The algorithm is very similar to the basic Gale-Shapley algorithm from the text. At any point in time, a student is either "committed" to a hospital or "free." A hospital either has available positions, or it is "full." The algorithm is the following:

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While some hospital h_i has available positions
  h_i offers a position to the next student s_j on its preference list
  if s_j is free then
    s_j accepts the offer
  else (s_j is already committed to a hospital h_k)
    if s_j prefers h_k to h_i then
      s_j remains committed to h_k
    else s_j becomes committed to h_i
         the number of available positions at h_k increases by one.
         the number of available positions at h_i decreases by one.
```

The algorithm terminates in $O(mn)$ steps because each hospital offers a positions to a student at most once, and in each iteration, some hospital offers a position to some student.

Suppose there are $p_i > 0$ positions available at hospital h_i. The algorithm terminates with an assignment in which all available positions are filled, because any hospital that did not fill all its positions must have offered one to every student; but then, all these students would be committed to some hospital, which contradicts our assumption that $\sum_{i=1}^{m} p_i < n$.

Finally, we want to argue that the assignment is stable. For the first kind of instability, suppose there are students s and s', and a hospital h as above. If h prefers s' to s, then h would have offered a position to s' before it offered one to s; from then on, s' would have a position at *some* hospital, and hence would not be free at the end — a contradiction.

For the second kind of instability, suppose that (h_i, s_j) is a pair that causes instability. Then h_i must have offered a position to s_j, for otherwise it has p_i residents all of whom it prefers to s_j. Moreover, s_j must have rejected h_i in favor of some h_k which he/she preferred; and s_j must therefore be committed to some h_ℓ (possibly different from h_k) which he/she also prefers to h_i.

[1] ex304.339.892