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lunedì 2 aprile 2018 19:20

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① $f(x) = x + \frac{3|x|}{x}$ asintoti in insieme di def.

dom f = $(-\infty, 0) \cup (0, +\infty)$

- $\lim_{x\to-\infty} x - \frac{3\cancel{x}}{\cancel{x}} = \boxed{-\infty}$ → No orizz.
- $\lim_{x\to+\infty} x + \frac{3\cancel{x}}{\cancel{x}} = \boxed{+\infty}$ ↗
- $\lim_{x\to 0^+} x + 3 = \boxed{3}$ → No verticale
- $\lim_{x\to 0^-} x - 3 = \boxed{-3}$ ↗
- $\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} 1 + \frac{3\cancel{x}}{x^{\cancel{2}}} = \boxed{1} = m$

  $\lim_{x\to+\infty} f(x) - mx = \lim_{x\to+\infty} \cancel{x} + 3 - \cancel{x} = \boxed{3}$ } $y = x+3$ obliquo
- $\lim_{x\to-\infty} \frac{f(x)}{x} = \lim_{x\to-\infty} \cancel{1} - \frac{3}{x} = \boxed{1} = m$ } $y = x-3$ obliquo
- $\lim_{x\to-\infty} f(x) - mx = \lim_{x\to-\infty} \cancel{x} - 3 - \cancel{x} = \boxed{-3}$

$\boxed{B}$ ✓

②

$f(x) = \left(1+\frac{1}{x}\right)^{x^2} = e^{x^2\log\left(1+\frac{1}{x}\right)}$ $D\left[1+\frac{1}{x}\right] = -x^{-2} = \boxed{-\frac{1}{x^2}}$

$f'(x) = e^{x^2\log\left(1+\frac{1}{x}\right)} \cdot \left(2x\log\left(1+\frac{1}{x}\right) + x^2\frac{1}{1+\frac{1}{x}} \cdot \left(-\frac{1}{x^2}\right)\right)$

$= e^{x^2\log\left(1+\frac{1}{x}\right)}\left(2x\log\left(1+\frac{1}{x}\right) - \frac{x^2}{x^2+x}\right)$

$= \left(1+\frac{1}{x}\right)^{x^2}\left(2x\log\left(1+\frac{1}{x}\right) - \frac{x^{\cancel{2}1}}{\cancel{x}(x+1)}\right) \to \boxed{C}$ ✓

③

$f: \mathbb{R} \to \mathbb{R}$ $f(x) \begin{cases} \frac{1-\cos x}{x^2} & \text{se } x \neq 0 \\ \frac{1}{2} & \text{se } x = 0 \end{cases}$ max, min, limitata?

$\lim_{x\to-\infty} \frac{1-\cos x}{x^2} = \frac{1-[-1,1]}{+\infty} = \boxed{0}$ → funzione limitata ~~C~~

$\lim_{x\to+\infty} \frac{1-\cos x}{x^2} = \frac{\text{limitata}}{+\infty} = \boxed{0}$ ↗

Ha max e min ⟹ D ⓥ

④

$$a_n = \frac{(n+1)! - n!}{n^2\,(n-1)!} \qquad n \geq 1 \quad \text{limite}$$

$$\frac{(n+1)\,n! - n!}{n^2\,(n-1)!} = \frac{n!\,((n+1)-1)}{(n-1)!\,n^2} = \frac{\cancel{n^2}\cdot\cancel{(n-1)!}}{\cancel{(n-1)!}\,\cancel{n^2}} = \boxed{1}$$

↓ Ⓑ ✓

⑤
$$\lim_{n\to\infty} \frac{(n-1)^n}{e^{n\log n}} = \frac{(n-1)^n}{n^n} = \left(1 - \frac{1}{n}\right)^n = [1^\infty] = e^{-1} = \boxed{\frac{1}{e}} \Rightarrow D ✓$$

⑥ $F: \mathbb{R} \to \mathbb{R}$
$$F(x) = \int_1^x \frac{\arctan(t^2-1)}{1+t^2}\,dt$$

$$\boxed{F'(x) = \frac{\arctan(x^2-1)}{1+x^2}} - \cancel{\frac{\arctan(0)}{2}} \to 0$$

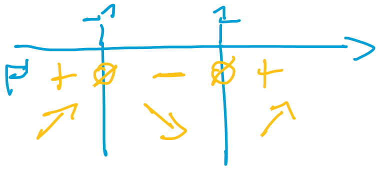
$$\boxed{F'(x) > 0} \quad \frac{\arctan(x^2-1)}{1+x^2} > 0 \iff \arctan(x^2-1) > 0$$

$$\iff x^2 - 1 > 0 \iff x < -1 \lor x > 1$$

$(x+1)(x-1) = 0$

$x = 1$

$x = -1$



$x = -1$ punto di max
$x = 1$ punto di min
} $\boxed{A}$ ✓