Machine Learning

Linear Equation

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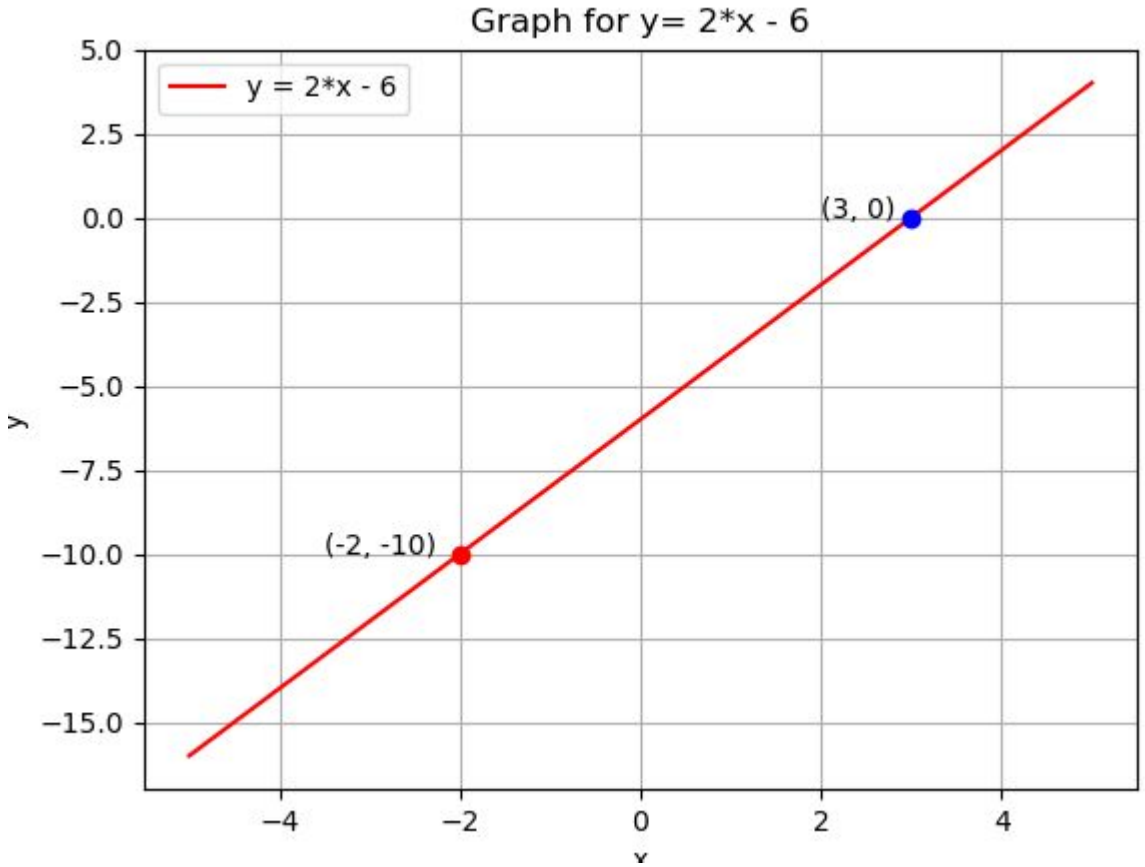
Linear Equation

- We are about to study **‘Linear Regression’**, which depends on the **‘Linear Equation’**
  - *Calculus is study of change while geometry is study of shapes*
- **Linear equations** can be represented in several ways
- We will use the **slope-intercept way (y =** mx + c)
  - The **m** variable is called the slope
  - **c** is the y-intercept (hence x = 0)
  - **x** is called an *independent* variable and **y** is a *dependent* variable

Linear Equation (optional)

- Assume we have a line passing through 2 points:
  - (3, 0) and (-2, -10)
- Compute the m and c ⇒ 2 and -6
  - **m** = (y2-y1) / (x2-x1) = 2
  - To compute **c** in the equation y = mx + c
    - c = y - mx
    - Substitute m with 2
    - Use one of the points, e.g. (3, 0)
    - Solve and get c = -6
- Equation is: y = 2*x - 6



What does the intercept c = 0 mean?

Code Snippet (optional)

```
6   def slope(p1, p2):
7       dx = p2[0] - p1[0]
8       dy = p2[1] - p1[1]
9       return dy / dx      # if dx is not zero!
10
11  def y_intercept(slope, p):
12      # y = mx + c
13      # c = y - mx
14      x, y = p
15      return y - slope * x
16
17  point1 = (3, 0)
18  point2 = (-2, -10)
19
20  m = slope(point1, point2)   # 2
21  c = y_intercept(m, point1)  # -6
22  #print(m, c) # 2.0 -6.0  ==> 2*x - 6
```

Online Calculator

Slope (m) $= \frac{\Delta Y}{\Delta X} = \frac{2}{1} = 2$

$\theta = \arctan(\frac{\Delta Y}{\Delta X}) + 180° = 243.43494882292°$

$\Delta X = -2 - 3 = -5$

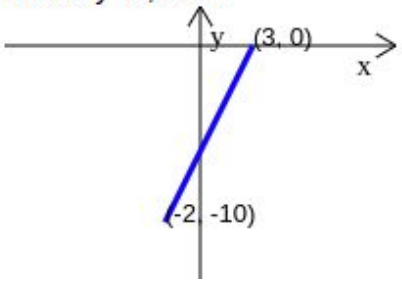
$\Delta Y = -10 - 0 = -10$

Distance (d) $= \sqrt{\Delta X^2 + \Delta Y^2} = \sqrt{125} = 11.180339887499$

Equation of the line:

$y = 2x - 6$

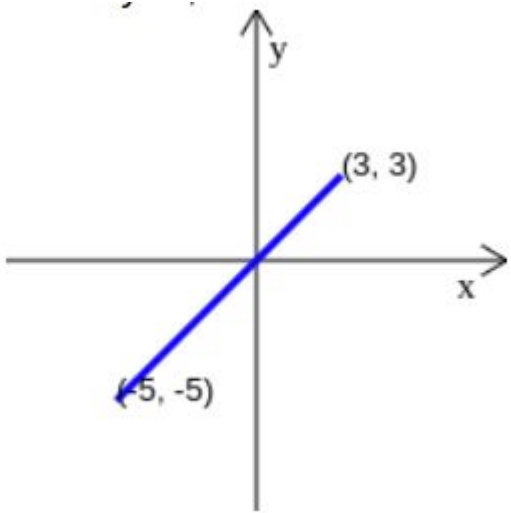
When x=0, y = -6
When y=0, x = 3

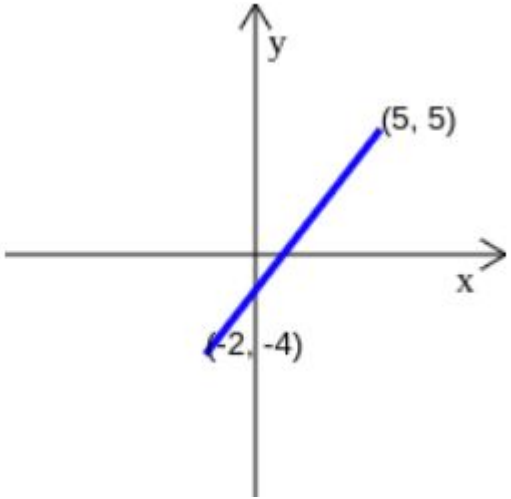


- Intuitively, the slope tells us with every **1 unit** in x direction how much the **y value rise?**
- Intuitively, for every **1 unit** in the x-direction, the slope tells us how much the **y-value** rises

Question! Which line has intercept = 0?

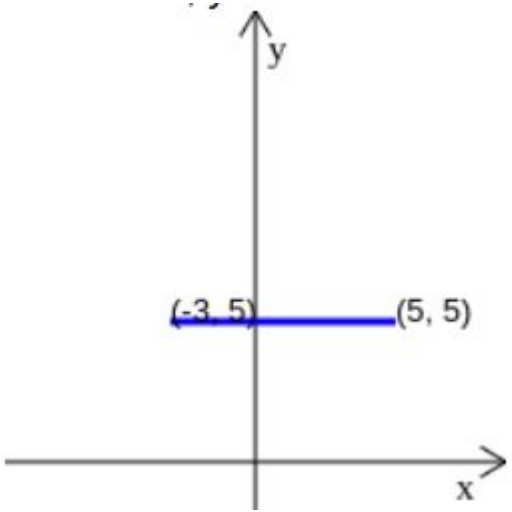
- When the intercept is 0, we mean the line must pass through the origin
- This is a big restriction for the line

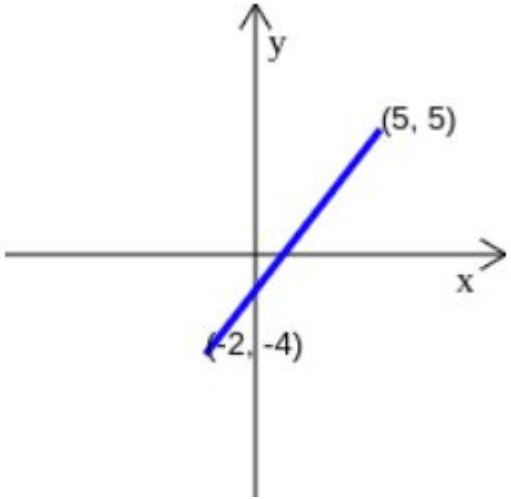




Question! Which line has slope = 0?

- The horizontal line is even more limited than intercept = 0



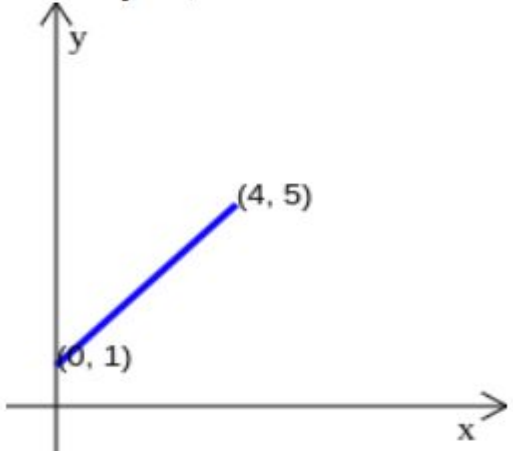


Slope Steepness

- The **slope** of the line tells us how **steep the line** is
- The bigger the slope, the **steeper** the line

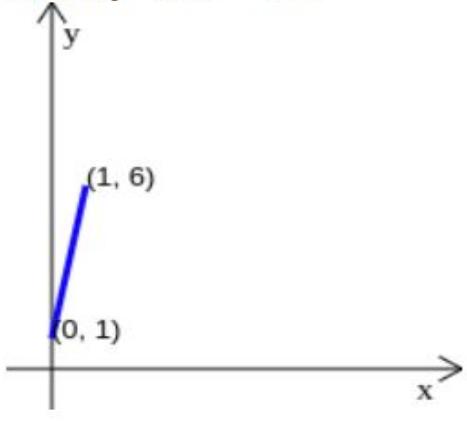
y = x + 1

When x=0, y = 1
When y=0, x = -1



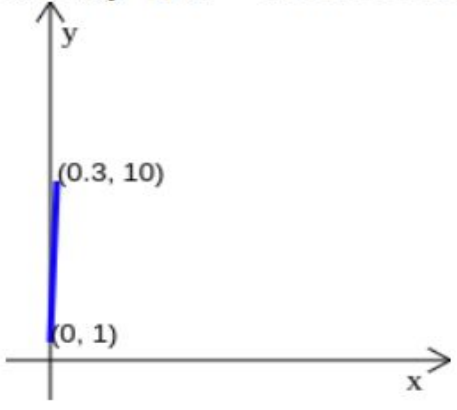
y = 5x + 1

When x=0, y = 1
When y=0, x = -0.2



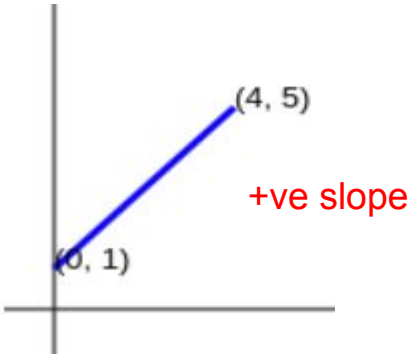
y = 30x + 1

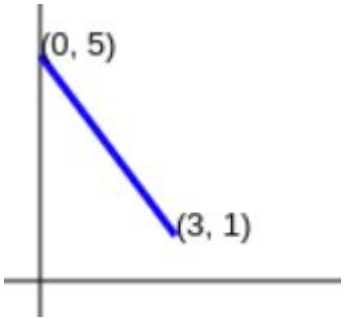
When x=0, y = 1
When y=0, x = -0.033333333

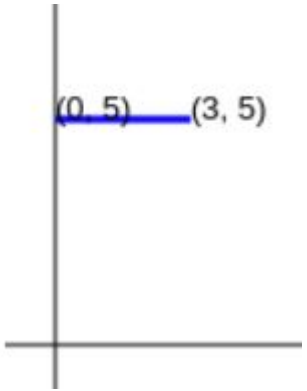


Interpreting the Slope

- A slope can be positive, negative or zero
- **Positive slope**: when x increases, y increases (and the opposite)
  - A upward line from left to right
- **Negative slope:** when x increases, y decreases (and the opposite)
  - A downward line from left to right
- **Zero Slope**: Just a horizontal line
  - **Vertical line** has an **undefined** slope [dy/dx and dx = 0]



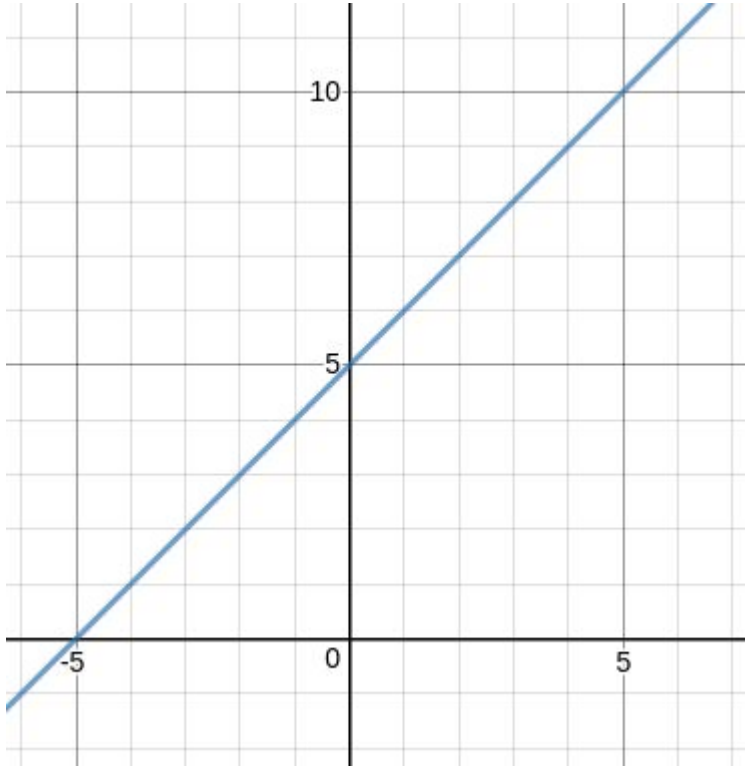




Question!

- What is the y-intercept?
- What is the slope?

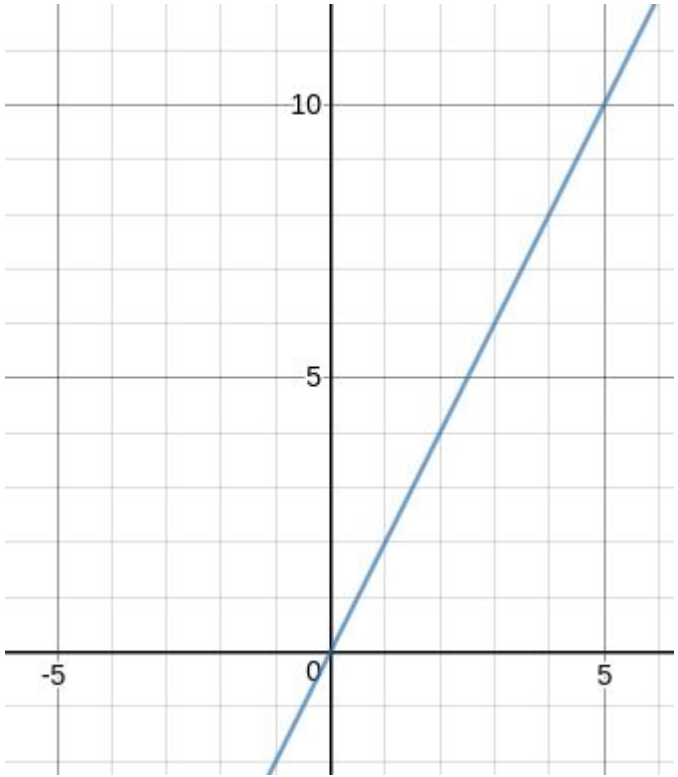
- Visually, we intercept the y-axis at 5
  - Intercept = 5
- Visually, there's 1 unit on the y-axis for each unit on the x-axis
  - 1 / 1 = 1
- Equation: y = x + 5



Question!

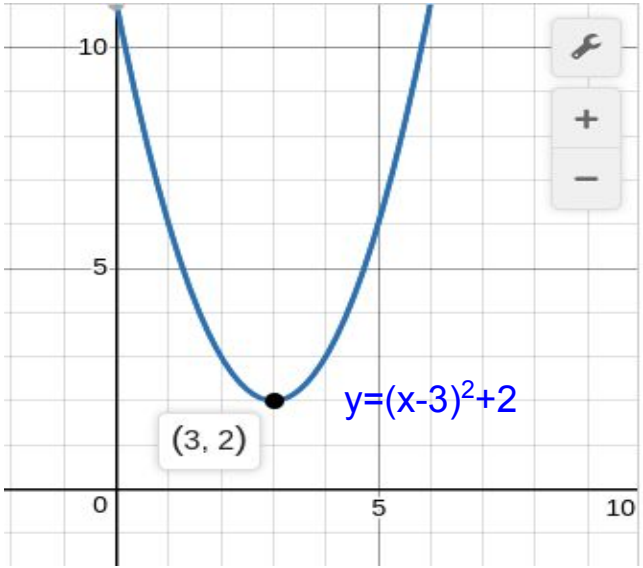
- What is the y-intercept?
- What is the slope?

- Visually, we intercept the y-axis at 0
  - Intercept = 0
- Visually, we see there's 10 units on the y-axis for 5 units on the x-axis
  - 10 / 5 = 2
- Equation: y = 2x



Question! Is positive, negative or zero slope?

- What is the slope sign of a tangent line at (3, 2)?
- What is the slope sign of a tangent line for points before (3, 2)?
- What is the slope sign of a tangent line for points after (3, 2)?



Slope vs Gradient

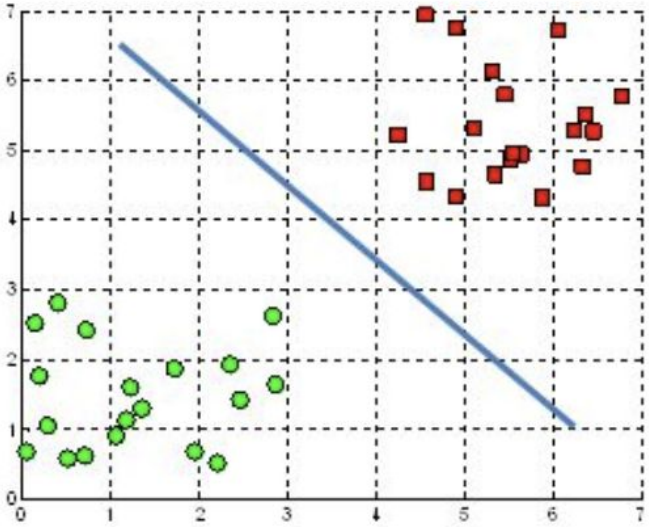
- Functions:
  - A **single** variable function takes a single value, e.g. sqrt(25) ⇒ 5
  - A **multivariable** variable function takes several values, e.g. sum(3, 5) = 8
- **Slope** is a scalar (e.g. 6.2) relevant to a single variable function
- **Gradient** is a vector of partial derivatives (slopes) for a multivariable function
  - gradient = [slope1, slope2, etc]
- In machine learning, we deal with multivariable functions, so we usually use the word gradient, not slope

Hyperplane

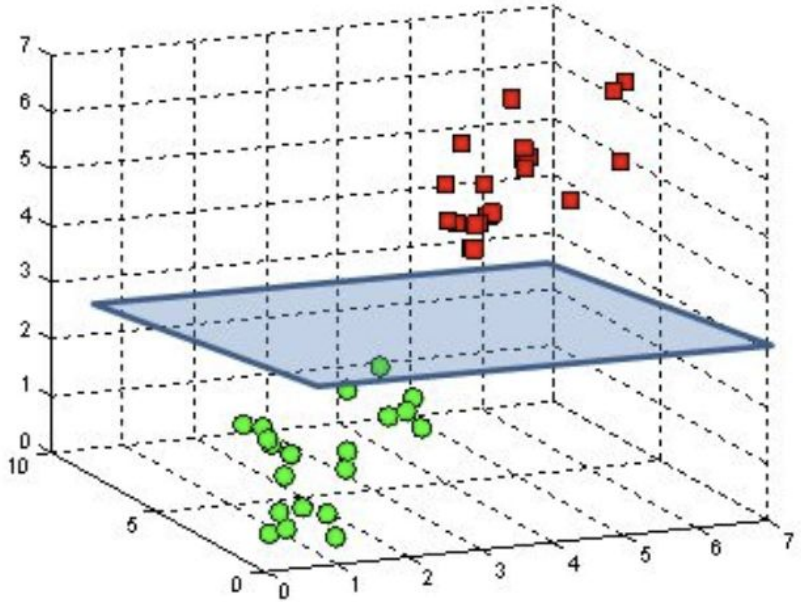
- The hyperplane **separates** space into **two separate regions**
- A **point** is a hyperplane in 1-dimensional space
- A **line** is a hyperplane in 2-dimensional space
- A **plane** is a hyperplane in 3-dimensional space
  - A line in 3-dimensional does not separate the space into two regions!
  - Line in 3D is similar to the point in 2D; separating nothing!

Visualization: Line vs Plane

A hyperplane in $\mathbb{R}^2$ is a line



A hyperplane in $\mathbb{R}^3$ is a plane



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"Acquire knowledge and impart it to the people."

"Seek knowledge from the Cradle to the Grave."