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① $\lim_{x\to-\infty} \dfrac{\sqrt[4]{1+\frac{1}{x}}-1}{(\sin x)^2+1}\left(1-\cos\frac{1}{x}\right) = \dfrac{0}{[-1,1]^2+1}\cdot 0 = 0 \Rightarrow \boxed{A}$ ✓

$[-1,1]^2+1 \to \neq 0$

② $f(x)=\dfrac{\log x}{x-1}$ asintoti?

$\mathrm{dom}(f): \begin{cases} x\neq 1 \\ x>0 \end{cases}$ $\boxed{\mathrm{dom}(f)=(0,1)\cup(1,+\infty)}$

- $\lim_{x\to 0^+} \dfrac{\log x}{x-1} = \dfrac{-\infty}{-1} = +\infty \to \boxed{x=0 \text{ verticale}}$
- $\lim_{x\to 1^-} \dfrac{\log x}{x-1} = \left[\dfrac{0}{0}\right] \overset{\text{Hopital}}{=} \lim_{x\to 1^-} \dfrac{1}{x} = \dfrac{1}{1^-} = 1$
- $\lim_{x\to 1^+} f(x) \overset{\text{Hopital}}{=} \dfrac{1}{1^+} = 1 \to$ Nada
- $\lim_{x\to+\infty} f(x) = 0$ $\boxed{y=0 \text{ asintoto orizzontale}}$
- $\lim_{x\to\infty} \dfrac{f(x)}{x} = \dfrac{\log x}{x^2-x} = \dfrac{\log x}{x^2\left(1-\frac{1}{x}\right)} = 0 \Rightarrow$ $\boxed{\text{No obliquo!}}$ ($\frac{1}{x}\to 0$)

$\Rightarrow \boxed{D}$ ✓

③

$f:(0,+\infty)\to\mathbb{R}$ $f(x)=\dfrac{2\cos x-\sin(x^4)-2}{x^3}$ è limitata?

$\sin x = x + o(x^2)$

- $\lim_{x\to 0^+} f(x) = \dfrac{2-0-2}{0^+} = \left[\dfrac{0}{0}\right]$

$$\frac{2\left(1-\frac{x^2}{2}+\frac{x^4}{24}+o(x^4)\right)-x^2+o(x^4)-2}{x^3}$$

$$\frac{\cancel{2}-x^2+\frac{x^4}{12}+\cancel{o(x^4)}-x^2+\cancel{o(x^4)}-\cancel{2}}{x^3} = \frac{-2x^2+\frac{x^4}{12}+o(x^4)}{x^3}$$

$$= \frac{\cancel{x^2}\left(-2+\frac{x^2}{12}+\frac{o(x^4)}{x^2}\right)}{x^{\cancel{3}}} = \frac{-2}{0^+} = -\infty \to$$

$\frac{x^2}{12}\to 0$, $\frac{o(x^4)}{x^2}\to 0$

è limitata sup. ma non inf.

- $\lim_{x\to+\infty} f(x) = \left[\dfrac{2[-1,1]-[-1,1]-2}{+\infty}\right] = \left[\dfrac{\text{limitata}}{+\infty}\right] = 0$ $\Rightarrow \boxed{C}$ ✓

④

$a_n=\left(\dfrac{n^2+2}{n^2-3n}\right)^{3n^2+1}$ con $n\geq 4$ max, min?

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$$\lim_{n\to\infty} \left(1-(-1)^n\right) n^{\left((-1)^n\right)} =$$

- $b_n = 2n$ (pari) $\Rightarrow \lim_{n\to\infty} \underbrace{(1-1)}_{\text{fa } 0 \text{ senza dim!}} \cdot 2n = 0 \to 0 \to \boxed{A}$
- $k_n = 2n+1$ (dispari) $\Rightarrow \lim_{n\to\infty} (1+1)\cdot(2n+2)^{-1} = \frac{1}{+\infty} = 0$

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$a_n = \frac{n^3 - n!}{e^n - n^4}$ max, min?

$$\lim a_n = \frac{n!\left(\cancel{\frac{n^3}{n!}}^{\to 0} - 1\right)}{e^n\left(1 - \cancel{\frac{n^4}{e^n}}_{\to 0}\right)} \to -\infty \Rightarrow$$

Ha max ma non ha min

$\Rightarrow \boxed{D}$ V

⑦ $\int_0^{\pi/4} \frac{\cos(2x)}{\cos x + \sin x}\,dx$ $\qquad \boxed{\cos(2x) = \cos^2 x - \sin^2 x}$

$F(x) = \int \frac{\cos(2x)}{\cos x + \sin x}\,dx = \int \frac{\cos^2 x - \sin^2 x}{\cos x + \sin x}\,dx = \int \frac{(\cos x - \sin x)\cdot\cancel{(\cos x + \sin x)}}{\cancel{\cos x + \sin x}}\,dx = \boxed{\sin x + \cos x + c}$

$F(\pi/4) - F(0) = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - (0 + 1) = \boxed{\sqrt{2} - 1} \rightarrow \boxed{C}$ ✓

$\int \frac{f'(x)}{\cos^2 f(x)}\,dx = \tan f(x) + c$

⑧
$\int_0^{\frac{\sqrt{\pi}}{2}} \frac{x}{\cos^2(x^2)}\,dx =$

$F(x) = \frac{1}{2}\int \frac{2x}{\cos^2(\underbrace{x^2}_{2x})} = \boxed{\frac{\tan x^2}{2} + c}$

$F(\frac{\sqrt{\pi}}{2}) - F(0) = \frac{\tan\frac{\pi}{4}}{2} - (0) = \boxed{\frac{1}{2}} \Rightarrow \boxed{A}$ ✓

⑨
$\begin{cases} y'' + 4y' + 4y = 0 \\ y(0) = 2 \\ y'(0) = 1 \end{cases}$ $\qquad y(3) = ?$

$\lambda^2 + 4\lambda + 4 = 0$

$\lambda_{1,2} = \frac{-4 \pm \sqrt{16 - 4(4)}}{2} = \frac{-4 \pm 0}{2} = -2 =$

$y(x) = C_1 e^{\lambda x} + x C_2 e^{\lambda x} = \boxed{C_1 e^{-2x} + x C_2 e^{-2x}}$

$y'(x) = -2C_1 e^{-2x} + C_2 \left[\underbrace{e^{-2x} + x(-2e^{-2x})}_{e^{-2x} - 2xe^{-2x}}\right] = \boxed{-2C_1 e^{-2x} + C_2 e^{-2x} - 2xe^{-2x}}$

$\boxed{y(0) = 2}$ $\boxed{y'(0) = 1}$ $\begin{cases} C_1 = 2 \\ -2C_1 + C_2 = 1 \end{cases}$ $\begin{cases} C_1 = 2 \\ -4 + C_2 = 1 \end{cases}$ $\begin{cases} C_1 = 2 \\ C_2 = 5 \end{cases}$

$\boxed{y(x) = 2e^{-2x} + x\,5\,e^{-2x}}$

$\Rightarrow y(3) = 2e^{-6} + 15e^{-6} = \frac{2 + 15}{e^6} = \frac{17}{e^6} \rightarrow \boxed{A}$ ✓

⑩
$\begin{cases} y' = 1 - \frac{y}{x} \\ y(2) = -5 \end{cases}$ $\qquad y(5) = ?$

$\boxed{y'(x) = ay + b}$ $\qquad y' = \underbrace{-\frac{1}{x}}_{a(x)} y + \underbrace{1}_{b(x)}$ $\qquad A(x) = -\int \frac{1}{x}\,dx = \boxed{-\log(x)}$

$$y(x) = e^{-\log x}\left(\int e^{\log x}\cdot 1\,dx + c\right) = \frac{1}{x}\left(\frac{x^2}{2} + c\right) = \boxed{\frac{x}{2} + \frac{c}{x}}$$

$$\boxed{y(2) = -5} \Rightarrow 1 + \frac{c}{2} = -5 \Leftrightarrow \frac{2+c}{2} = \frac{-10}{2} \Leftrightarrow \boxed{c = -12}$$

$$\boxed{y(x) = \frac{x}{2} - \frac{12}{x}} \Rightarrow y(5) = \frac{5}{2} - \frac{12}{5} = \frac{25-24}{10} = \boxed{\frac{1}{10}} \rightarrow \boxed{B} \checkmark$$