

Assume we have three men  $m_1$  to  $m_3$  and three women  $w_1$  to  $w_3$  with preferences as given in the table below. Column  $w_3$  shows true preferences of woman  $w_3$ , while in column  $w'_3$  she pretends she prefers man  $m_3$  to  $m_1$ .

$m_1$	$m_2$	$m_3$	$w_1$	$w_2$	$w_3$	$(w'_3)$
$w_3$	$w_1$	$w_3$	$m_1$	$m_1$	$m_2$	$m_2$
$w_1$	$w_3$	$w_1$	$m_2$	$m_2$	$m_1$	$m_3$
$w_2$	$w_2$	$w_2$	$m_3$	$m_3$	$m_3$	$m_1$

First let us consider one possible execution of the G-S algorithm with the true preference list of  $w_3$ .

$m_1$	$w_3$			$w_3$
$m_2$		$w_1$		$w_1$
$m_3$			$[w_3][w_1]w_2$	$w_2$

First  $m_1$  proposes to  $w_3$ , then  $m_2$  proposes to  $w_1$ . Then  $m_3$  proposes to  $w_2$  and  $w_1$  and gets rejected, finally proposes to  $w_2$  and is accepted. This execution forms pairs  $(m_1, w_3)$ ,  $(m_2, w_1)$  and  $(m_3, w_2)$ , thus pairing  $w_3$  with  $m_1$ , who is her second choice.

Now consider execution of the G-S algorithm when  $w_3$  pretends she prefers  $m_3$  to  $m_1$  (see column  $w'_3$ ). Then the execution might look as follows:

$m_1$	$w_3$		—	$w_1$		$w_1$
$m_2$		$w_1$		—	$w_3$	$w_3$
$m_3$			$w_3$		—	$[w_1]w_2$

Man  $m_1$  proposes to  $w_3$ ,  $m_2$  to  $w_1$ , then  $m_3$  to  $w_3$ . She accepts the proposal, leaving  $m_1$  alone. Then  $m_1$  proposes to  $w_1$  which causes  $w_1$  to leave her current partner  $m_2$ , who consequently proposes to  $w_3$  (and that is exactly what  $w_3$  wants). Finally, the algorithm pairs up  $m_3$  (recently left by  $w_3$ ) and  $w_2$ . As we see,  $w_3$  ends up with the man  $m_2$ , who is her true favorite. Thus we conclude that by falsely switching order of her preferences, a woman may be able to get a more desirable partner in the G-S algorithm.