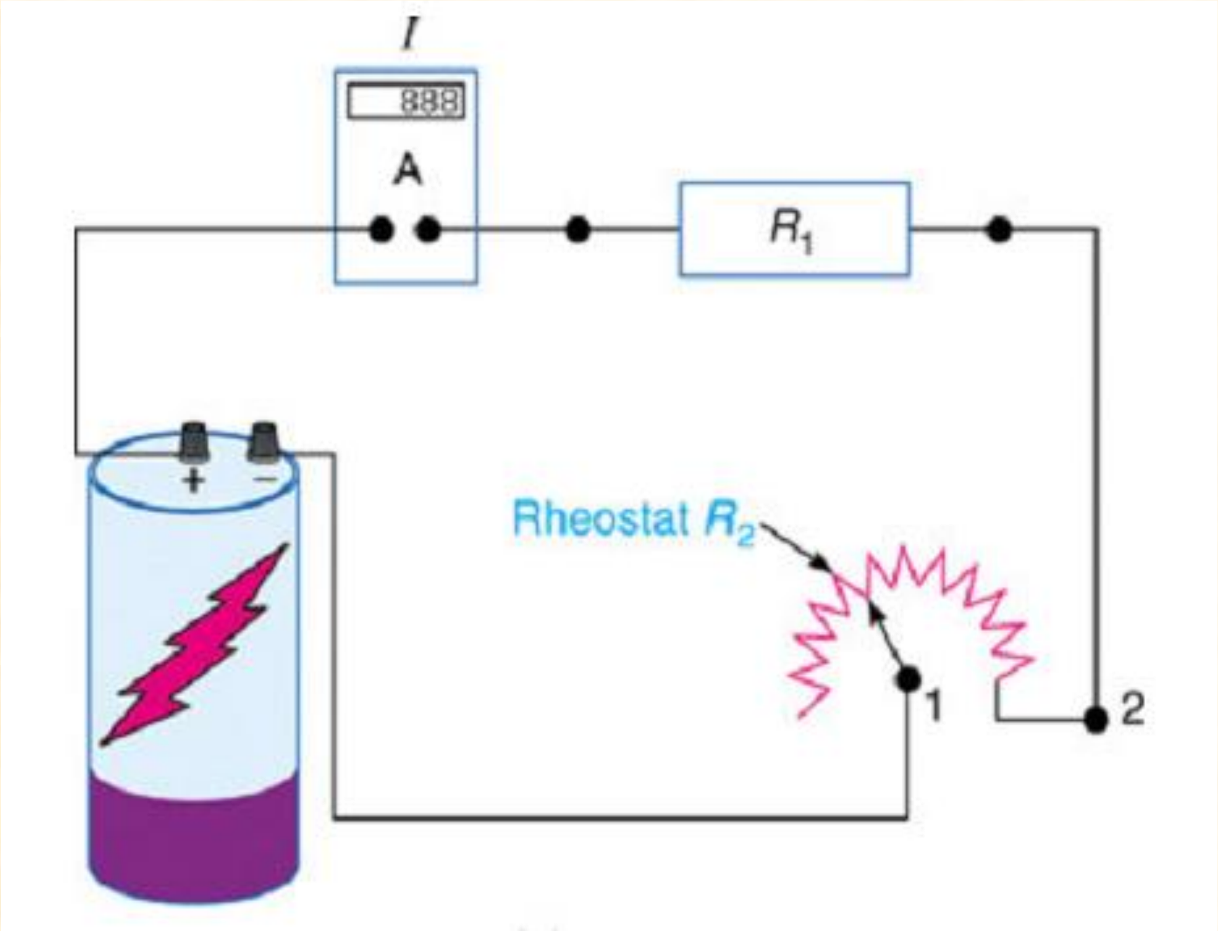


Variable Resistor Construction

This is a normal resistor with an additional arm contact that can move along the resistive material and tap off the desired resistance.



Variable Resistor Construction



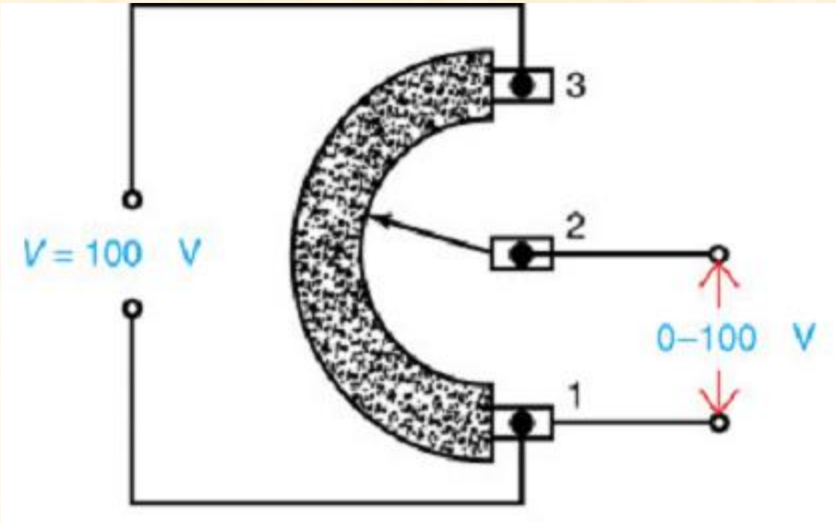
Variable Resistor Construction

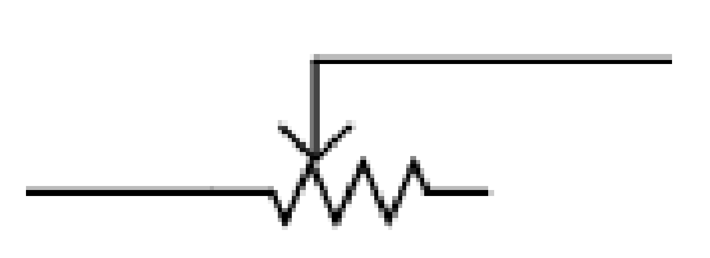
Potentiometers:

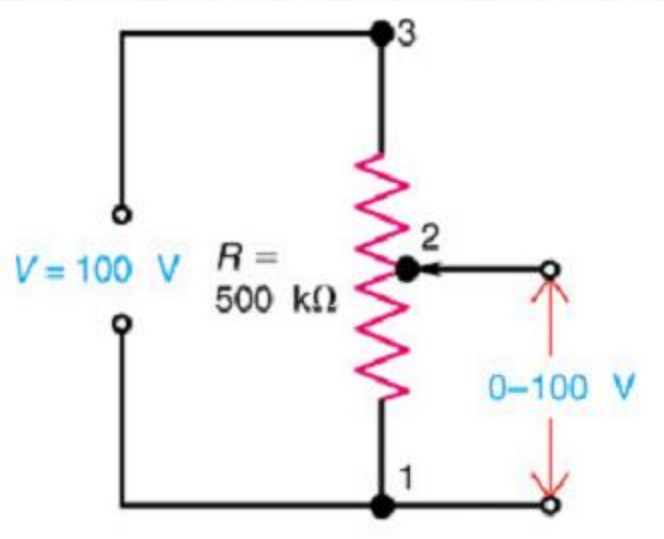
Three terminals.

Ends connected across the voltage source.

Third variable arm taps off part of the voltage.



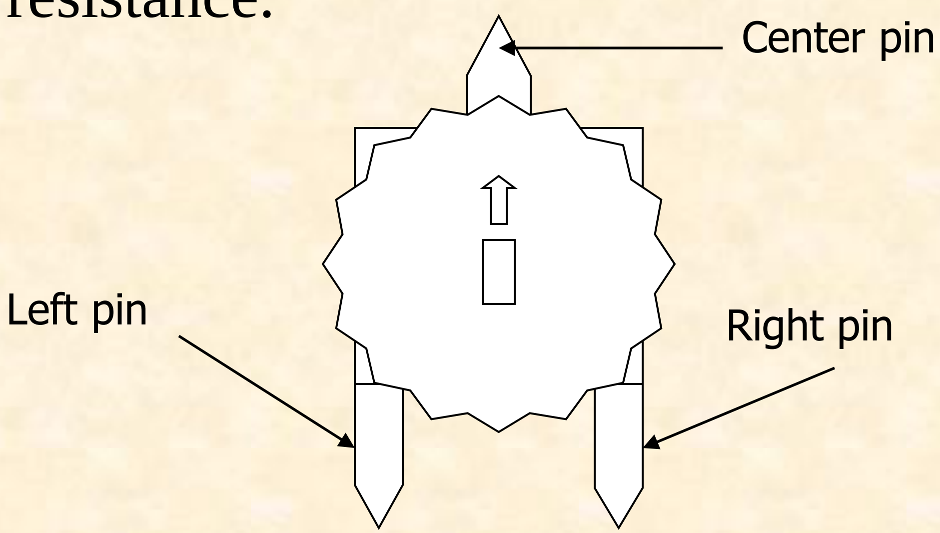




Variable Resistor Operation

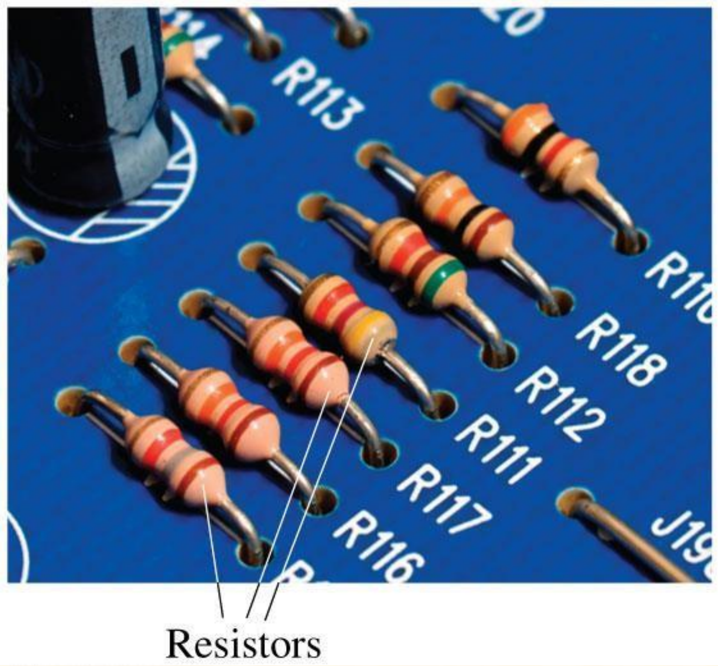
The dial on the variable resistor moves the arm contact and sets the resistance between the left and center pins. The remaining resistance of the part is between the center and right pins.

For example, when the dial is turned fully to the left, there is minimal resistance between the left and center pins (usually 0W) and maximum resistance between the center and right pins. The resistance between the left and right pins will always be the total resistance.



Resistor applications

Circuit Elements



Inside many electronic devices is a circuit board with many small cylinders. These cylinders are resistors that help control currents and voltages in the circuit. The colored bands on the resistors indicate their resistance values.

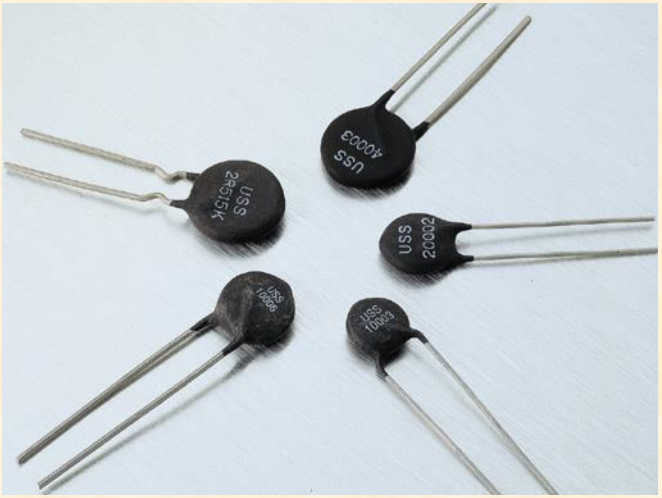
Resistor applications

Sensor Elements (Photoresistor)

A resistor whose resistance changes in response to changing circumstances can be used as sensor. The resistance of this night-light sensor changes when daylight strikes it. A circuit detects this change and turns off the light during the day.

Resistor applications

Heating Elements (Thermistor)



As charges move through a resistive wire, their electric energy is transformed into thermal energy, heating the wire. Wires in a toaster, a stove burner, or the rear window defroster of a car are practical examples of this electric heating.