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## Compitino

① $a_n = \sqrt[n]{2^n + 1}$ max, min?

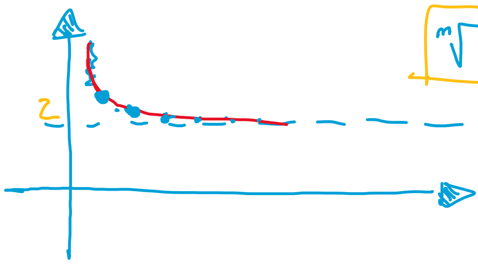
Ha max ma no min → $\boxed{B}$

C. Rapp/Rad

$b_n = 2^n + 1 \leadsto \frac{b_{n+1}}{b_n} = \boxed{\frac{2^{n+1}+1}{2^n+1}} \to 2 \Rightarrow \boxed{a_n \to 2}$

C. Rapp/Rad.

$\sqrt[n]{2^n+1} > 2 \quad 2^n + 1 > 2^n \Leftrightarrow \forall n \in N$



Ha ma ma non ha min → $\boxed{A}$ ✓

② $\lim_{n \to \infty} \frac{1}{2^n} + \frac{1}{(-1)^n 5^n} = 0 + 0 = 0 \to \boxed{D}$ ✓

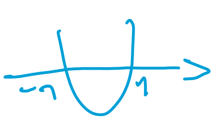
③ $x^3 - 3x + 5 = 0$

$f(x) = x^3 - 3x + 5 \qquad dom(f): \mathbb{R}$

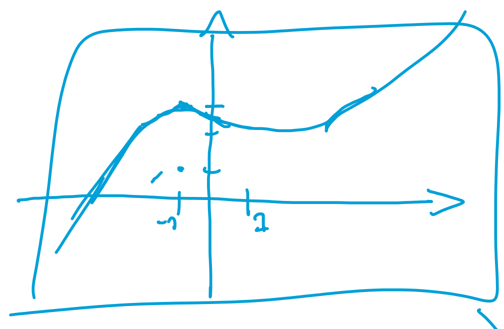
$f'(x) = 3x^2 - 3$

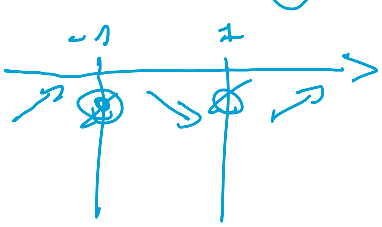
$f'(x) \geq 0: \quad 3x^2 - 3 \geq 0 \Leftrightarrow x^2 - 1 \geq 0 \Leftrightarrow (x+1)(x-1) \geq 0$

$x = \pm 1$



$\Rightarrow x \leq -1 \vee x \geq 1$





$\lim_{x \to +\infty} x^3 \cdot \left(1 - \frac{3}{x^2} + \frac{5}{x^3}\right) = +\infty$

$\lim_{x \to -\infty} x^3 \cdot \left(1 - \frac{3}{x^2} + \frac{5}{x^3}\right) = -\infty$

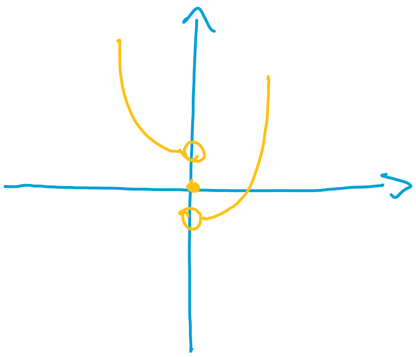
Ha una soluzione

$\boxed{B}$ ✓

④

$\begin{cases} 0 & se\ x = 0 \end{cases} \qquad \begin{cases} 0 & x \neq 0 \end{cases}$

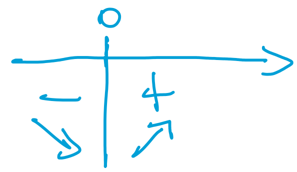
$f(x) \begin{cases} x\left(x - \frac{1}{|x|}\right) & \text{se } x \neq 0 \end{cases} \rightsquigarrow f(x) \begin{cases} x^2 - 1 & x \geq 0 \\ x^2 + 1 & x < 0 \end{cases}$



$\Rightarrow$ • $f: \mathbb{R} \to (-1, +\infty)$

• $f(x)$ non è continua in $x_0 = \emptyset$

• $f'(x) \geq 0$ $\quad f' \begin{cases} 0 & x = 0 \\ 2x & x \geq 0 \\ 2x & x < 0 \end{cases}$



Non ha punti di min o max locali $\to$ $\boxed{C}$ V

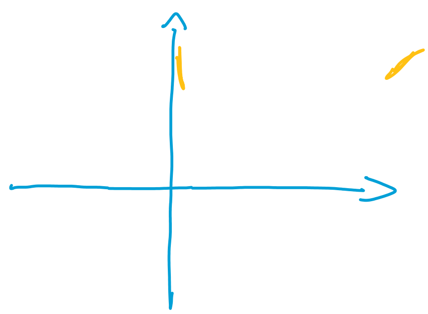
⑤

$f: (0, +\infty) \to \mathbb{R} \qquad f(x) = x\, e^{\frac{1}{\sqrt{x}}}$

• $\lim_{x \to +\infty} x\, e^{\frac{1}{\sqrt{x}}} = +\infty \, e^0 = +\infty$

• $\lim_{x \to 0} x\, e^{\frac{1}{\sqrt{x}}} = [0 \cdot +\infty]$

$\lim_{x \to 0} \frac{e^{\frac{1}{\sqrt{x}}}}{\frac{1}{x}} = \lim_{t \to +\infty} \frac{e^{\sqrt{t}}}{t} = +\infty$



Ha min, ma non max

$\boxed{C}$ V

⑥ $\int_e^{e^2} \frac{(\log t)^2}{t}\, dt = *$

$F(x) = \int \frac{(\overbrace{\log t}^{\frac{1}{t}})^2}{t}\, dt = \boxed{\frac{\log^3 t}{3} + c}$

$* = F(e^2) - F(e) = \frac{\log^3(e^2)}{3} - \frac{\log^3 e}{3} = \frac{8}{3} - \frac{1}{3} = \boxed{\frac{7}{3}} \to \boxed{A}$ V

⑦ $f(x) = \int_x^{2x} \frac{\sin t}{t}\, dt$ allora $\lim_{x \to 0^+} \frac{f'(x)}{x} = *$

$$f'(x) = \frac{\sin 2x}{\cancel{2}x}\cdot \cancel{2} - \frac{\sin x}{x} = \boxed{\sin 2x - \sin x}$$

$$\lim_{x\to 0^+} \frac{\sin 2x - \sin x}{x^2} = \boxed{\frac{2x + o(x) - x + o(x)}{x^2}}$$

$$\lim_{x\to 0^+} \frac{\cancel{x}\cdot\left(1 + \cancel{\frac{o(x)}{x}}\right)}{x^{\cancel{2}}} = +\infty \to \boxed{B}$$

8) $\lim_{x\to +\infty} \sin\left(\int_0^x t\, e^{-t^2} dt\right) = (*)$

$\to D: -2t$

• $\int t\, e^{-t^2} dt = -\frac{1}{2}\int -2t\, e^{-t^2} dt = \boxed{-\frac{1}{2} e^{-t^2} + c}$

. $F(x) - F(0) = -\frac{1}{2} e^{-x^2} + \frac{1}{2} e^0 = -\frac{1}{2} e^{-x^2} + \frac{1}{2} = \boxed{\frac{1}{2}\left(-e^{-x^2} + 1\right)}$

$(*) = \lim_{x\to +\infty} \sin\left(\frac{1}{2}\left(-e^{-x^2} + 1\right)\right) = \lim_{x\to+\infty} \sin\left(\frac{1}{2}\left(-\frac{1}{e^{x^2}} + 1\right)\right) \to \boxed{\sin\frac{1}{2}}$

V $\boxed{D}$

9)

$$\begin{cases} y' = y + 1 + x \\ y(1) = 1 \end{cases} \qquad y(0) = ?$$

$f = e^{-x}$ $F = -e^{-x}$

$*$ $g = x$ $g' = 1$

$= -xe^{-x} + \int e^{-x} = \boxed{-xe^{-x} - e^{-x}}$

$y' = \boxed{1}^{\,a(x)} y + \overbrace{(1+x)}^{f(x)}$

$A(x) = x$

$y(x) = e^x\left(\int (1+x)\, e^{-x} dx + c\right) = e^x\left(\int e^{-x} dx + \boxed{\int x e^{-x} dx} + c\right)$

$= e^x\left(-e^{-x} - xe^{-x} - e^{-x} + c\right) = \boxed{-2 - x + ce^x}$

$\boxed{y(1) = 1} \Rightarrow -2 - 1 + ce = 1$

$ce = 4 \Leftrightarrow \boxed{c = \frac{4}{e}}$

$y(0) = \boxed{-2 + \frac{4}{e}} \Rightarrow \boxed{D}$ V n°10

$$\begin{cases} y'' = 10y' - 9y \\ y(0) = 0 \\ y'(0) = 4 \end{cases} \qquad y(1) =$$

$y'' - 10y' + 9y = 0$

$\lambda^2 - 10\lambda + 9 = 0 \qquad \Delta = 100 - 4(9) = 100 - 36 = 64 \Rightarrow$

$\lambda_{1,2} = \frac{10 \pm 8}{2} \to 1 = \lambda_1, \; 9 = \lambda_2$

$$\begin{cases} y(x) = C_1 e^{\lambda_1 x} + C_2 e^{\lambda_2 x} = \boxed{C_1 e^{x} + C_2 e^{9x}} \\ y'(x) = C_1 e^{x} + 9 C_2 e^{9x} \end{cases}$$

$\boxed{y(0) = 0} \Rightarrow \begin{cases} C_1 + C_2 = 0 \\ C_1 + 9C_2 = 4 \end{cases} \quad \begin{cases} C_1 = -C_2 \\ 8C_2 = 4 \end{cases} \quad \begin{cases} C_1 = -\frac{1}{2} \\ C_2 = \frac{1}{2} \end{cases}$

$\boxed{y(0)' = 4} \Rightarrow$

$$\boxed{y(x) = -\frac{1}{2} e^{x} + \frac{1}{2} e^{9x}}$$

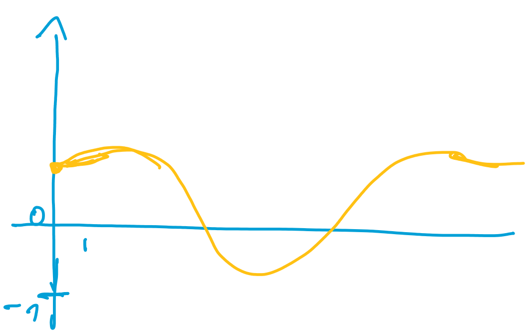
$\Rightarrow y(1) = -\frac{1}{2} e + \frac{1}{2} e^{9} = \boxed{\frac{e^9 - e}{2}} \to \boxed{D}$ V

# Scritto

n° 1

$f(x) = (\sin x)^2 + \cos x \quad$ in $x \in [0, 2\pi]$

max, min, punt. max/min, convessità, grafico, soluz. $f(x) = K$ con $K \in \mathbb{R}$



$f(x) = 2\sin x \cos x - \sin x = \sin x\,(2\cos x - 1)$

$f'(x) \geq 0 : \sin x\,(2\cos x - 1) \geq 0$

- $\sin x \geq 0 \iff 0 < x < \pi$
- $2\cos x - 1 > 0 \iff \cos x > \frac{1}{2} \iff x \in [0, \pi/3) \cup (\frac{5}{3}\pi, 2\pi)$

| | 0 | | π/3 | | π | | 5/3 π | | 2π |
|---|---|---|---|---|---|---|---|---|---|
| $F_1$ | 0 | + | | + | 0 | − | | − | |
| $F_2$ | | + | 0 | − | | − | 0 | + | |
| $f'$ | | + | | − | | + | | − | |

- in $[0, \pi/3]$ $f$ è strett. crescente
- in $[\pi/3, \pi]$ $f$ è strett. decresc.
- in $[\pi, \frac{5}{3}\pi]$ $f$ è strett. crescente
- in $[\frac{5}{3}\pi, 2\pi]$ $f$ è strett. decresc.

$x_1 = 0$, $x_2 = \pi$, $x_3 = 2\pi$ } Punti di min locale

$x_4 = \pi/3$, $x_5 = 5/3\pi$ } Punti di max locale

- $f(0) = 1$ ; $f(\pi) = -1$ ; $f(2\pi) = 1$ Max locali

$f(\frac{\pi}{3}) = \frac{1}{2} + \frac{3}{4} = \frac{2+3}{4} = \frac{5}{4}$ ; $f(\frac{5}{3}\pi) = -\frac{5}{4}$ Min locali

## Convessità

$\boxed{\sin^2 = 1 - \cos^2 x}$

$f''(x) = (f'(x))' = D[\sin x\,(2\cos x - 1)] =$

$= \cos x\,(2\cos x - 1) + \sin x\,(-2\sin x) = 2\cos^2 x - \cos x - 2\sin^2 x$ (*)

$= 2\cos^2 x - \cos x - 2 + 2\cos^2 x = \boxed{4\cos^2 x - \cos x - 2}$

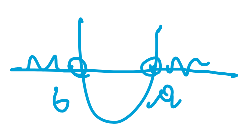
$f''(x) > 0 \iff 4\cos^2 x - \cos x - 2 > 0$ con $t = \cos x$

$4t^2 - t - 2 > 0$

$t_{1,2} = \frac{1 \pm \sqrt{1 - 4(-8)}}{8} = \frac{1 \pm \sqrt{33}}{8}$

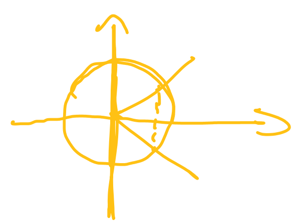
$\frac{1+\sqrt{33}}{8} = a$

$\frac{1-\sqrt{33}}{8} = b$



$t < \frac{1-\sqrt{33}}{8} \lor t > \frac{1+\sqrt{33}}{8}$

$\cos x < \frac{1-\sqrt{33}}{8} \lor \cos x > \frac{1+\sqrt{33}}{8}$



$\underline{\underline{n^{o}2}}$

$$\int_0^1 \frac{1}{(\sin^2 x) + (\cosh x)^2} \, dx$$

$$\int \frac{1}{k^2 + g^2(x)}$$