Result figures

May 4, 2011

In this paper, I will do every test based on combinations of two different line search seperately.

- 1st approach of line search: choose α_0 as the minimum of feasible steps based on every variable, denoted as $lin1$
- 2nd approach of line search: Fix $\alpha_0 = 1$, then apply Armijo, denoted as $lin2$
- 3rd approach of line search: Fix $\alpha_0 = 1$, then apply Armijo. Also fix the binding points without moving, denoted as $lin3$.

1 TN: lin1; MG/OPT: lin3

In Fig 3, figure 7a illurstrates how close among the exact original prolem solution (green), solution of shifted problem (red), approximation from fine solution (blue); figure 7b is showing the error with these three solution.

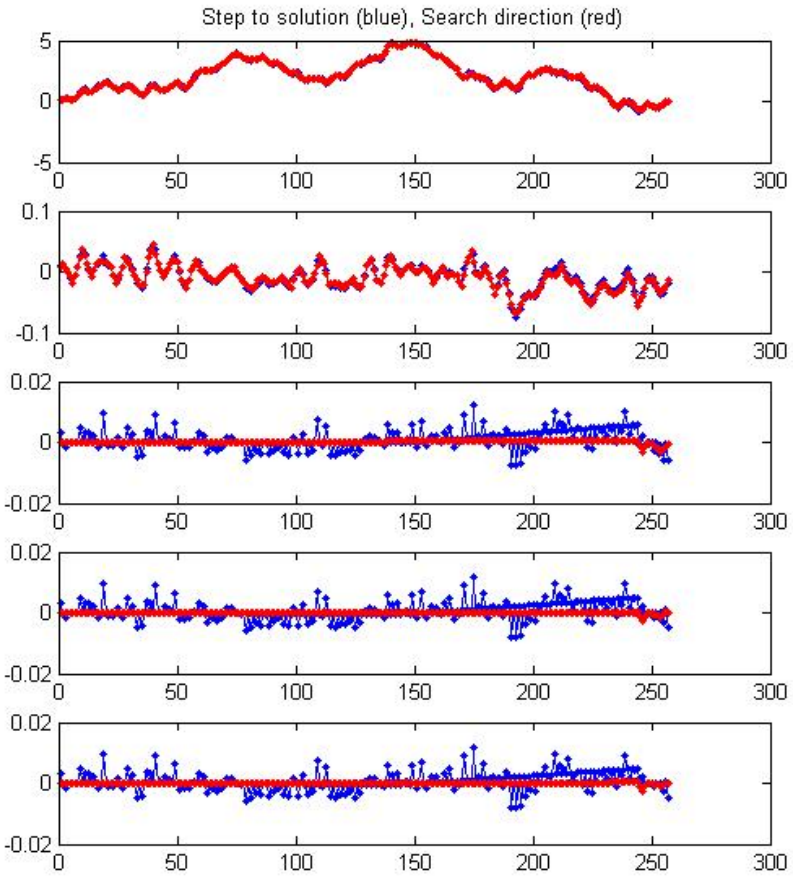


Figure 1: 2-side bounds: TN: lin1; MG/OPT: lin3; Compare step to solution with search direction on fine level: discretization level: $[257, 128]$

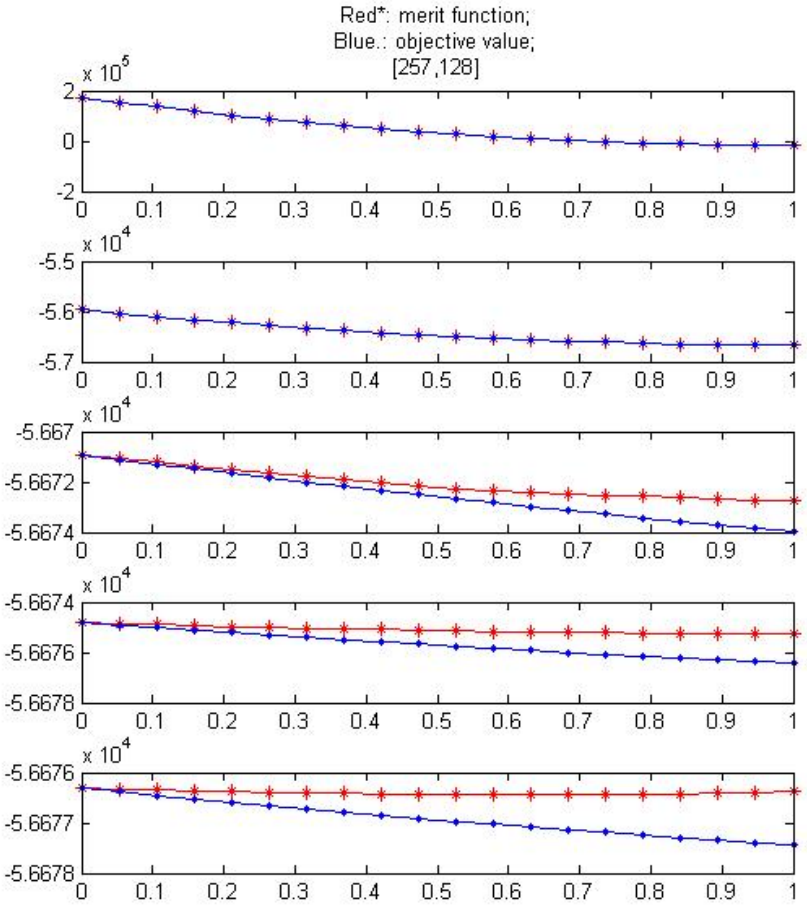
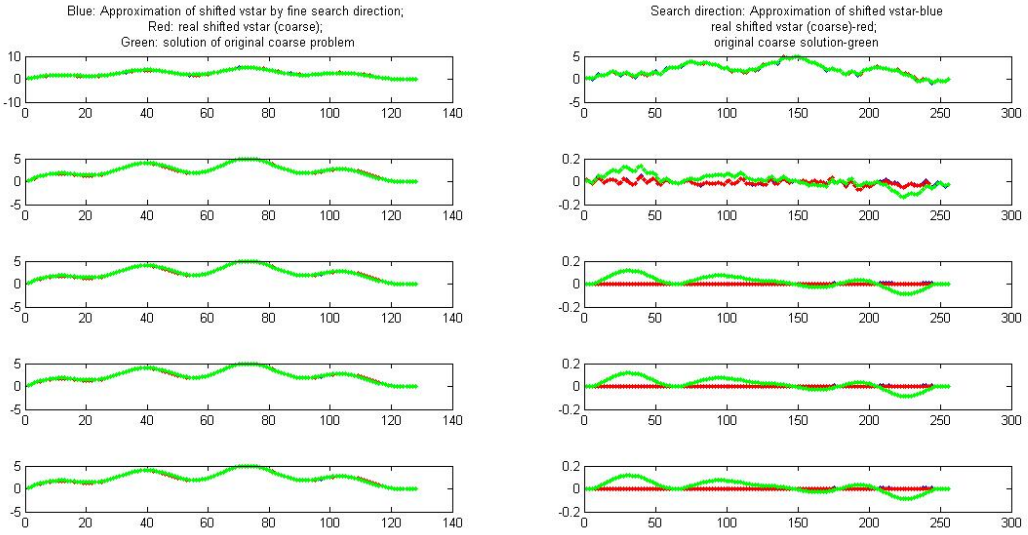


Figure 2: 2-side bounds: TN: lin1; MG/OPT: lin3; penalty $\rho = 1$, merit and objective function based on different α



(a) solution (b) search direction

Figure 3: 2-side bounds: TN: lin1; MG/OPT: lin3; Compare solution in coarse grid: discretization level: $[257, 128]$

Output of 5th cycle of MG/OPT:

```
In  mgrid: n = 257
  it      nf      cg           f            |g|
   0       1       0   -5.66763126e+004   1.4e+003
   1       2       2   -5.66763126e+004   1.4e+003

###################################
### NO EXTRA BOUND CONSTRAINT ###
###################################

In  mgrid: n = 128
  it      nf      cg           f            |g|
   0       1       0   -2.82421049e+004   4.1e+002
   1       2       4   -2.82423697e+004   4.1e+002
   2       3      54   -2.82424262e+004   2.8e+002
   3       4      57   -2.82424520e+004   1.3e+002
   4       5      59   -2.82425721e+004   7.7e+001
   5       6      61   -2.82426652e+004   6.3e+000
   6       7      63   -2.82426666e+004   1.5e+000
   7       8      67   -2.82426668e+004   5.5e-001
   8       9      76   -2.82426668e+004   4.3e-001
```

```
   9     10     87   -2.82426669e+004   5.1e-001
  10     11    130   -2.82426669e+004   5.5e-001
  11     12    180   -2.82426669e+004   4.7e-001
  12     13    228   -2.82426669e+004   4.0e-001
  13     14    250   -2.82426669e+004   4.2e-001
  14     15    270   -2.82426669e+004   4.3e-001
  15     16    305   -2.82426669e+004   2.0e-001
  16     17    338   -2.82426669e+004   2.5e-001
  17     18    341   -2.82426669e+004   2.5e-001
  18     19    364   -2.82426669e+004   6.5e-002
  19     20    367   -2.82426669e+004   6.1e-002
  20     21    380   -2.82426669e+004   5.3e-002
  21     22    384   -2.82426669e+004   5.5e-002
  22     23    397   -2.82426669e+004   8.2e-002
  23     24    409   -2.82426669e+004   4.1e-002
  24     25    419   -2.82426669e+004   5.0e-002
  25     26    432   -2.82426669e+004   4.1e-002
  26     27    445   -2.82426669e+004   3.7e-002
  27     28    448   -2.82426669e+004   2.9e-002
  28     29    460   -2.82426669e+004   2.5e-002
  29     30    474   -2.82426669e+004   1.2e-002
  30     31    491   -2.82426669e+004   8.1e-003
  31     32    505   -2.82426669e+004   2.2e-003
  32     33    524   -2.82426669e+004   1.8e-003
  33     34    535   -2.82426669e+004   1.2e-003
  34     35    564   -2.82426669e+004   6.4e-004
  35     36    589   -2.82426669e+004   5.6e-004
  36     37    592   -2.82426669e+004   4.0e-004
MG/Opt line search: alpha =  1.00000000e+000
In  mgrid: n = 257
  it     nf     cg           f           |g|
   0      1      0   -5.66769813e+004   1.4e+003
   1      2      2   -5.66769813e+004   1.4e+003

-------------------
Optimization costs per grid
-------------------
N:        257   128
-------------------
NF(it):    10   196
NF(nf):    20   215
NF(cg):    20  3241
-------------------
```

2 TN: lin3; MG/OPT:lin3

Output of 1st cycle of MG/OPT working on $[257, 128]$**:**

```
In  mgrid: n = 257
  it     nf     cg           f           |g|
   0      1      0    4.62493224e+006   2.9e+005
   1      2      2    1.73085726e+005   2.3e+004

###################################
### NO EXTRA BOUND CONSTRAINT ###
###################################

In  mgrid: n = 128
  it     nf     cg           f           |g|
   0      1      0    6.40004590e+004   1.4e+004
   1      2      2   -1.71052746e+003   2.6e+003
   2      3     10   -1.81946179e+004   3.5e+003
   3      4     21   -1.83931932e+004   3.6e+003
   4      5     32   -1.93133518e+004   3.5e+003
   5      7     34   -1.93683526e+004   3.5e+003
   6      8     41   -1.99913485e+004   4.0e+003
   7      9     48   -2.07860842e+004   3.3e+003
   8     10     55   -2.14226947e+004   3.4e+003
   9     11     61   -2.18994247e+004   3.2e+003
  10     12     65   -2.19230171e+004   3.8e+003
  11     13     75   -2.25369984e+004   3.2e+003
  12     14     81   -2.27217905e+004   3.2e+003
  13     15     86   -2.29434215e+004   3.2e+003
  14     16     90   -2.30641974e+004   3.0e+003
  15     17     94   -2.31568892e+004   3.0e+003
  16     18     98   -2.32879819e+004   3.0e+003
alpha0 =
     1
Error in Line Search (lmqnbcm.m)
    ncg1   = 4
    alpha  =   0.00000000
    alpha0 =   1.00000000
    g'p    = -2.9169e+003
    |g|     =  1.0321e+004
    |p|     =  5.9334e-001
MG/Opt line search: alpha =  1.00000000e+000
In  mgrid: n = 257
  it     nf     cg           f           |g|
   0      1      0   -1.35218465e+004   2.2e+004
   1      2      2   -4.28115105e+004   3.6e+003
```

```
------------------
Optimization costs per grid
------------------
N:        257   128
------------------
NF(it):     2    16
NF(nf):     4    18
NF(cg):     4   102
------------------
```

Output of 5th cycle of MG/OPT:

```
In  mgrid: n = 257
  it     nf     cg           f           |g|
   0      1      0   -5.68043929e+004   7.0e+001
   1      2      4   -5.68045459e+004   2.8e+001

###################################
### NO EXTRA BOUND CONSTRAINT ###
###################################

In  mgrid: n = 128
  it     nf     cg           f           |g|
   0      1      0   -2.82490033e+004   3.9e+002
   1     10     50   -2.82490566e+004   4.6e+002
alpha0 =
     1
Error in Line Search (lmqnbcm.m)
    ncg1   = 2
    alpha  =   0.00000000
    alpha0 =   1.00000000
    g'p    = -2.4109e+002
    |g|     =  4.8517e+002
    |p|     =  3.5303e+000
MG/Opt line search: alpha =  1.25000000e-001
In  mgrid: n = 257
  it     nf     cg           f           |g|
   0      1      0   -5.68045463e+004   2.8e+001
   1      2      7   -5.68046214e+004   1.3e+001

------------------
Optimization costs per grid
------------------
N:        257   128
------------------
NF(it):    10    80
```

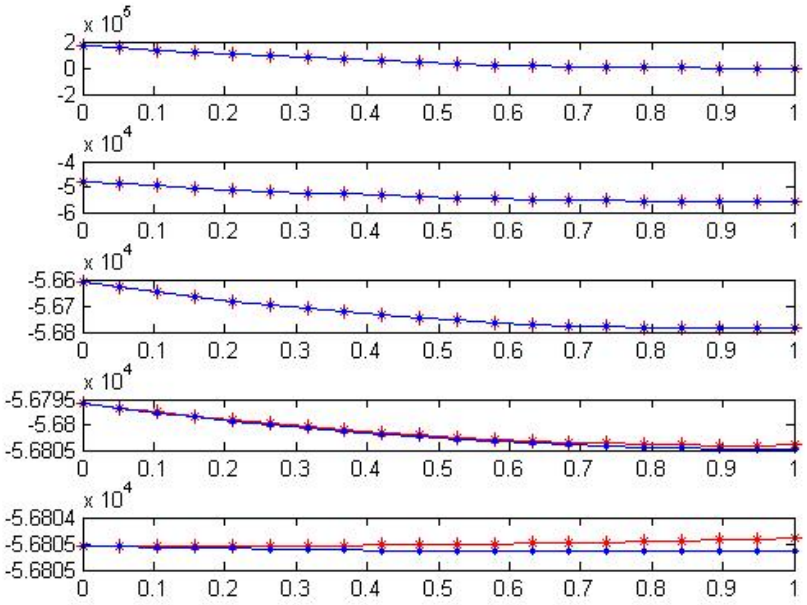


Figure 4: 2-side bounds: TN: lin3; MG/OPT: lin3; penalty $\rho = 1$, merit and objective function based on different α

```
NF(nf):     20    108
NF(cg):     56    541
-------------------
```

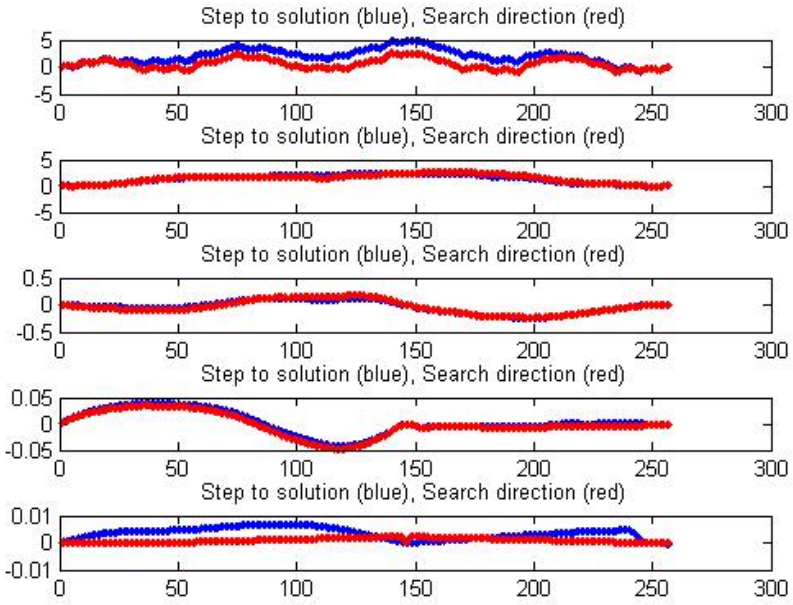
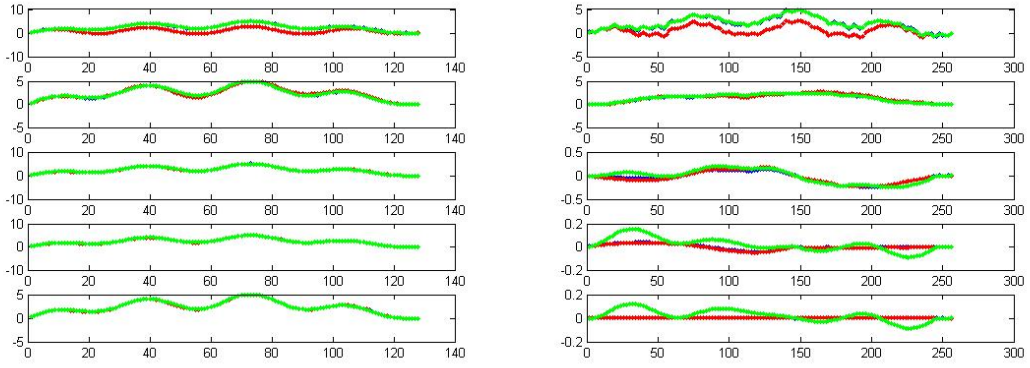


Figure 6: 2-side bounds: TN: lin3; MG/OPT: lin3; Compare step to solution with search direction on fine level: discretization level: $[257, 128]$



(a) solution

(b) search direction

Figure 5: 2-side bounds: TN: lin3; MG/OPT: lin3; Compare solution in coarse grid: discretization level: $[257, 128]$

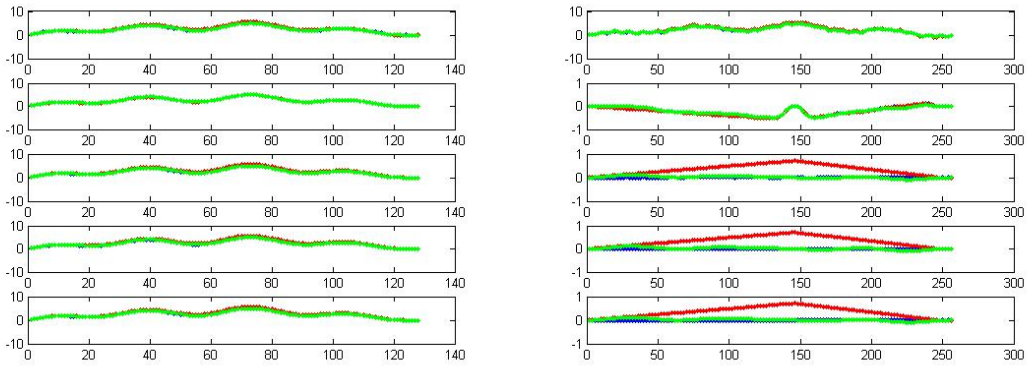
3 TN: lin2; MG/OPT: lin3

Output of 5th cycle:

```
In  mgrid: n = 257
  it     nf     cg           f           |g|
   0      1      0   -5.68046940e+004   8.6e+000
   1      2     21   -5.68047254e+004   4.4e+000

###################################
### NO EXTRA BOUND CONSTRAINT ###
###################################

In  mgrid: n = 128
  it     nf     cg           f           |g|
   0      1      0   -2.82491963e+004   4.0e+002
   1      2     50   -2.83680035e+004   2.0e+002
   2      3     75   -2.83862763e+004   6.5e+001
   3      4    101   -2.83912259e+004   5.1e+001
   4      5    117   -2.83916011e+004   4.6e+001
   5      6    134   -2.83916553e+004   4.3e+001
   6      7    170   -2.83917090e+004   6.0e+000
   7      8    211   -2.83917212e+004   3.6e+000
   8      9    240   -2.83917252e+004   3.7e+000
   9     10    262   -2.83917278e+004   6.1e+000
  10     11    282   -2.83917308e+004   4.1e+000
  11     12    312   -2.83917363e+004   2.0e+000
  12     13    325   -2.83917371e+004   1.6e+000
  13     14    347   -2.83917378e+004   1.1e+000
  14     15    361   -2.83917382e+004   1.0e+000
  15     16    374   -2.83917386e+004   6.7e-001
  16     17    394   -2.83917390e+004   1.2e+000
  17     18    409   -2.83917393e+004   4.6e-001
  18     19    426   -2.83917395e+004   4.1e-001
  19     20    440   -2.83917395e+004   2.6e-001
  20     21    474   -2.83917398e+004   3.7e-001
  21     22    498   -2.83917399e+004   2.5e-001
  22     23    519   -2.83917400e+004   1.4e-001
  23     24    569   -2.83917401e+004   2.2e-001
  24     25    604   -2.83917401e+004   1.0e-001
  25     26    630   -2.83917401e+004   6.3e-002
  26     27    650   -2.83917401e+004   4.7e-002
  27     28    668   -2.83917401e+004   2.7e-002
  28     29    680   -2.83917401e+004   7.1e-003
  29     30    704   -2.83917401e+004   9.6e-003
  30     31    724   -2.83917401e+004   3.6e-003
```

```
  31     32    740   -2.83917401e+004   1.8e-003
  32     33    751   -2.83917401e+004   1.1e-003
  33     34    770   -2.83917401e+004   1.3e-003
  34     35    785   -2.83917401e+004   4.4e-004
In  mgrid: n = 257
  it     nf     cg           f            |g|
   0      1      0   -5.68047254e+004   4.4e+000
   1      2     20   -5.68047317e+004   1.4e+000

-------------------
Optimization costs per grid
-------------------
N:        257   128
-------------------
NF(it):    10   185
NF(nf):    20   196
NF(cg):    73  3781
-------------------
```

As the above output shown, we can see that coarse level approximation couldn't provide a descent direction to fine level even though $e' * Gvmg < 0$.



(a) solution (b) search direction

Figure 7: 2-side bounds: TN: lin2; MG/OPT: lin3; Compare solution in coarse grid: discretization level: $[257, 128]$

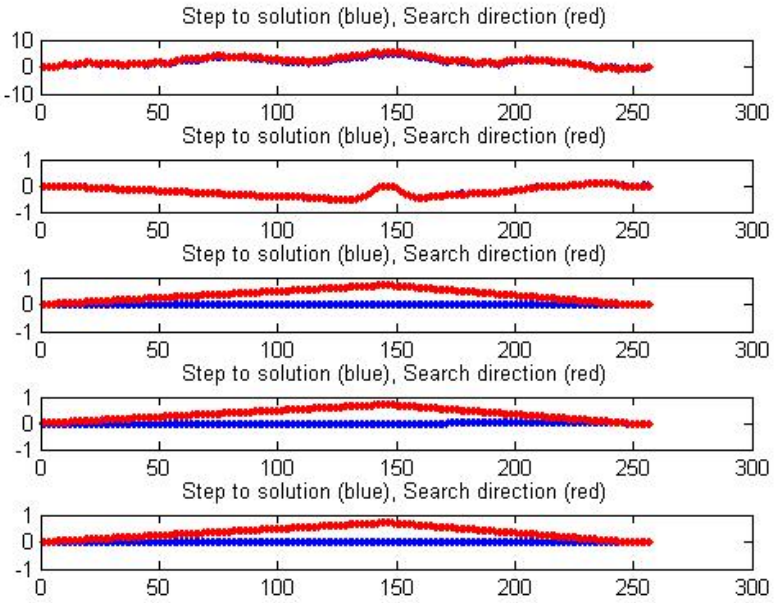


Figure 8: 2-side bounds: TN: lin2; MG/OPT: lin3; Compare step to solution with search direction on fine level: discretization level: $[257, 128]$

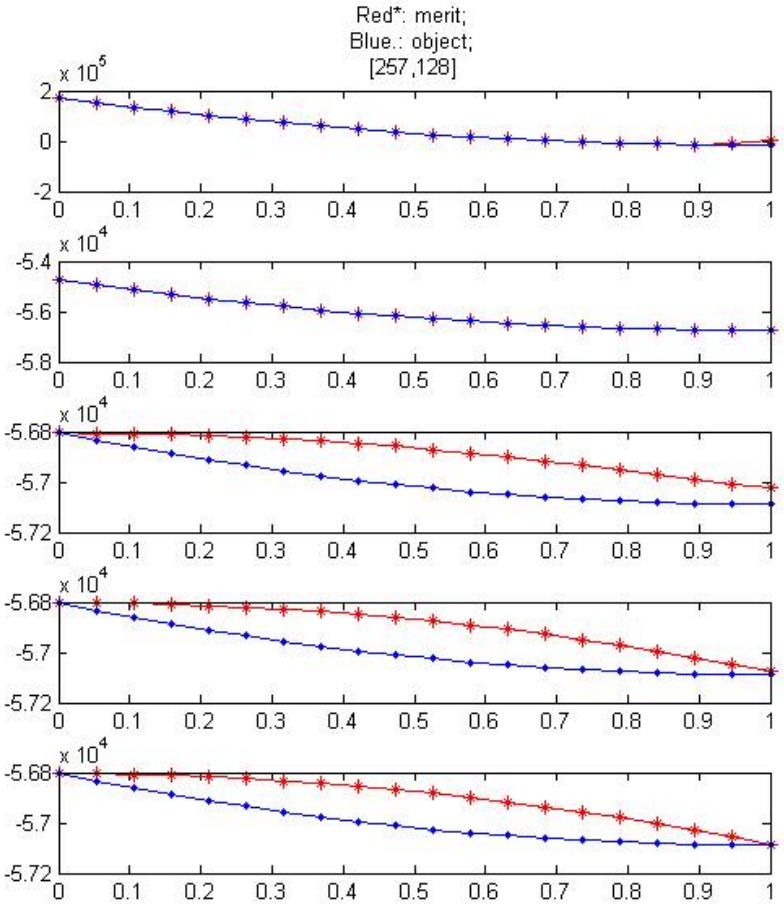
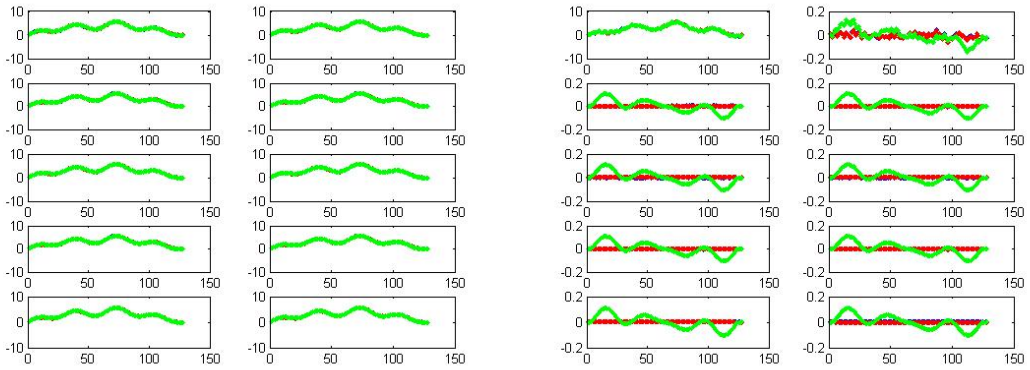


Figure 9: 2-side bounds: TN: lin2; MG/OPT: lin3; penalty $\rho = 1$, merit and objective function based on different α

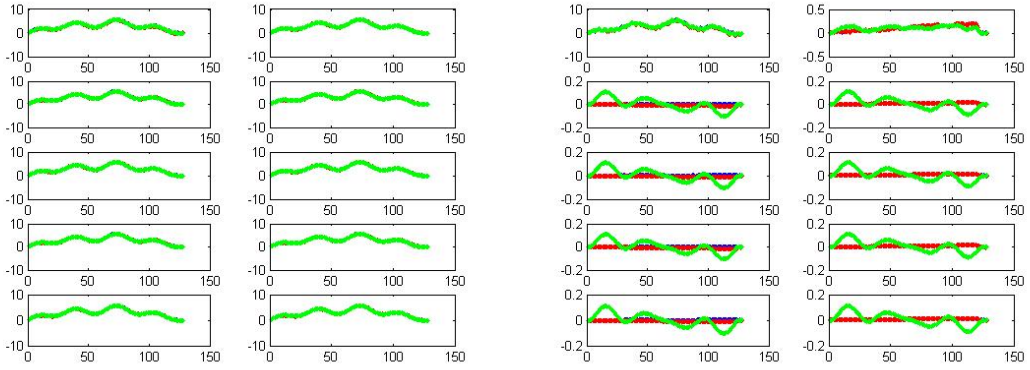
4 1-side bound: TN: lin1; MG/OPT: lin2



(a) solution (b) search direction

Figure 10: 1-side bound constraint: TN: lin1; MG/OPT: lin2; Compare solution in coarse grid: discretization level: $[257, 128]$

5 1-side bound: TN: lin2; MG/OPT: lin2



(a) solution (b) search direction

Figure 13: 1-side bound constraint: TN: lin2; MG/OPT: lin2; Compare solution in coarse grid: discretization level: $[257, 128]$

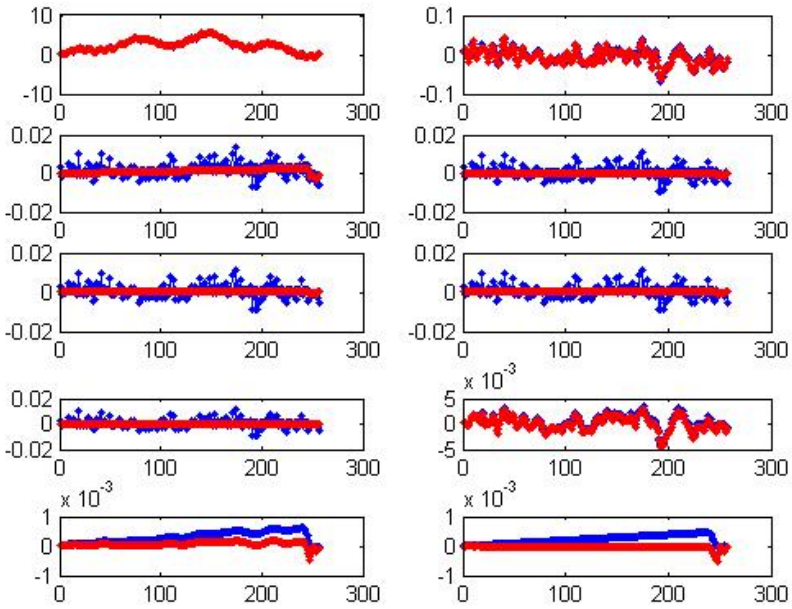


Figure 11: 1-side bound constraint: TN: lin1; MG/OPT: lin2; Compare step to solution with search direction on fine level: discretization level: $[257, 128]$

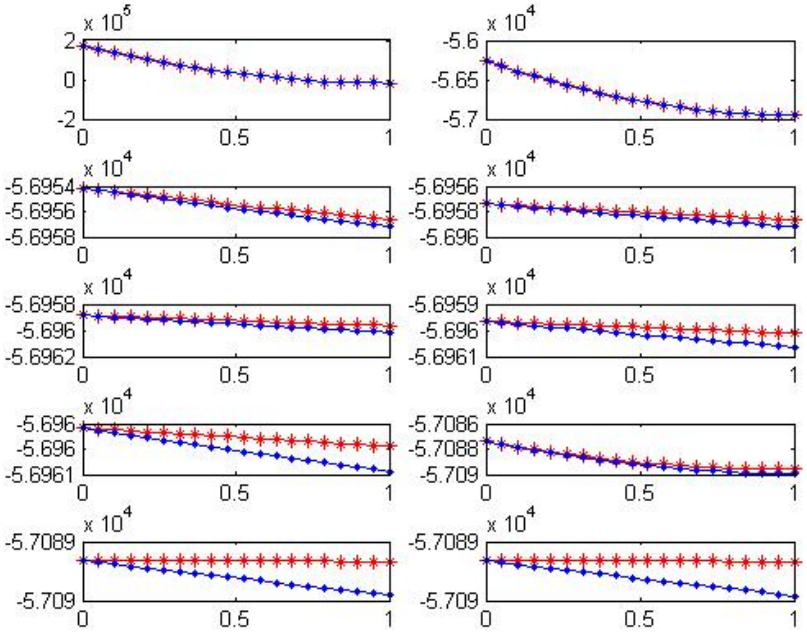


Figure 12: 1-side bound constraint: TN: lin1; MG/OPT: lin2; penalty $\rho = 1$, merit and objective function based on different α

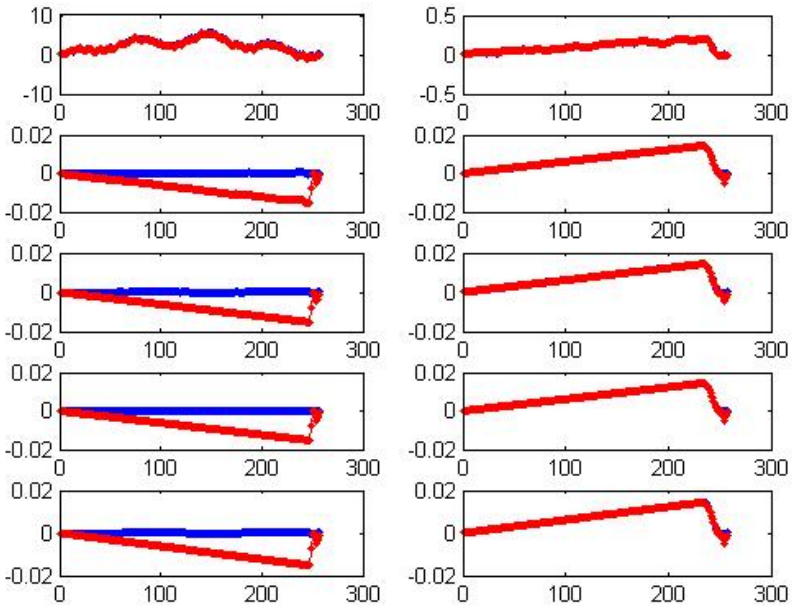


Figure 14: 1-side bound constraint: TN: lin2; MG/OPT: lin2; Compare step to solution with search direction on fine level: discretization level: $[257, 128]$

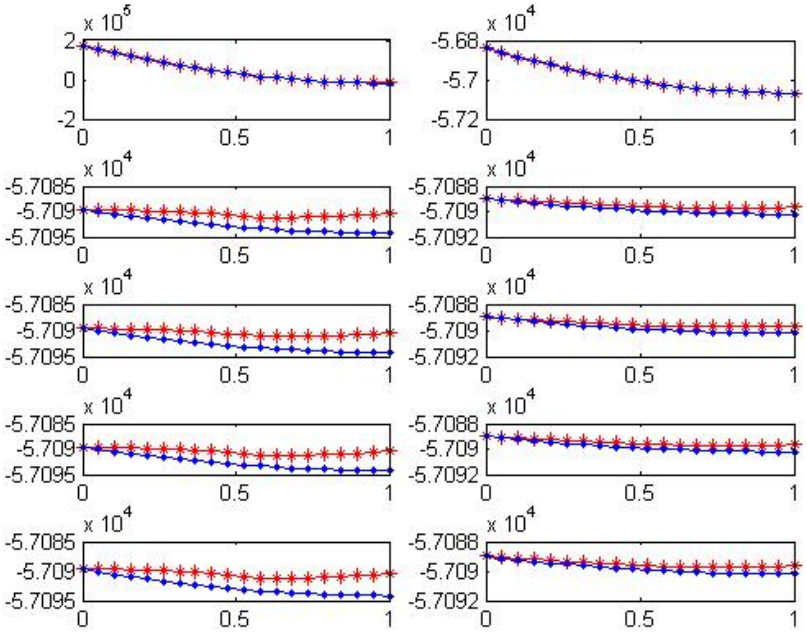
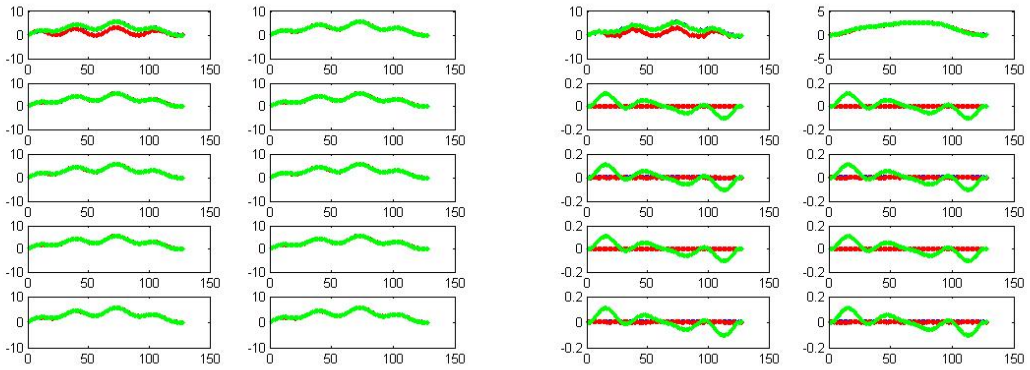


Figure 15: 1-side bound constraint: TN: lin2; MG/OPT: lin2; penalty $\rho = 1$, merit and objective function based on different α

6 1-side bound: TN: lin3; MG/OPT: lin2



(a) solution (b) search direction

Figure 16: 1-side bound constraint: TN: lin3; MG/OPT: lin2; Compare solution in coarse grid: discretization level: $[257, 128]$

Output of 10th cycle: TN: lin3; MG/OPT: lin2

```
In  mgrid: n = 257
  it     nf     cg          f           |g|
   0      1      0   -5.70896641e+004   1.4e-002
   1      2     50   -5.70896641e+004   2.7e-002

###################################
### NO EXTRA BOUND CONSTRAINT ###
###################################

In  mgrid: n = 128
  it     nf     cg          f           |g|
   0      1      0   -2.83884805e+004   2.6e+002
   1      4      3   -2.83885960e+004   2.7e+002
alpha0 =
     1
Error in Line Search (lmqnbcm.m)
    ncg1   = 2
    alpha  =   0.00000000
    alpha0 =   1.00000000
    g'p    = -3.4021e+000
    |g|     =  3.0168e+002
    |p|     =  1.3446e-002
```

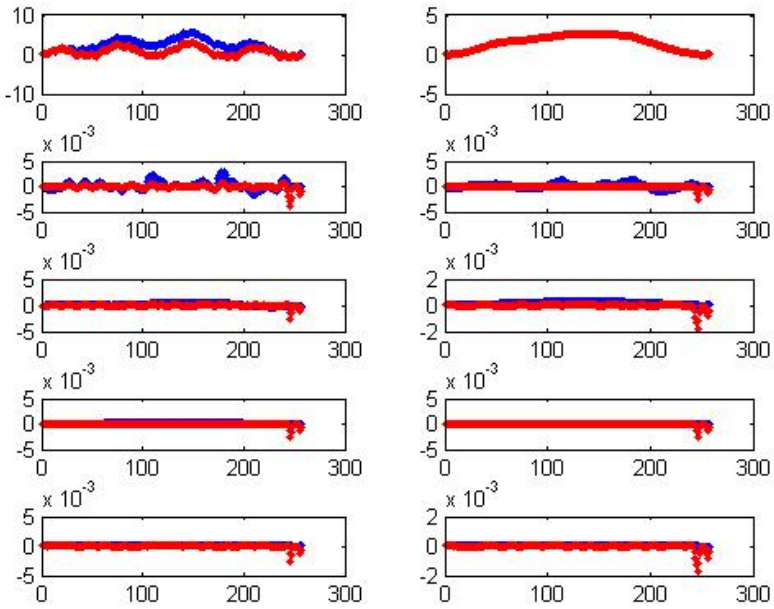


Figure 17: 1-side bound constraint: TN: lin3; MG/OPT: lin2; Compare step to solution with search direction on fine level: discretization level: $[257, 128]$

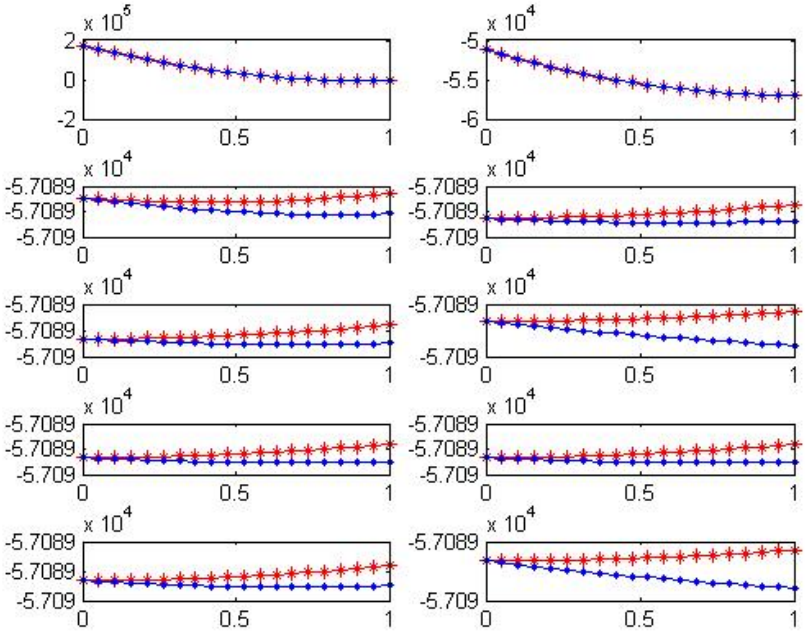


Figure 18: 1-side bound constraint: TN: lin3; MG/OPT: lin2; penalty $\rho = 1$, merit and objective function based on different α

```
only lower violation
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MG/Opt line search: alpha =  2.44140625e-004
In  mgrid: n = 257
  it     nf     cg           f           |g|
   0      1      0   -5.70896641e+004   2.7e-002
   1      2     30   -5.70896641e+004   1.1e-002

-------------------
Optimization costs per grid
-------------------
N:        257   128
-------------------
NF(it):    20    92
NF(nf):    40   113
NF(cg):   417   998
```
