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Scritto

$$f(x) = \log\left((4-x^2)(1-x)\right)$$

- Insieme di def

$$\mathrm{dom}(f):\ (4-x^2)(1-x) > 0 \iff \underset{①}{(2-x)}\underset{②}{(2+x)}\underset{③}{(1-x)} > 0$$

① >0: $2-x>0 \iff x<2$

② >0: $2+x>0 \iff x>-2$

③ >0: $1-x>0 \iff x<1$

		-2	1	2
①	+	+	+	0 −
②	−	+	+	+
③	+	+	−	−
S	−	+	−	+

$$\boxed{\mathrm{dom}(f) = x \in (-2,1) \cup (2;+\infty)}$$

Asintoti

$$\lim_{x\to -2} \log\left((4-x^2)(1-x)\right) = -\infty$$

$$\lim_{x\to 1} \log\left((4-x^2)(1-x)\right) = -\infty$$

$$\lim_{x\to 2} \log\left((4-x^2)(1-x)\right) = -\infty$$

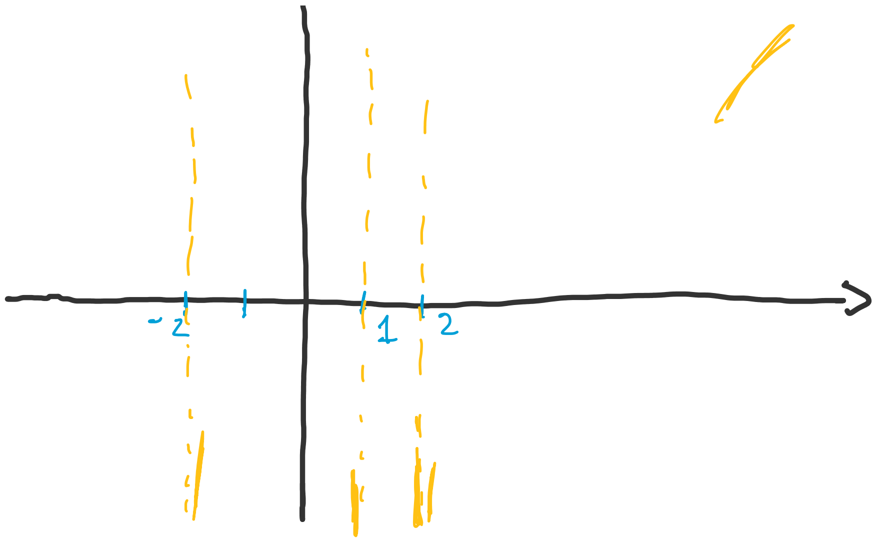
$$\lim_{x\to +\infty} \log\left((4-x^2)(1-x)\right) = \left[\log(+\infty-\infty)\right] \text{ f. indet.}$$

$$= \lim_{x\to+\infty} \log\left(4-4x-x^2+x^3\right) = \lim_{x\to+\infty} \log\left(x^3\left(\cancel{\frac{4}{x^3}}^{0} - \cancel{\frac{4}{x^2}}^{0} - \cancel{\frac{1}{x}}^{0} + 1\right)\right)$$

$$= +\infty$$

- $\displaystyle\lim_{x\to+\infty} \frac{f(x)}{x} = \lim_{x\to+\infty} \frac{\log\left(x^3(\sim)\right)}{x} = 0 \rightarrow$ NO obliquo!

- $x=-2;\ x=1,\ x=2$ sono asintoti verticali
- NON ha obliqui nè orizzontali



$\inf(f) = -\infty$; $\sup(f) = +\infty \Rightarrow$ NON ha max né min

Punti di massimo/minimo locali

$$f'(x) = f'(g(x)) \cdot g'(x) = \frac{1}{(4-x^2)(1-x)} \cdot \left(-2x(1-x) + (4-x^2)\cdot -1\right)$$

$$= \frac{-2x + 2x^2 - 4 + x^2}{(4-x^2)(1-x)} = \boxed{\frac{3x^2-2x-4}{(4-x^2)(1-x)}}$$

$$f'(x) > 0 \iff \frac{3x^2-2x-4}{(4-x^2)(1-x)} > 0$$

16 | 4
4 | 4
1 |

$9 = 3^2$

144

52 | 2
26 | 2
13 | 13
1 |

$(N>0)$: $3x^2 - 2x - 4 > 0$

$$x_{1,2} = \frac{+2 \pm \sqrt{4-4(-12)}}{6} = +\frac{1}{3} \pm \frac{\sqrt{4+48}}{6} = \frac{1}{3} \frac{\sqrt{2^2 \cdot 13}}{6}$$

$$= \frac{1}{3} \pm \frac{\sqrt{13}}{3} = \begin{cases} \frac{1}{3} + \frac{\sqrt{13}}{3} \\ \frac{1}{3} - \frac{\sqrt{13}}{3} \end{cases}$$

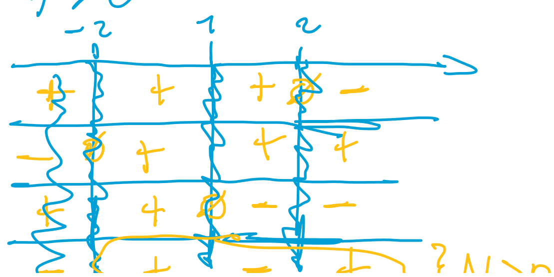
$$\boxed{N > 0 \iff x < \frac{1-\sqrt{13}}{3} \lor x > \frac{1+\sqrt{13}}{3}}$$

$(D>0)$: $(2-x)(2+x)(1-x) > 0$

$2 - x > 0 \iff x < 2$

$2 + x > 0 \iff x > -2$

$1 - x > 0 \iff x < 1$



$\mathrm{dom}(f) = x \in (-2,1) \cup (2,+\infty)$ ... $N>0$

$\boxed{\text{dom}(f):\ x \in (-2,1) \cup (2;+\infty)}$

	-2		$\frac{1-\sqrt{13}}{3}$		1		$\frac{1+\sqrt{13}}{3}$		2	
N		+		−		−		+		+
D		+		+		−		−		+
f'		+		−		+		−		+
	$\nexists f'$				$\nexists f'$				$\nexists f'$	

f strettamente decrescente in $\left(-2, \frac{1-\sqrt{13}}{3}\right) \cup \left(1, \frac{1+\sqrt{13}}{3}\right)$

f strettamente crescente in $\left(\frac{1-\sqrt{13}}{3}, 1\right) \cup \left(\frac{1+\sqrt{13}}{3}, 2\right)$

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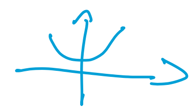
n°2

$$\int_{-1}^{1} \frac{|x| + (\sin x)^3}{1+x^2}\,dx = \underbrace{\int_{-1}^{1} \frac{|x|}{1+x^2}\,dx}_{①} + \underbrace{\int \frac{(\sin x)^3}{1+x^2}\,dx}_{②}$$



① $f(x) = \frac{|x|}{1+x^2} \rightsquigarrow f(-x) = \frac{|-x|}{1+(-x)^2} = \frac{x}{1+x^2} = f(x) \Rightarrow$ è pari

Essendo pari possiamo stud. integrale in $[0,1)$ o $(-1,0]$

$$\Rightarrow \int_0^1 \frac{x}{\underbrace{1+x^2}_{2x}} = \left[\frac{1}{2}\log(1+x^2)\right]_0^1 = \frac{1}{2}\log(2)$$

② $\int \frac{\sin^3 x}{1+x^2}\,dx \quad f(x) = \frac{\sin^3 x}{1+x^2} \rightsquigarrow f(-x) = \frac{\sin^3 -x}{1+(-x)^2} = -\frac{\sin x}{1+x^2} = -f(x)$

$\Rightarrow$ È dispari!

$= 0$!

$$\boxed{\text{Soluzione} = \frac{\log 2}{2}}$$

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n°3

$$\begin{cases} y'' + 3y' + 2y = x^2 + 2x \\ y(0) = 9/4 \\ y'(0) = 5/2 \end{cases}$$

1) $\lambda^2 + 3\lambda + 2 = 0$ $\qquad -1 = \lambda_1$

$\lambda_{1,2} = \frac{-3 \pm \sqrt{9-4(2)}}{2} = \frac{-3 \pm 1}{2}$ $\nearrow$ $-1 = \lambda_1$, $\searrow$ $-2 = \lambda_2$

$\Rightarrow y_0(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} = \boxed{C_1 e^{-x} + C_2 e^{-2x}}$

2) $f(x) = e^{\alpha x} [p(x)\cos(\beta x) + q(x)\sin(\beta x)]$ $f(x) = x^2 + 2x$

$\alpha = 0$, $\beta = 0$, $p(x) = x^2 + 2x$ $\alpha + i\beta$ non è rad. $\Rightarrow \boxed{m = 0}$

$\bar{y}(x) = Ax^2 + Bx + C$

$\bar{y}'(x) = 2Ax + B$

$\bar{y}''(x) = 2A$

Sostituisco:

$2A + 3(2Ax + B) + 2(Ax^2 + Bx + C) = x^2 + 2x$

$2A + 6Ax + 3B + 2Ax^2 + 2Bx + 2C = x^2 + 2x$

$2Ax^2 + x(6A + 2B) + (2A + 3B + 2C) = x^2 + 2x$

$\begin{cases} 2A = 1 \\ 6A + 2B = 2 \\ 2A + 3B + 2C = 0 \end{cases}$ $\begin{cases} A = \frac{1}{2} \\ 3 + 2B = 2 \\ 1 + 3B + 2C = 0 \end{cases}$ $\begin{cases} A = \frac{1}{2} \\ B = -\frac{1}{2} \\ 2C = -1 + \frac{3}{2} \end{cases}$ $\begin{cases} = \\ = \\ C = \frac{1}{4} \end{cases}$

$\Rightarrow \boxed{\bar{y}(x) = \frac{1}{2}x^2 - \frac{1}{2}x + \frac{1}{4}}$

$\boxed{y(x) = y_0 + \bar{y} = C_1 e^{-x} + C_2 e^{-2x} + \frac{x^2}{2} - \frac{x}{2} + \frac{1}{4}}$ Sol generale

$\boxed{y'(x) = -C_1 e^{-x} - 2C_2 e^{-2x} + x - \frac{1}{2}}$

$y(0) = \frac{9}{4} \Rightarrow$ $\begin{cases} C_1 + C_2 + \frac{1}{4} = \frac{9}{4} \\ -C_1 - 2C_2 - \frac{1}{2} = \frac{5}{2} \end{cases}$ $\begin{cases} C_1 = 2 - C_2 \\ -2 + \underbrace{C_2 - 2C_2}_{-C_2} - \frac{1}{2} = \frac{5}{2} \end{cases}$ $\begin{cases} C_1 = 2 - C_2 \\ -C_2 = \frac{\cancel{10}\,5}{\cancel{2}\,1} \end{cases}$

$y'(0) = \frac{5}{2} \Rightarrow$

$\begin{cases} C_1 = 2 + 5 \\ C_2 = -5 \end{cases}$ $C_1 = 7$ $C_2 = -5$

$\boxed{y(x) = -7e^{-x} + 5e^{-2x} + \frac{x^2}{2} - \frac{x^2}{2} + \frac{1}{4}}$ ✓