

(a) The answer is Yes. A simple way to think about it is to break the ties in some fashion and then run the stable matching algorithm on the resulting preference lists. We can for example break the ties lexicographically — that is if a man m is indifferent between two women w_i and w_j then w_i appears on m 's preference list before w_j if $i < j$ and if $j < i$ w_j appears before w_i . Similarly if w is indifferent between two men m_i and m_j then m_i appears on w 's preference list before m_j if $i < j$ and if $j < i$ m_j appears before m_i .

Now that we have concrete preference lists, we run the stable matching algorithm. We claim that the matching produced would have no strong instability. But this latter claim is true because any strong instability would be an instability for the match produced by the algorithm, yet we know that the algorithm produced a stable matching — a matching with no instabilities.

(b) The answer is No. The following is a simple counterexample. Let $n = 2$ and m_1, m_2 be the two men, and w_1, w_2 the two women. Let m_1 be indifferent between w_1 and w_2 and let both of the women prefer m_1 to m_2 . The choices of m_2 are insignificant. There is no matching without weak stability in this example, since regardless of who was matched with m_1 , the other woman together with m_1 would form a weak instability.

¹ex734.923.393