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Compitino

1) $\lim_{n\to\infty} \frac{\sqrt[n]{e^{n+1}-e^{n-1}}}{\log(n^2+e^{2n})} = \left[\frac{\sqrt{+\infty-\infty}}{+\infty}\right] = 0$ $\boxed{C}$

5) $\int x \log x \, dx$ per parti $\boxed{F\cdot g - \int F g' \, dx}$

$f = x \to F = \frac{x^2}{2}$

$g = \log x \to g' = \frac{1}{x}$

$\frac{x^2}{2}\log x - \int \frac{x^{\cancel{2}}}{2}\frac{1}{\cancel{x}}\,dx = \frac{x^2}{2}\log x - \frac{x^2}{4} = \frac{2x^2\log x - x^2}{4}$

$\boxed{\frac{x^2(2\log x - 1)}{4}} \to \boxed{D}$ ✓

8) $\begin{cases} y'' - 4y = 0 \\ y(5) = 0 \\ y'(5) = -2 \end{cases}$ $y(-3) = ?$

$y'' - 4y = 0$ eq. omog.

$\lambda^2 - 4 = 0$

$\lambda^2 = 4 \to \begin{cases} \lambda_1 = +2 \\ \lambda_2 = -2 \end{cases}$

- $y(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} = \boxed{C_1 e^{2x} + C_2 e^{-2x}}$
- $y'(x) = \boxed{2C_1 e^{2x} - 2C_2 e^{-2x}}$

$\boxed{y(5)=0} \Rightarrow \begin{cases} C_1 e^{10} + C_2 e^{-10} = 0 \\ 2C_1 e^{10} - 2C_2 e^{-10} = -2 \end{cases}$
$\boxed{y'(5)=-2} \Rightarrow$

$\begin{cases} C_1 = -C_2 e^{-20} \\ -2C_2 e^{-10} - 2C_2 e^{-10} = -2 \\ -4C_2 e^{-10} = -2 \end{cases} \Rightarrow \begin{cases} C_1 = -\frac{e^{10}}{2} e^{-20} \\ C_2 = \frac{e^{10}}{2} \end{cases} \Rightarrow \begin{matrix} C_1 = -\frac{1}{2e^{10}} \\ C_2 = \frac{1}{2}e^{10} \end{matrix}$

$y(x) = -\frac{1}{2e^{10}} e^{2x} + \frac{1}{2} e^{10} \cdot e^{-2x}$ solu. cauchy

$$y(-3) = -\frac{1}{2e^{10}} e^{-6} + \frac{1}{2} e^{10} \cdot e^{6} = -\frac{1}{2e^{16}} + \frac{1}{2} e^{16} =$$

n° 9

$$\begin{cases} y' = (\cos y)^2 \sin x \\ y\left(\frac{\pi}{4}\right) = \frac{\pi}{4} \end{cases}$$

$y' = a($

n° 10

$$\begin{cases} y' = \frac{x+1}{x} y + x \\ y(5) = -1 \end{cases} \qquad y(1) = ?$$

$y' = a(x) y + b(x)$

- $A(x) = \int \frac{x+1}{x} dx = \int 1 dx + \int \frac{1}{x} dx = x + \ln|x|$

$y(x) = e^{x+\ln|x|} \left(\int x e^{x+\ln|x|} dx + c \right)$

$\int x e^{x+\ln|x|} dx$ per parti

$f = e^{x+\ln|x|} \rightarrow F = \int e^{x+\ln|x|}$

$y = x$

$f(x) = (x-1)\, e^{\frac{1}{x^2-1}}$

- dom,
- eventuali asintoti
- sup, inf (o max e min)
- Num di punti di max e min locali
- grafico

Dom

$x^2 - 1 \neq 0$

$x^2 \neq 1$

$x \neq \pm 1 \Rightarrow \boxed{dom(f) = (-\infty, -1) \cup (-1, 1) \cup (1; +\infty)}$

Asintoti

- $\lim_{x \to -\infty} (x-1)\, e^{\frac{1}{x^2-1}} = -\infty \cdot e^0 = -\infty$
- $\lim_{x \to -1^-} (x-1)\, e^{\frac{1}{x^2-1}} = -2\, e^{\frac{1}{1^+-1}} = -2\, e^{+\infty} = -\infty$ $y = -1$ asintoto verticale
- $\lim_{x \to -1^+} (x-1)\, e^{\frac{1}{x^2-1}} = -2\, e^{\frac{1}{1^- -1}} = 2\, e^{-\infty} = 0$
- $\lim_{x \to 1^-} (x-1)\, e^{\frac{1}{x^2-1}} = 0 \cdot e^{\frac{1}{0^-}} = 0 \cdot e^{-\infty} = 0$
- $\lim_{x \to 1^+} (x-1)\, e^{\frac{1}{x^2-1}} = 0 \cdot e^{\frac{1}{0^+}} = [0 \cdot \infty]$

$\lim_{x \to 1^+} (x-1)\, e^{\frac{1}{x-1} \cdot \frac{1}{x+1}}$ $\Rightarrow 1$ $\quad t = \frac{1}{x-1} \quad x \to 1^+ \Rightarrow t \to +\infty$

$\lim_{t \to +\infty} \frac{1}{t}\, e^{t}$