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n° 1

$f(x) = \sin(\log(x^2))$ $\quad dom(f) = (-\infty, 0) \cup (0, +\infty)$

- $\lim_{x \to +\infty} f(x) = [-1, 1] \rightsquigarrow$ Non è iniettiva perchè non cresce sempre. $\cancel{A}$ NO
- $\lim_{x \to 0} f(x) = \sin(-\infty) = [-1, 1] \to$ è lim sup e inf. $\cancel{D}$ NO

f(x) è una dilatazione orizzontale di sen x. Più x cresce e più le oscillazioni si stringono. Ha di stessi punti di max e min di sin x: che sono infiniti $\rightsquigarrow$ $\cancel{B}$ NO $\to$ $\boxed{C}$ V

n° 2

$$f(x) = \frac{\log(1 + 3\sqrt[3]{x})}{x + 2x^4 + x^2} \qquad \boxed{x > 0}$$

$$\frac{1}{3} - \frac{3}{3} = -\frac{2}{3}$$

$$\lim_{x \to +\infty} f(x) = \emptyset$$

$$\lim_{x \to 0^+} f(x) = \left[\frac{0}{0}\right] \qquad \boxed{\log(1+x) = (-1)^{n-1}\frac{x^n}{n} + o(x^n) = x + o(x)}$$

$$\lim_{x \to 0^+} \frac{3\sqrt[3]{x} + o(x^{1/3})}{\sim} = \lim \frac{\sqrt[3]{x}\left(3 + \frac{o(x^{1/3})}{x^{1/3}}\right)}{x \cdot (1 + 2x^3 + x)} =$$

$$\lim_{x \to 0^+} \frac{\left(3 + \cancel{\frac{o(x^{1/3})}{x^{1/3}}}\right)}{\sqrt[3]{x^2}\,(1 + 2x^3 + x)} = +\infty \qquad \cancel{A} \text{ NO } \cancel{\text{è lim sup}}$$

$\frac{o(x^{1/3})}{x^{1/3}} \to 0$

f(x) è decrescente $\to$ $\cancel{B}$

$f(x)$ è lim. inferiormente $\to \cancel{C} \to \boxed{D}$

n° 3

$$f(x) \begin{cases} \dfrac{e^{x^2}-1}{\log(x^2+1)} & x>0 \\ \dfrac{2x^2+x^3}{(x-1)^2} & x\le 0 \end{cases} \quad \text{in } x=0$$

$f(0) = 0$

$\lim_{x\to 0^-} f(x) = 0$

$f(0) = \lim_{x\to 0^-} f(x) \neq \lim_{x\to 0^+} f(x) \to$ è continua solo a sx $\downarrow$ $\boxed{A}$

$$\lim_{x\to 0^+} \frac{e^{x^2}-1}{\log(x^2+1)} \overset{\left[\frac{0}{0}\right]}{=} \lim_{x\to 0^+} \frac{e^{x^2}-1}{x^2}\cdot\frac{x^2}{\log(x^2+1)} = 1$$

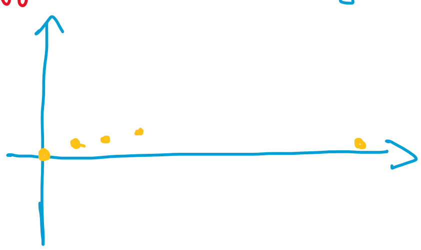
n° 4

$a_n = \dfrac{n^3}{2^n}$ $\quad n\ge 0$

$0 < l < 1$

$a_n \to 0$ $\left(\text{infatti } \dfrac{(n+1)^3}{2^{n+1}}\cdot\dfrac{2^n}{n^3} = \dfrac{(n+1)^3}{2n^3} = \dfrac{1}{2} \Rightarrow a_n \to 0\right)$

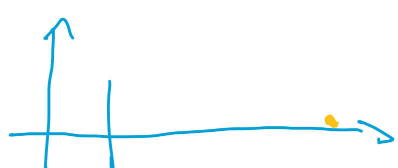
$\cancel{D}$ NO



$\Rightarrow$
- è lim inf $\cancel{A}$ NO
- ha min e max $\boxed{C}$ ✓
- $\cancel{B}$

n° 5

$a_n = \dfrac{\left(\frac{1}{2}\right)^n + (-1)^n}{n\log n}$ $\quad n \ge 2$



$a_n \to \dfrac{0+1}{\infty} = 0 \to \cancel{A} \to \boxed{C}$

$\boxed{n \ge 2} \Rightarrow$ è limitata! $\cancel{B}$

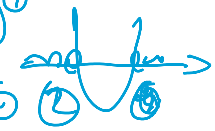
n° 6

$$\begin{array}{r|l} 24 & 2 \\ 12 & 2 \\ 6 & 2 \\ 3 & 3 \end{array}$$

$A = \{n \in \mathbb{N} : n^2 + 2n > 5\}$ max, min ?

$n^2 + 2n - 5 > 0$

$n_{1,2} = \dfrac{-2 \pm \sqrt{4 - 4(-5)}}{2} = \dfrac{-2 \pm 2\sqrt{6}}{2} = \begin{cases} -1 + \sqrt{6} \\ -1 - \sqrt{6} \end{cases}$



$n < -1 - \sqrt{6} \quad \wedge \quad n > -1 + \sqrt{6}$

$\sqrt{6} \simeq 2{,}4$

$A = (-\infty, \cancel{-1-\sqrt{6}}) \cup (-1+\sqrt{6}, +\infty)$ in $\mathbb{R}$

$A = (-1+2, +\infty) \leadsto A = [2 ; +\infty) \Rightarrow$ ha min ma no max

$\hookrightarrow \boxed{B}$ V

n° 7

$\displaystyle\int_e^{e^2} \frac{1 + \log x}{x \log x}\, dx = *$

$F(x) = \displaystyle\int \frac{1 + \log x}{x \log x}\, dx = \int \frac{1}{x} \cdot \frac{1}{\log x}\, dx + \int \frac{\cancel{\log x}}{x \cancel{\log x}}\, dx = \log(\log x) + \log x + c$

$\hookrightarrow D: \frac{1}{x}$

$* = F(e^2) - F(e) = \log(\log e^2) + \log e^2 - (\log(\log e) + \log e)$

$= \log(2) + 2 - (0 + 1) = \boxed{\log(2) + 1} \to \boxed{C}$ V

n° 8 ???

$\displaystyle\lim_{x \to 0} \frac{1}{x^3} \int_0^x \sin(\sin(t^2))\, dt$

$g = 1 \to F = t$

$g = \sin(\sin t^2) \to g' = \cos(\sin t^2) \cdot \cos t^2 \cdot 2t$

$D[\cos(\sin t^2)] = -\sin(\sin t^2) \cdot \cos t^2 \cdot 2t$

$\displaystyle\int \sin(\underbrace{\sin(t^2)}_{D:\, -2t\cos t^2})\, dt = t \sin(\sin t^2) - \int \cos(\sin t^2) \cdot \cos t^2 \cdot 2t^2\, dt$

n° 9

$\displaystyle\int y' = \frac{x}{y}$

$y(1) = \ldots$

$y(2) = 10$

$y' = \frac{x}{y} \to \frac{dy}{dx} = \frac{x}{y} \to \int y\,dy = \int x\,dx$ $\boxed{y = 10!}$

$\frac{y^2}{2} = \frac{x^2}{2} \rightsquigarrow y^2 = x^2 + c \to \boxed{y = \pm\sqrt{x^2+c}} \to y = +\sqrt{x^2+c}$

$\boxed{y(2)=10} \to \sqrt{4+c} = 10$

$c = 100 - 4 \Rightarrow \boxed{c = 96}$

$\boxed{y(x) = \sqrt{x^2+96}} \Rightarrow \boxed{y(1) = \sqrt{97}} \to \boxed{A}$ V

n° 10

$$\begin{cases} y'' + 5y' - \frac{11}{4} y = 0 \\ y(0) = 0 \\ y'(0) = 3 \end{cases}$$

$y(2) = \dots$

$\lambda^2 + 5\lambda - \frac{11}{4} = 0 \to \Delta = 25 - 4\left(-\frac{11}{4}\right) = 25 + 11 = 36$

$\lambda_{1,2} = \frac{-5 \pm \sqrt{36}}{2} = \frac{-5 \pm 6}{2}$ → $\frac{1}{2}$, $-\frac{11}{2}$

- $y(x) = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x} = \boxed{c_1 e^{\frac{1}{2}x} + c_2 e^{-\frac{11}{2}x}}$

$y'(x) = \boxed{\frac{1}{2} c_1 e^{\frac{1}{2}x} - \frac{11}{2} c_2 e^{-\frac{11}{2}x}}$

$\boxed{y(0)=0} \Rightarrow \begin{cases} c_1 + c_2 = 0 \\ \frac{1}{2}c_1 - \frac{11}{2}c_2 = 3 \end{cases}$ $\begin{cases} c_1 = -c_2 \\ -\frac{1}{2}c_2 - \frac{11}{2}c_2 = 3 \end{cases}$ $\begin{cases} \sim \\ -6c_2 = 3 \end{cases}$

$\boxed{y'(0)=3} \Rightarrow$

$$\begin{cases} c_1 = \frac{1}{2} \\ c_2 = -\frac{1}{2} \end{cases} \rightsquigarrow y(x) = \frac{1}{2}e^{\frac{1}{2}x} - \frac{1}{2}e^{-\frac{11}{2}x}$$

$\Rightarrow y(2) = \frac{1}{2}e - \frac{1}{2}e^{-11} = \boxed{\frac{e^{12} - 1}{2e^{11}}}$ V

Scritto

$$f(x) = \frac{x \log x}{(1-\log x)^2}$$

- dom: $\begin{cases} x > 0 \\ 1 - \log x \neq 0 \end{cases}$ $\begin{cases} x > 0 \\ \log x \neq \log e^1 \end{cases}$ $\begin{cases} x > 0 \\ x \neq e \end{cases}$ dom(f) ✓ $\boxed{(0, e) \cup (e, +\infty)}$

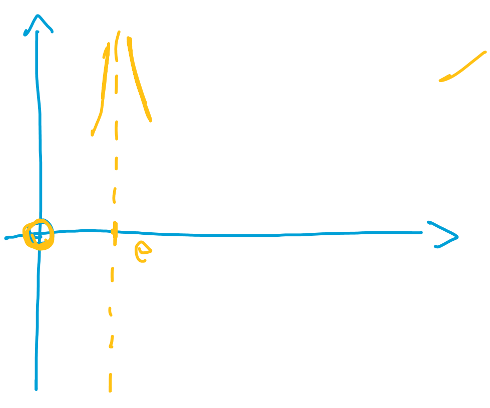
- $\lim_{x \to 0^+} \frac{x \log x}{(1-\log x)^2} = \frac{[0 \cdot (-\infty)]}{+\infty}$

$\lim_{x \to 0^+} \frac{\log x}{(1-\log x)} \cdot \frac{x}{1-\log x} = \frac{0}{-\infty} = 0$ → 1

- $\lim_{x \to e^{\pm}} \frac{x \log x}{(1-\log x)^2} = \frac{e}{(1-\cancel{1})^2} = \frac{e}{0^+} = +\infty$ $\boxed{\underline{x=e} \text{ verticale}}$

- $\lim_{x \to +\infty} \frac{x \log x}{(1-\log x)^2} = \lim_{x \to +\infty} \frac{x \cancel{\log x}}{\log^2 x} = +\infty$ 1

- $\lim_{x \to +\infty} \frac{\cancel{x} \log x}{(1-\log x)^2} \cdot \frac{1}{\cancel{x}} = \lim_{x \to +\infty} \frac{1}{\log x} = 0$ → $\boxed{\underline{\text{no obliquo}}}$



$\sup(f) = +\infty \to \nexists \max(f)$

se $\nexists\, x_0.\ f(x_0) < 0 \Rightarrow \inf(f) = 0$

Segno $f(x)$ in dom(f)

$\frac{x \log x}{(1-\log x)^2} < 0 \iff x \log x < 0 \iff \log x < 0$

$(1-\log x)^2 > 0\ \forall x \in \text{dom}(f)$; $x > 0\ \forall x \in \text{dom}(f)$

$\iff \log x < \log e^0 \iff 0 < x < 1 \Rightarrow \boxed{0 < x < 1}$

$\iff \log x < \log e \iff x < 1 \rightsquigarrow \boxed{0 < x < 1}$

- $f(x)$ ha min, troviamolo usando la sua derivata

$$f'(x) = \frac{f'g - fg'}{g^2} =$$

$$= \frac{\left(\log x + \cancel{x}\frac{1}{\cancel{x}}\right)(1-\log x)^2 + 2\cancel{x}\log x(1-\log x)\cancel{\frac{1}{x}}}{(1-\log x)^4}$$

$$= \frac{\cancel{(1-\log x)}\left((\log x+1)(1-\log x)+2\log x\right)}{(1-\log x)^3}$$

$$= \frac{\cancel{\log x} - \log^2 x + 1 \cancel{-\log x} + 2\log x}{(1-\log x)^3} = \boxed{\frac{1-\log^2 x + 2\log x}{(1-\log x)^3}}$$

Studio $f'(x)$ ($\mathrm{dom}\, f' = \mathrm{dom}\, f$) $\mathrm{dom}\, f' = (0, e) \cup (e, +\infty)$

$f'(x) > 0$

$N > 0$: $1 - \log^2 x + 2\log x > 0 \iff \log^2 x - 2\log x - 1 < 0$ $\quad t = \log x$

$\iff t^2 - 2t - 1 < 0 \quad t_{1,2} = \frac{+2 \pm \sqrt{8}}{2}$ $\rightarrow 1+\sqrt{2}$ t_2, $\rightarrow 1-\sqrt{2}$ t_1

t_1 t_2 $\Rightarrow 1-\sqrt{2} < t < 1+\sqrt{2} \iff \boxed{e^{1-\sqrt{2}} < x < e^{1+\sqrt{2}}}$

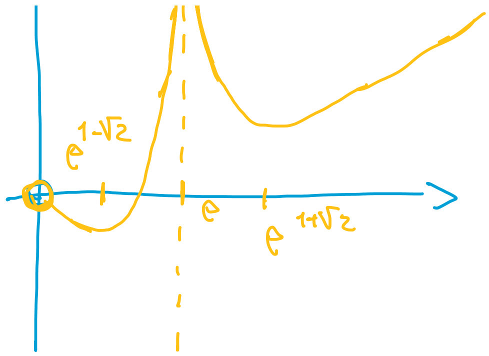
$D > 0$: $(1-\log x)^3 > 0 \iff 1 - \log x > 0 \iff \log x < 1$

$x < e$

	0		$e^{1-\sqrt{2}}$		e		$e^{1+\sqrt{2}}$	
N		$-$	0	$+$		$+$	0	$-$
D		$+$		$+$		$-$		$-$
S		$-$		$+$		$-$		$+$
		$\searrow$		$\nearrow$		$\searrow$		$\nearrow$

$$\begin{cases} x_1 = e^{1+\sqrt{2}} \\ x_2 = e^{1-\sqrt{2}} \end{cases}$$ punti di min locale di $f(x)$

$$\boxed{\min(f) = f(x_2) = \frac{e^{1-\sqrt{2}}(1-\sqrt{2})}{2}}$$



n° 2

$$\int_0^1 \frac{\arctan(e^x)}{e^x+e^{-x}}\,dx = (*) \qquad \boxed{t = e^x \rightsquigarrow x = \log t \rightsquigarrow dx = \frac{1}{t}dt}$$

$$F(x) = \int \frac{\arctan(e^{\log t})}{t+\frac{1}{t}}\frac{1}{t}\,dt = \int \frac{\arctan(t)}{t^2+1}\,dt \qquad \boxed{\begin{array}{l} z = \arctan t \\ dz = \frac{1}{1+t^2}dt\end{array}}$$

$$\int z\,dz = \frac{z^2}{2}+c = \frac{(\arctan t)^2}{2}+c = \boxed{\frac{(\arctan e^x)^2}{2}+c}$$

$$F(1)-F(0) = \frac{(\arctan e)^2}{2} - \frac{(\arctan 1)^2}{2} = \boxed{\frac{(\arctan e)^2}{2} - \frac{\pi^2}{32}}$$

n° 3

$$\begin{cases} y' + \frac{y}{\sqrt{x}} = 1 \\ y(1) = \frac{e^2+2}{2e^2}\end{cases} \qquad \min(y(x)) \text{ per } x \in (0,+\infty)$$

$$y' = \underbrace{-\frac{1}{\sqrt{x}}}_{a}y + \underbrace{1}_{b}$$

- $A(x) = -\int x^{-\frac{1}{2}}dx = -2x^{\frac{1}{2}} = \boxed{-2\sqrt{x}}$

$$y(x) = e^{A(x)}\left(\int b(x)e^{-A(x)}dx + c\right)$$

$$y(x) = e^{-2\sqrt{x}}\left(\boxed{\int e^{2\sqrt{x}}}^{*} dx + c\right) =$$

$$* \int e^{2\sqrt{x}}dx \qquad t = \sqrt{x} \rightsquigarrow \cancel{dt = \frac{1}{2\sqrt{x}}}$$

$$x = t^2 \rightsquigarrow dx = 2t\,dt$$

Per parti:

$f = e^{2t}$ $\quad F = \frac{1}{2}\int e^{2t}\cdot 2 = \frac{e^{2t}}{2}$

$g = 2t$ $\quad g' = 2$

$$\int e^{2t}\,2t\,dt = e^{2t}t - \int e^{2t}dt = e^{2t} - \frac{e^{2t}}{2} = \boxed{\sqrt{x}e^{2\sqrt{x}} - \frac{e^{2\sqrt{x}}}{2}}$$

$$y(x) = e^{-2\sqrt{x}}\left(\sqrt{x}e^{2\sqrt{x}} - \frac{e^{2\sqrt{x}}}{2} + c\right) = \sqrt{x}e^{0} - \frac{e^{0}}{2} + e^{-2\sqrt{x}}c = \sqrt{x} - \frac{1}{2} + e^{-2\sqrt{x}}c$$

$$\boxed{y(x) = \sqrt{x} - \frac{1}{2} + ce^{-2\sqrt{x}}}$$

$$\boxed{y(1) = \frac{e^2+2}{2e^2}} \Rightarrow 1 - \frac{1}{2} + ce^{-2} = \frac{e^2+2}{2e^2} \iff \frac{1}{2} + ce^{-2} = \frac{e^2+2}{2e^2}$$

$$\iff \frac{e^2+2c}{2e^2} = \frac{e^2+2}{2e^2} \iff \boxed{c = 1}$$

$$\boxed{y(x) = -\frac{1}{2} + \sqrt{x} + e^{-2\sqrt{x}}}$$

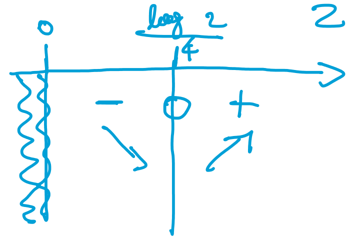
$$y'(x) = \frac{1}{2\sqrt{x}} + \left(-\cancel{2}\cdot\frac{1}{\cancel{2}\sqrt{x}}\right)\cdot e^{-2\sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}} - \frac{e^{-2\sqrt{x}}}{\sqrt{x}}}$$

$$\boxed{y'(x) \geq 0}$$

$$\frac{1}{2} \geq e^{-2\sqrt{x}} \iff \log\frac{1}{2} \geq -2\sqrt{x} \iff +\frac{\log 2}{2} \leq \sqrt{x}$$

$$\boxed{x \geq \frac{\log^2 2}{4}}$$

2



$x_0 = \frac{\log^2 2}{4}$ punto di minimo

$$y(x_0) = -\frac{1}{2} + \frac{\log 2}{2} + e^{-\cancel{2}\frac{\log 2}{\cancel{2}}} = -\frac{1}{2} + \frac{\log 2}{2} + e^{\log 2^{-1}}$$

$$= -\frac{1}{2} + \frac{\log 2}{2} + \frac{1}{2} = \boxed{\frac{\log 2}{2}}$$