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Problem I:

[Petrol Station Multi-Channel Queue]

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TABLE OF CONTENT

PARTS		PAGE
0	• Cover Sheet	1
	• Table of content	2
1	• Problem Formulation & Objectives	3-4
2	• System Component	5
	• System Analysis:	
	▪ Car category distribution	6
	▪ Inter – arrival time distribution	6
	▪ Category A &B service time distribution	6
	▪ Category C service time distribution	7
	▪ Simulation Table	8
3	• Experimental Design Parameters:	9
	▪ Car Category probabilities Scenarios & Justification	9
	▪ Time between arrivals probabilities Scenarios & justifications	10
	▪ Probabilities of service time for category A&B Scenarios & Justifications	11
	▪ Probabilities of service time for category C Scenarios & justifications	12
4	• Results Analysis:	13
	▪ Average service time for car categories	13
	▪ Average waiting time in the queues for each pump and for all cars	13
	▪ Average maximum queue length for each pump	14
	▪ Probability that car waits for each pump	14
	▪ Portion of idle for each pump	15
	▪ Theoretical average vs experimental average	16
	▪ investigating the addition of one extra pump	16
	▪ Conclusion	16-17

- # Problem formulation:

we need to study and simulate the queue of the three pumps (90 octane ,95 octane, Gas) considering some factors:

1. **Car Categories:** (Table 1)

A. fuel type:

1. Category A: Use 95 Octane petrol only.
2. Category B: Use 90 or 95 Octane petrol.
3. Category C: Use Gas or 90 Octane petrol.

Probabilities of each car category:

1. Category A: 20%
2. Category B: 35%
3. Category C: 45%

B. Decision making:

1. *Category B:* Choose the 95 Octane pump with probability of 60% if the number of cars at 90 Octane pump is more than 3 cars.

2. Category C: Choose the 90 Octane pump with probability of 40% if the number of cars at Gas pump is more than 4 cars.

2. Arrival time:

Cars arrive the petrol station depending on statistical inter arrival time distribution table (Table 2).

3. Service time:

Service time depends on car category and follow service time distribution table

I. Category A & B: follow service time distribution in (Table 3).

II. Category C: follow service time distribution in (Table 4).

4. Cost:

Drivers wants the cheaper fuel option (Gas is cheaper than 90 Octane and both are cheaper than 95 Octane) but there is a queue condition:

I. Category B prefer 90 Octane unless there are more than 3 cars.
II. Category C prefer Gas unless there are more than 4 cars.

Objectives:

We want to create a simulation program to simulate the queues at the Petrol Station to optimize operations and analyse some factors.

1. Calculate system performance statistics:

1. Calculate the average service time for cars in each category (A,B and C).
2. Calculate the average waiting time in the queues for each pump (90, 95 and Gas) and for all cars.
3. Determent the length of the queue for each pump.
4. Calculate the probability that a car waits for each pump.
5. Calculate the idle time portion for each pump.

2. Validate Theoretical vs Experimental results:

1. check if the theoretical average service time matches the experimental average service time for each car category.
2. check if the theoretical average inters arrival matches the experimental one.
3. Analyse the impact of adding extra pump:
   Determine which type of pump (95, 90, Gas) will result in the most decrease in the average waiting time for all cars.

System Components:

System	Entity	Attribute	Activity	State	Events
Petrol Station	• Cars • Pumps	➢ For cars: • Category • Fuel type • Arrival Time • Service time • Waiting time • Departure time ➢ For pump: • Queue length • Category • Idle time	• Car arrival • Pump selection • Car departure	• Number of queues in each Pump • Pump Status (Idle or Busy) • Number of each car category in each pump	• Car arrival • Service Start • Service end • Pump selection • Car Departure

System Analysis:

- Car category distribution:

Car category	Probability	Cumulative Probability	Random Digit Assignment
A	0.2	0.2	0.01 – 0.2
B	0.35	0.55	0.21 – 0.55
C	0.45	1	0.56 – 1

(Table 1)

- Inter – arrival time distribution:

Time between Arrivals	Probability	Cumulative Probability	Random Digit Assignments
0	0.17	0.17	0.01 - 0.17
1	0.23	0.4	0.18 - 0.4
2	0.25	0.65	0.41 – 0.65
3	0.35	1	0.66 – 1

(Table 2)

- Category A &B service time distribution:

Category A & B Service time	Probability	Cumulative Probability	Random Digit Assignment
1	0.2	0.2	0.01 - 0.2
2	0.3	0.5	0.21 - 0.5
3	0.5	1	0.51 – 1

(Table 3)

- Category C service time distribution:

Category C Service time	Probability	Cumulative Probability	Random Digit Assignment
3	0.2	0.2	0.01 - 0.2
5	0.5	0.7	0.21 - 0.7
7	0.3	1	0.71 – 1

(Table 4)

Simulation table (20 Cars):

Car Number	Rand. No. for car category	Car Category	Rand. no. for Time Between Arrivals	Time Between Arrivals	Arrival Time	Service Start 95 Octane	Service Start 90 Octane	Service Start Gas	Rand. No. for service time	Service Time	Leave Time	Time car waits in queue	Time car waits in petrol station	Fuel Type	Idle Time 95	Idle Time 90	Idle Time Gas
1	0.83	C	-	-	0	-	-	0	0.32	5	5	0	5	Gas	-	-	0
2	0.85	C	0.03	0	0	-	-	5	0.73	7	12	5	12	Gas	-	-	0
3	0.6	C	0.15	0	0	-	-	12	0.9	3	15	12	15	Gas	-	-	0
4	0.97	C	0.2	1	1	-	-	15	0.47	5	20	15	20	Gas	-	-	0
5	0.43	B	0.84	3	4	-	4	-	0.48	3	7	0	3	90 Octane	-	4	-
6	0.7	C	0.95	3	7	-	-	20	0.85	7	27	13	20	Gas	-	-	0
7	0.68	C	0.34	1	8	-	-	27	0.6	5	32	19	24	Gas	-	-	0
8	0.57	C	0.3	1	9	-	-	32	0.71	7	39	23	30	Gas	-	-	0
9	0.23	B	0.16	0	9	-	9	-	0.96	3	12	0	3	90 Octane	-	2	-
10	0.91	C	0.72	3	12	-	-	39	0.39	5	44	27	32	Gas	-	-	0
11	0.93	C	0.51	2	14	-	14	-	0.1	3	17	0	3	90 Octane	-	2	-
12	0.33	B	0.54	2	16	-	17	-	0.48	2	19	1	3	90 Octane	-	0	-
13	0.84	C	0.18	1	17	-	19	-	0.3	3	22	2	5	90 Octane	-	0	-
14	0.5	B	0.09	0	17	-	22	-	0.67	3	25	5	8	90 Octane	-	0	-
15	0.03	A	0.7	3	20	20	-	-	0.49	2	22	0	2	95 Octane	20	-	-
16	0.81	C	0.67	3	23	-	-	44	0.15	3	47	21	24	Gas	-	-	0
17	0.13	A	0.6	2	25	25	-	-	0.3	2	27	0	2	95 Octane	3	-	-
18	0.67	C	0.8	0	25	-	-	47	0.7	3	50	22	25	Gas	-	-	0
19	0.43	B	0.74	3	28	-	28	-	0.38	2	30	0	2	90 Octane	-	3	-
20	0.31	B	0.98	3	31	-	31	-	0.43	2	33	0	2	90 Octane	-	1	-

Experimental Design Parameters:

- **Car category probabilities:**

  For 200 simulated cars and 1000 runs, the results of changing car category probabilities are as follows:

Scenario	P(A)	P(B)	P(C)	Avg. waiting for all cars	Avg. waiting time on 95	Avg. waiting time on 90	Avg. waiting time on gas
Theoretical	0.2	0.35	0.45	10.4736	0.325844	2.7088	24.8885
Scenario 1.1	0.3	0.3	0.4	6.89835	0.530495	1.48765	17.6811
Scenario 1.2	0.4	0.25	0.35	4.57454	0.83884	0.797824	12.1694
Scenario 1.3	0.5	0.2	0.3	3.19516	1.40394	0.41391	7.85887
Scenario 2.1	0.15	0.5	0.35	5.08743	0.231036	1.93517	12.287
Scenario 2.2	0.1	0.6	0.3	3.69962	0.191604	2.1996	7.95587
Scenario 2.3	0.05	0.7	0.25	3.19125	0.228599	2.88965	4.79274
Scenario 3.1	0.15	0.25	0.6	27.8467	0.231283	4.59878	57.8921
Scenario 3.2	0.1	0.2	0.7	48.071	0.160813	6.17709	89.2834
Scenario 3.3	0.05	0.15	0.8	73.0834	0.0828115	7.69363	121.905
Scenario 4	0.33	0.33	0.34	4.24545	0.610454	0.967445	11.3818

  - Scenario set 1: increase category A.
  - Scenario set 2: increase category B.
  - Scenario set 3: increase category C.
  - Scenario 4: Balance probabilities.

The best scenario is 2.3 (P(A)=0.05, P(B)=0.7, P(C)=0.25).

- **Justification:**

1.Lowest overall waiting time:

In scenario 2.3 the average waiting time for all cars is 3.19125 which is the lowest.

2.pump usage:

95 Octane pump:

Average Waiting time is 0.228599 which is good.

90 Octane pump:

Average Waiting time is 2.88965 which is manageable.

Gas pump:

Average Waiting time is 4.79274 which is the lowest in all scenarios.

- **Time between arrivals probabilities:**

For 200 simulated cars and 1000 runs, the results of changing time between arrivals probabilities are as follows:

Scenario	P(0)	P(1)	P(2)	P(3)	Avg. waiting for all cars	Avg. waiting time on 95	Avg. waiting time on 90	Avg. waiting time on gas
Theoretical	0.17	0.23	0.25	0.35	10.2985	0.319917	2.73362	24.5019
Scenario 1	0.3	0.3	0.2	0.2	20.5129	1.28365	9.13555	48.8646
Scenario 2	0.1	0.15	0.25	0.5	6.27719	0.159542	0.866147	14.3644
Scenario 3	0.15	0.35	0.35	0.15	15.3721	0.545057	5.43781	37.1732
Scenario 4	0.25	0.25	0.25	0.25	14.9454	0.66371	5.51718	35.993

- Scenario 1: Short interval.
- Scenario 2: long interval.
- Scenario 3: mid-range.
- Scenario 4: Balance probabilities.

The best scenario is 2 (P(0)=0.1, P(1)=0.15, P(2)=0.25, P(3)=0.5).

Justification:

1.Lowest overall waiting time:

In scenario 2 the average waiting time for all cars is 6.27719 which is the lowest.

2.pump usage:

95 Octane pump:

Average Waiting time is 0.159542 which is good.

90 Octane pump:

Average Waiting time is 0.866147 which is good.

Gas pump:

Average Waiting time is 6.27719 which is manageable and the lowest in all scenarios.

Probabilities of service time for category A&B:

For 200 simulated cars and 1000 runs, the results of changing service time probabilities are as follows:

Scenario	P(1)	P(2)	P(3)	Avg. waiting for all cars	Avg. waiting time on 95	Avg. waiting time on 90	Avg. waiting time on gas
Theoretical	0.2	0.3	0.5	10.3673	0.321764	2.62409	24.7418
Scenario 1	0.6	0.3	0.1	9.79837	0.125092	1.60302	24.4997
Scenario 2	0.1	0.2	0.7	10.5235	0.429434	3.2292	24.4887
Scenario 3	0.25	0.5	0.25	9.98437	0.225966	2.16528	24.2904
Scenario 4	0.33	0.33	0.34	10.0441	0.244134	2.30243	24.3512

- Scenario set 1: High short service time.
- Scenario set 2: High long service time.
- Scenario set 3: mid-service time.
- Scenario 4: Balance probabilities.

The best scenario is 1 (P(1)=0.6, P(2)=0.3, P(3)=0.1).

- **Justification:**

1.Lowest overall waiting time:

In scenario 1 the average waiting time for all cars is 9.79837 which is the lowest.

2.pump usage:

95 Octane pump:

Average Waiting time is 0.125092 which is the lowest.

90 Octane pump:

Average Waiting time is 1.60302 which is the lowest.

Gas pump:

Average Waiting time is 24.4997 which is the lowest.

- **Probabilities of service time for category C:**

  For 200 simulated cars and 1000 runs, the results of changing service time probabilities are as follows:

Scenario	P(3)	P(5)	P(7)	Avg. waiting for all cars	Avg. waiting time on 95	Avg. waiting time on 90	Avg. waiting time on gas
Theoretical	0.2	0.5	0.3	10.0288	0.320708	2.66118	23.8316
Scenario 1	0.7	0.2	0.1	3.6479	0.289465	0.89636	7.55019
Scenario 2	0.1	0.2	0.7	17.4905	0.372236	4.90765	43.9998
Scenario 3	0.25	0.5	0.25	9.00224	0.309445	2.33582	21.1031
Scenario 4	0.33	0.33	0.34	9.10904	0.315449	2.37893	21.4076

- Scenario set 1: High short service time.
- Scenario set 2: High long service time.
- Scenario set 3: mid-service time.
- Scenario 4: Balance probabilities.

The best scenario is 1 (P(1)=0.7, P(2)=0.2, P(3)=0.1).

- **Justification:**

1.Lowest overall waiting time:

In scenario 1 the average waiting time for all cars is 3.6479 which is the lowest.

2.pump usage:

95 Octane pump:

Average Waiting time is 0.289465 which is the lowest.

90 Octane pump:

Average Waiting time is 0.89636 which is the lowest.

Gas pump:

Average Waiting time is 7.55019 which is the lowest.

Results Analysis:

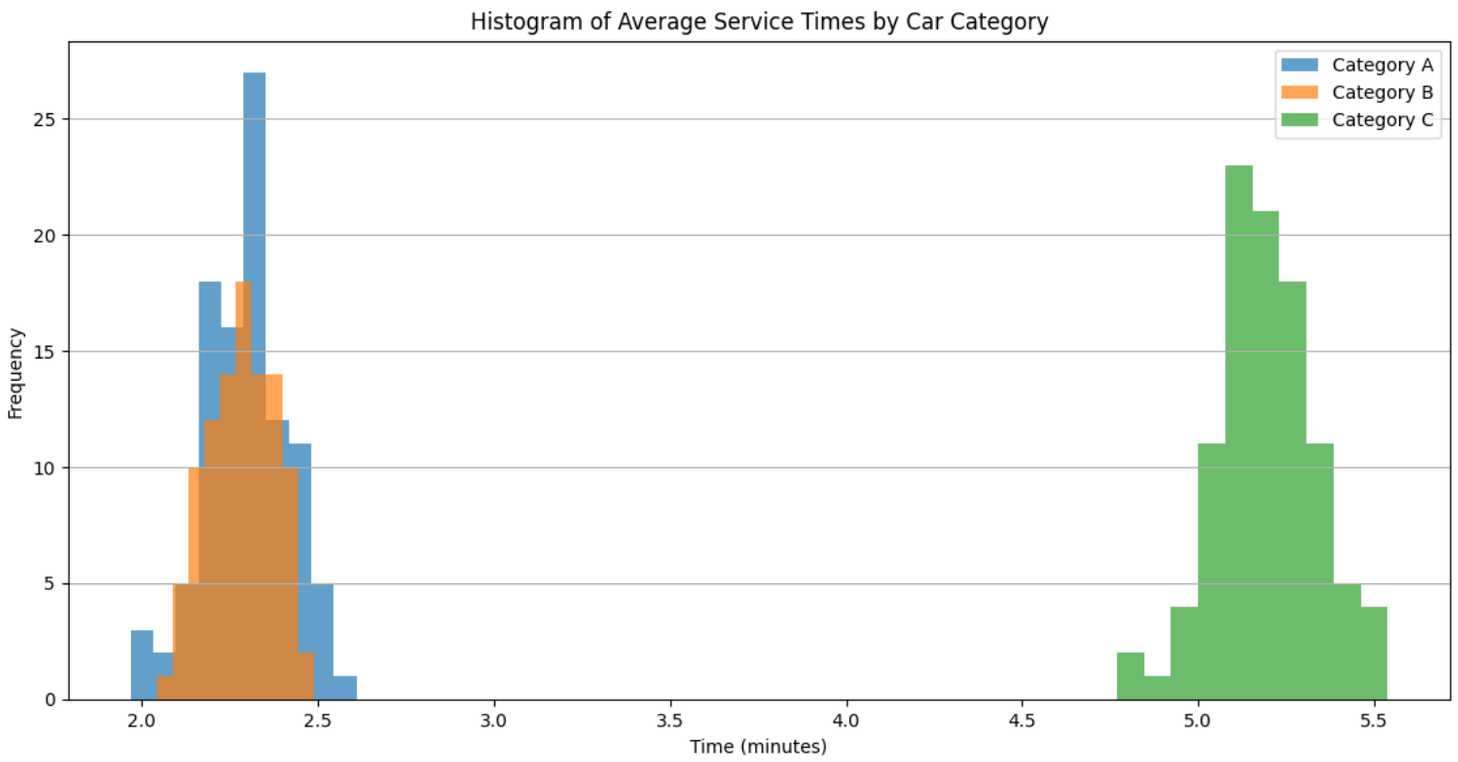
- **Average service time for car categories:**

  Numerical Results:

  Category A: 2.30039 mins/car

  Category B: 2.28182 mins/car

  Category C: 5.19361 mins/car



- **Average waiting time in the queues for each pump and for all cars:**

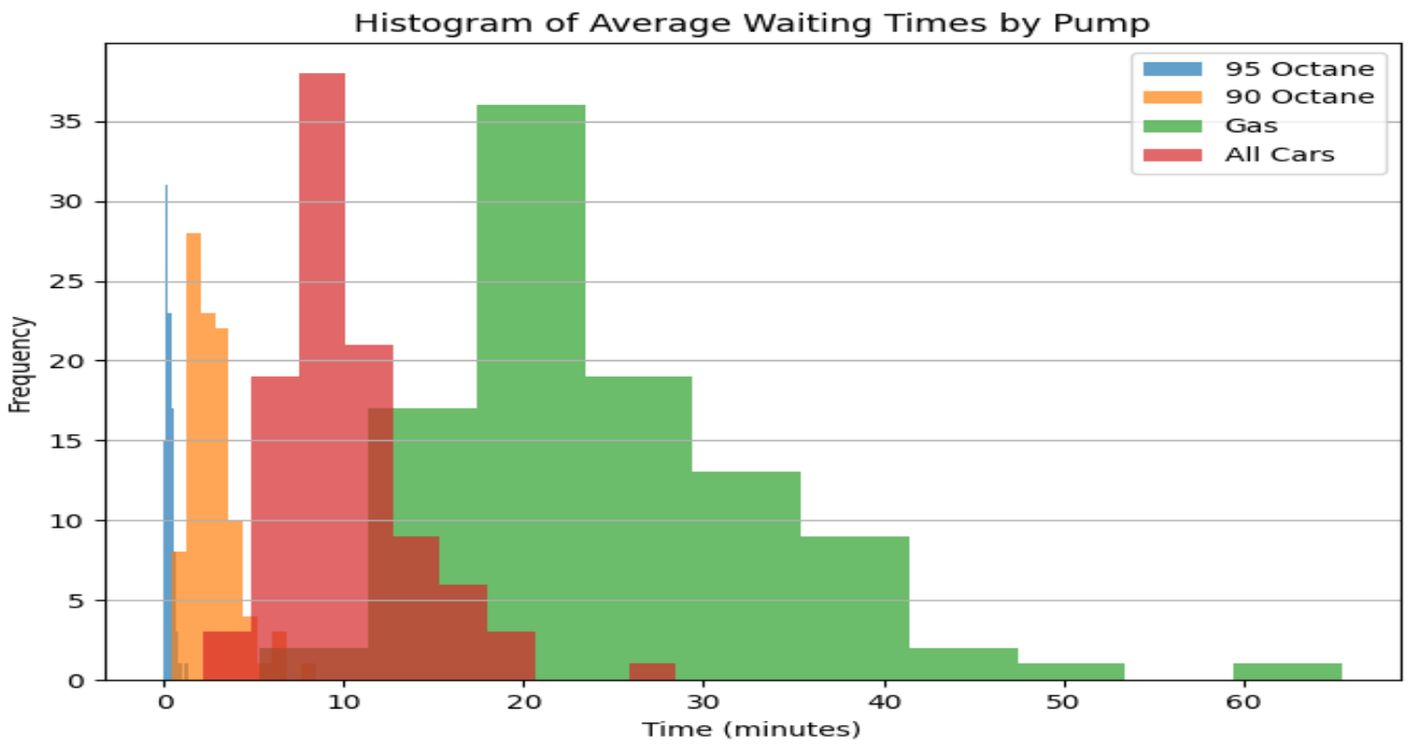
  Numerical Results:

  95 Octane: 0.329483 mins/car

  90 Octane: 2.69924 mins/car

  Gas: 24.4439 mins/car

  All Cars: 10.1969 mins/car



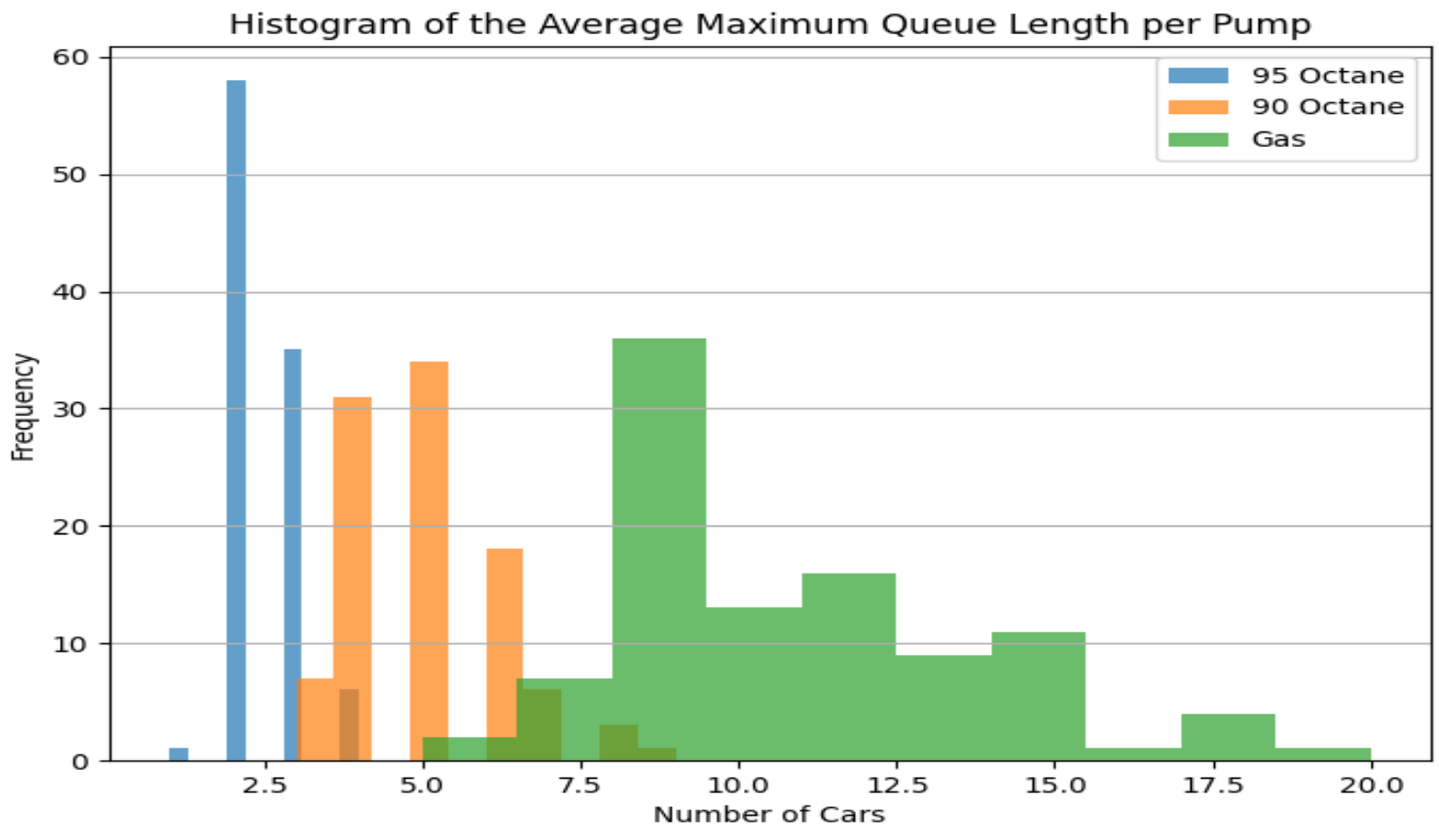
- **Average maximum queue length for each pump:**

  Numerical Results:

  95 Octane: 2.46 car

  90 Octane: 4.98 car

  Gas: 10.57 car



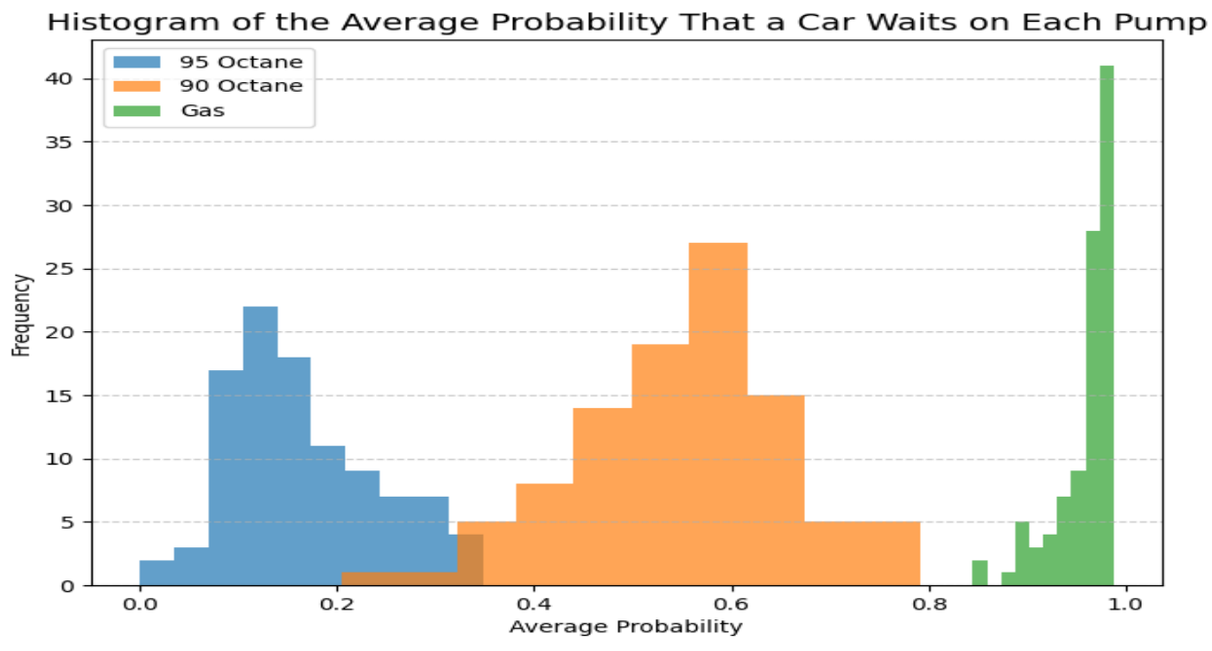
- **Probability that car waits for each pump:**

  Numerical Results:

  95 Octane: 0.164257

  90 Octane: 0.547213

  Gas: 0.960956



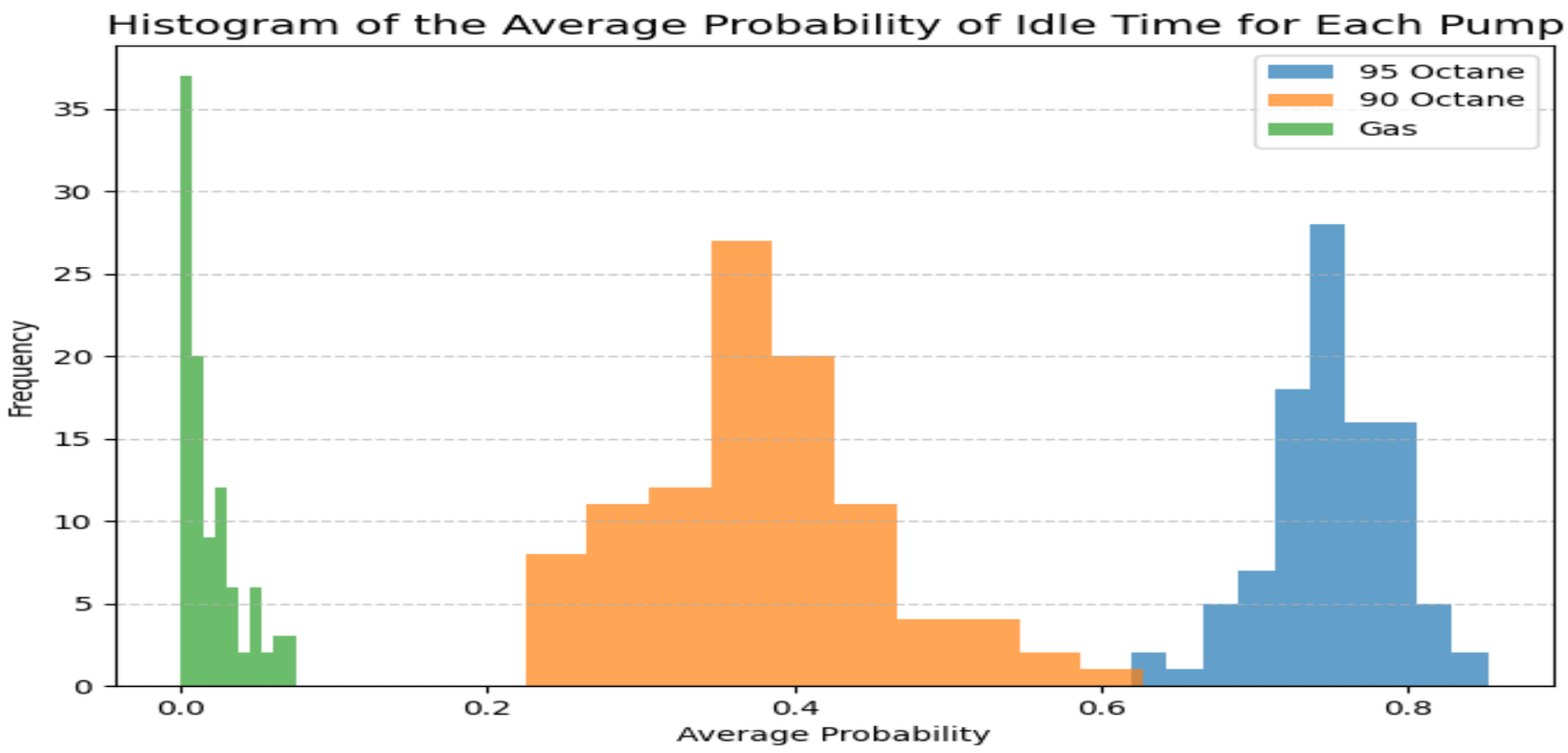
- **Portion of idle for each pump:**

  Numerical Results:

  95 Octane: 0.746416

  90 Octane: 0.375721

  Gas: 0.0179387



- **Theoretical average vs experimental average:**

Service time:

Category A & B:

Theoretical average service time = 2.3 min

Experimental average service time for category A = 2.30039 min/car

Experimental average service time for category B = 2.28182 min/car

Category C:

Theoretical average service time = 5.2 min

Experimental average service time for category C = 5.19361 min/car

Inter-arrival time:

Theoretical average inter arrival time = 1.78 min/car

Experimental average inter arrival time = 1.76255 min/car

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investigating the addition of one extra pump:

Results of Adding Extra Pumps :

1. Adding one extra Gas pump:

Average Waiting Time for all cars: 0.93865 mins/car

2. Adding one extra 90 Octane pump:

Average Waiting Time for all cars: 9.1344 mins/car

3. Adding one extra 95 Octane pump:

Average Waiting Time for all cars: 10.1284 mins/car

Conclusion:

Looking at service and queue metrics provides valuable information about how the system is running and how effective it has been.

1. Service Times:

The average service times for Categories A, B, and C from the experiment are very similar to their theoretical values, showing that the modelling is accurate and consistent.

2. Waiting Time:

The waiting time of cars differs considering fuel types, with Gas having the highest average waiting time at 24.4439 min/car and 95 Octane having the lowest at 0.329483 min/car.

The average waiting time for all cars is high (10.1969 min/car), which indicate problems at some pumps.

3. Queue Lengths:

The average maximum queue length at the Gas pump is 10.57 cars, much greater than that of the 95 Octane with 2.46 cars and 90 Octane with 4.98 cars, meaning higher demand and congestion at the Gas pump.

4. Probability of Waiting:

The probability of a car waiting varies across each pump: Gas has the highest chance at 0.960956, and 95 Octane has the lowest chance at 0.164257.

The most cars are served quickly at the 95 Octane pump, but it's pretty much a wait at the Gas pump.

5. Portion of idle for each pump:

The amount of idle time is very different. The Gas pump has very little idle time (0.0179387) when compared to 95 Octane (0.746416) and 90 Octane (0.375721).

This is underutilization at the 95 Octane pump and almost full utilization at the Gas pump.

6. Theoretical vs. Experimental Averages:

Theoretical and experimental averages for service times and inter-arrival times are very close to each other; the differences are very small. This shows that the theoretical model used for predictions is reliable.

Summary Recommendations:

Adding one extra Gas pump will result in the greatest decrease in the average waiting time for all cars compared to adding any other type of pump.