Report on Last week

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May 3, 2011

1 Lists to do

- Compare $\hat{v}_H \equiv \bar{v_H} + I_h^H(v^* - \bar{v_h})$ with the solution of shifted coarse problem, and expect $\hat{v_H}$ should satisfy the optimality test for the shifted coarse problem.
- With known binding subscripts $\{i_1, i_2, \dots\}$, modify $\hat{b}$ s.t.$f(x) = \frac{1}{2}x^T Ax - \hat{b}^T x$ is an unconstrained problem where $\hat{b} = b - \nabla c^T \lambda$, then test this problem both by OPT and MG/OPT to see if it's working well.

2 Work have been done on one-side bound

- Objective problem: $min\, f(x) = \frac{1}{2}x^T Ax - b^T x$ s.t. $-0.01 - x \leq 0$
- Result: From figure 1, MG/OPT shows enough benefit than OPT that can converge to error of magnitude 10^{-2}. Afterwards, the performance is stalled due to the full weighting downdate matrix will give the already binding fine point a negative direction so that variable will be infeasible.
  - 1st approach tried to resolve this issue: Downdate the variable simply by injection rather than full weighting, in this way the convergence can be improved but still couldn't show obvious benefit than opt to completely converge
  - 2nd approach is to apply the bound constraint introduced in paper from Toint and will also sligly improve the convergence although it is still not quick enough.
  - 3rd approach is to fix the binding without moving in the line search of MG/OPT with $\alpha_0 = 1$. But no α can satisfy $f(x + \alpha p) < f(x)$ even though $e^T Gvmg < 0$
- Question: The tests we did on our last meeting to compare the step to solution and search direction will become very different in only 3 cycles (actually comparing to step, search direction is too tiny) is I modified the way to approximate the lagrangian multiplier λ by simply pick the nonzero entries of λ as the components of variables which are binding in

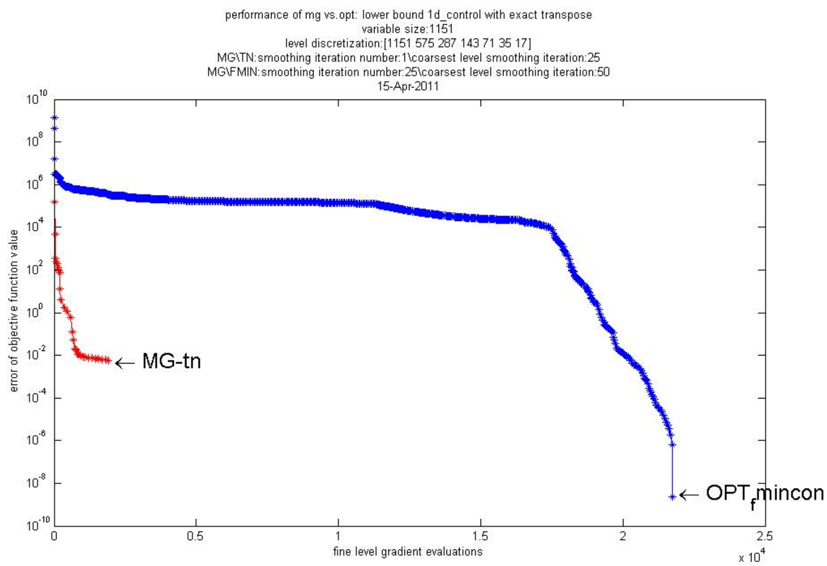

Figure 1: Test on Coarse approximation

'sfun'. Instead, if I use the entries as 'ipivot' assigned from TN, then it will show more similarity between step to solution and search direction for all cycles until error of 10^{-2} magnitude. So what's the real difference between these two ways.

3 2-side bound constrained problem

- Objective problem:

$$minf(x) = \frac{1}{2}x^T Ax - b^T x \, s.t. \left\{ \begin{array}{c} 1 - x \leq 0 \\ x - 5 \leq 0 \end{array} \right.$$

- Compare the original coarse solution with solution of shifted problem. The formula I used is to compare $v_s^* - \bar{v_H}$ with $v_H^* - \bar{v_H}$.
  - Result: it turns out that this compare has the same shape plot as the comparing we did before which is between the step to solution with search direction on the fine level. i.e. As applying more loop of MG/OPT, the search directions for both fine and coarse level become much more slight comparing to step to solution.
  - Conclustion: The coarse level approximation couldn't give a enough step for fine level to converge, that's why MG/OPT on the fine level didn't make progress after few cycles. Motivatived by this fact, I

found out that the primal reason for the stalling on fine level is because TN couldn't provide a positive step length α_0 to update $\bar{v_h}$ based on the computation of α_0 which is chosen as the minimum of feasible search direction p get from Conjugate-Gradient. Thus, once the variable hits the bounds, $\alpha_0 = 0$. Therefore, the progress for $\bar{v_h}$ can only be obtained from coarse level. i.e. $k_1 = k_2 = 0$.

   * Since $I_h^H I_H^h v_H \approx v_H$. In another word, the initial point for coarse shifted problem will be close to its solution and in turns bring a ting search direction with no moving on fine level.
   * Because of this reason, I tried another approach for α_0 that fix $\alpha_0 = 1$ and during the line search process, fix those binding points without moving to guarantee the feasibility, more progress can be obtained then. But couldn't completely converge yet due to the same issue happened for 1-side bound case about the full weighting downdate.

- [1] Test if $\hat{v_H} \equiv \bar{v_H} + I_h^H(v^* - \bar{v_h}) \approx v_s^*$ where v_s^* denotes the solution of shifted coarse problem.
  - Test problem discretization level: [257, 128]
  - TN line search: tried both version of α_0
  - When MG/OPT works well, then

$$v^* - \bar{v_h} \approx e_h = I_H^h e_H = I_H^h(v_H^+ - \bar{v_H})$$

$$I_h^H(v^* - \bar{v_h}) \approx I_h^H I_H^h e_H = I_h^H I_H^h(v_H^+ - \bar{v_H})$$

    Therefore

$$v_s^* \approx \bar{v_H} + (I_h^H I_H^h)^{-1} I_h^H(v^* - \bar{v_h})$$

    or roughly

$$v_s^* \approx \bar{v_H} + I_h^H(v^* - \bar{v_h})$$

  - Result: They perfectly match each other, so we can have the conclustion that coarse approximation is right, shown in Figure 2. From another view, the search direction from coarse grid will be very tiny.

- I also tried MG/FMINCON on this problem, the convergence is stalled too, due to the fact of inner fmincon which will increase the function value even at all feasible point. I still need to understand more on this.

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[1]**Note: All the tests have been done upon two different version of line search in TN:**

- **1st : α_0 is chose as the minimum of feasible search direction p get from Conjugate-Gradient**
- **2nd: Fix $\alpha_0 = 1$ and during the line search process, fix those binding points without moving to guarantee the feasibility**

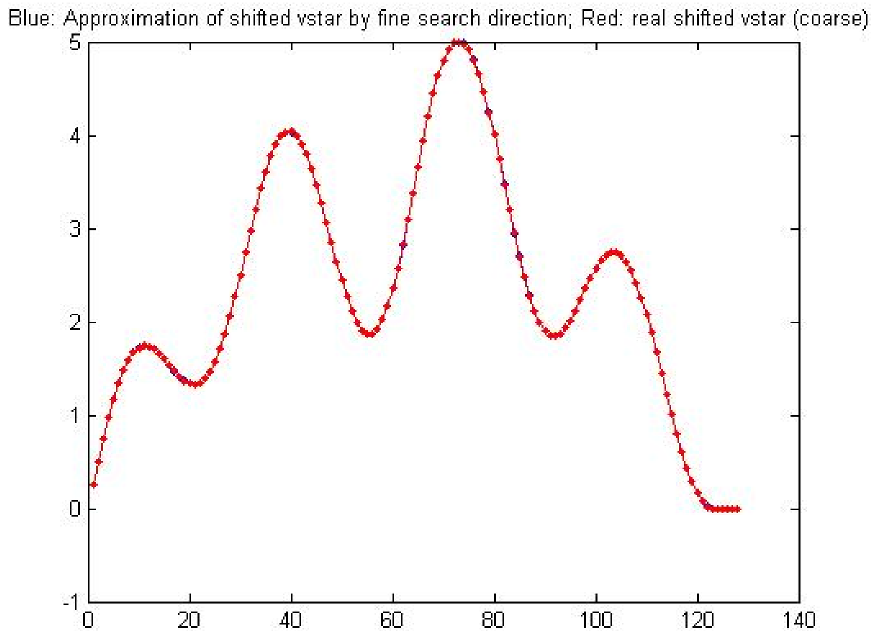

Figure 2: Test on Coarse approximation

- Then the next test is to treat constrained problem as unconstrained easily by setting $\hat{b}$ s.t. $f(x) = \frac{1}{2}x^T Ax - \hat{b}^T x$ where $\hat{b} = b - \nabla c^T \lambda$ with the known binding subscripts $\{i_1, i_2, \dots\}$.
  - test discretization level: $[17, 8]$
  - The performance of MG/OPT is just as expected for unconstraint problem.