Machine Learning
Probabilistic Homework

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Teaching, Training and Coaching for more than a decade!

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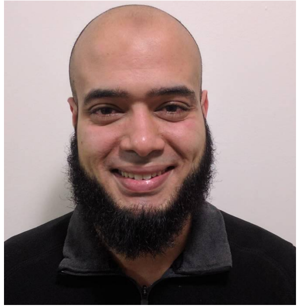

Problem #1: Log Gaussian

- It is common to assume a gaussian distribution for maximum likelihood
- Write the log of the gaussian with a single variable input
- Expand and simplify **into 3 terms**

$$P(x; \mu, \sigma) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right)$$

Background: scipy.optimize.minimize

- This function minimizes a scalar function of one or more variables
- The good part, it can find the minimum for a differentiable function without providing the derivative
  - The gradient is approximated numerically using finite differences

```
def objective_function(x):
    return (x - 3) ** 2

initial_guess = [0.5]
result = minimize(objective_function, initial_guess, method='BFGS')

print("Optimized parameters:", result.x)
print("Objective function value at minimum:", result.fun)
```

Problem #2: Gaussian with MLE

```
m, s, n = 160, 15, 1000
weights = np.random.normal(m, s, 1000)
```

- We generated 1000 numbers from a gaussian distribution
- Let’s try to find the original parameters (mean and sigma) using ML
- Define def neg_log_likelihood(params):
  - Input is a list with [mean, sigma]
  - The function evaluate the negative log likelihood on the global **weights** array
- Use scipy.optimize.minimize to find the optimal parameters for you

Problem #3: Analytical Solution

- Try to just print np.mean(weights) and np.std(weights) a
  - You will notice this **analytical formula** has very close solution as iterative one
  - That is why in GNB we just used mean/std for vars
- Prove that, given a dataset of a **univariate** X coming from a Gaussian distribution, the MLE is just the **sample** mean and variance of X
- Steps
  - Write down the log likelihood for solution to problem #7
  - Compute the partial derivative: Once for mean and once for the variance

"Acquire knowledge and impart it to the people."

"Seek knowledge from the Cradle to the Grave."