Progress Report until May 26

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1 Issues about active set method in TN

- Some coding issues in 'lmqnbcm' [1] may result in the maximum step length $\alpha_0 = 0$ for the line search
  - In 'lmqnbcm', the process is to update the active set first and then to calculate the maximum step length considering every variable in the active set. According to this, it may take the binding variable into consideration to calculate the maximum step length which may result into $\alpha_0 = 0$
- What I modified is to apply one more 'modz' before calculating α_0 which as follows:

```
if (abs(alpha-spe) <= 10*eps);
       ipivot0 = ipivot;
       newcon = 1;
       [ipivot, flast] = modz (x, p, ipivot, low, up, flast, f, alpha);
end;
spe = stpmax (stepmx, pe, x, p, ipivot, low, up);
alpha = step1 (f, gtp, spe);
alpha0 = alpha;
```

  In this way, it will exclude any current binding variable into account.
- In figure 1, it shows the improvement of MG/OPT on 2-side bound problem[2].

2 Some Analysis in 1-side bound problem

- My next task is to try to fix the issue happend in the error between downdate of variable and the original fine level which will force the fine-level variable out of bound after being updated by the coarse solution

[1] **Note:the same issues happen in 'lmqnbc' as well**
[2] **: Note: All the following tests are based on the original setting of line search in both TN and mgrid**

- Firstly, I did some analysis on the original problem to find a proper corresponding coarse problem constraint.
  - The original quadratic problem : $f(x) = 1/2x^T Ax - b^T x$ s.t.$x \geq l$
  - Our goal is to minimize

$$f(v_h + I_H^h(e_2 - v_H))$$

$$= 1/2(v_h+I_H^h(e_2-v_H))^T A_h(v_h+I_H^h(e_2-v_H))-b_h^T(v_h+I_H^h(e_2-v_H))$$

$$= 1/2v_h^T A_h v_h + 1/2(I_H^h(e_2 - v_H))^T A_h v_h + 1/2v_h^T A_h(I_H^h(e_2$$

$$= f(v_h) + \tilde{f}(e_2)$$

where $\tilde{f}(e_2) = 1/2(I_H^h(e_2-v_H))^T A_h(I_H^h(e_2-v_H))+\tilde{b}^T(I_H^h(e_2-v_H))$ in which $\tilde{b} = A_h v_h - b_h$

Since we have $I_H^h = 2(I_h^H)^T \implies (I_H^h)^T = 2I_h^H$. And also we know that in MG/OPT, the shifted coarse function is defined as

$$f_H^s = f_H(x) - (\nabla f_H(v_H) - I_h^H \nabla f_h(v_h))^T x$$

$$= 1/2x^T A_H x - b_H^T x + (I_h^H A_h v_h - A_H v_H)^T x$$

$$\tilde{f}(e_2) = (v_h^T - b_h^T)(I_H^h e_2) - (v_h^T A_h - b_h^T) I_H^h v_H + 1/2((I_H^h e_2)^T A_h(I_H^h e_2))$$

$$-((I_H^h V_H)^T A_h(I_H^h e_2)) + 1/2((I_H^h v_H)^T A_h(I_H^h v_H))$$

$$= v_h^T A_h I_H^h e_2 - b_h^T I_H^h e_2 + \frac{1}{2}2e_2^T A_H e_2 - \frac{1}{2}2e_2^T A_H v_H - \frac{1}{2}2v_H^T A_H e_2 + C$$

$$= 2f_H^s(e_2) + C$$

where $C = -(v_h^T A_h) I_H^h v_H + b_h^T I_H^h v_H + \frac{1}{2}2v_H^T A_H v_H$

On coarse level, we have $e_{2,0} = v_H$. Thus

$$f(v_h + I_H^h(e_2 - v_H)) = f(v_h) + \tilde{f}(e_2) \leq f(v_h) + \tilde{f}(v_H) = f(v_h)$$

Therefore, $min\, f(v_h + I_H^h(e_2 - v_H))$ is equivalent to $min\, \tilde{f}(e_2)$, then it turns out to be equivalent to minimize $f_H^s(e_2)$. Also, in order to force $v_h + I_H^h(e_2 - v_H) \geq l$, let the lower bound of coarse problem is l_H, then

$$l_{H,i} = max(l_{2i}\,,\, l_{2i} + 1/4v_{h,2i-1} - 1/2v_{h,2i} + 1/4v_{h,2i+1})$$

Proof:For even nods:

$$v_{h,2i} + I_H^h(e_{2,i} - v_{H,i})$$

$$\geq v_{h,2i} + I_H^h(l_{2i} + 1/4v_{h,2i-1} - 1/2v_{h,2i} + 1/4v_{h,2i+1} - v_{H,i})$$

$$= v_{h,2i}+l_{2i}+1/4v_{h,2i-1}-1/2v_{h,2i}+1/4v_{h,2i+1}-(1/4v_{h,2i-1}+1/2v_{h,2i}+1/4v_{h,2i+1})$$

$$= l_{2i}$$

For odd nods:

$$v_{h,2i-1} + 1/2(e_{2,i-1} - v_{H,i-1}) + 1/2(e_{2,i} - v_{H,i})$$

$$= v_{h,2i-1}+1/2(e_{2,i-1}+e_{2,i})-1/2(1/2v_{h,2i-3}+1/2v_{h,2i-2}+1/2v_{h,2i-1}+1/2v_{h,2i}+1/4v_{h,2i+1})$$

$$\geq (3/4 - 1/8 - 2/8 - 2/8 - 1/8)l_{h,2i-1} + 1/2(l_{H,i-1} + l_{H,i})$$

. Thus, choose $l_H = l_h$ will satisfy the bound constraint.

- According to following analysis, I found a mistake in the setting up of all parameters, before, I calculate b separately in different levels following the same fomular, acutally, $b_H = I_h^H b_h$.
- Then apply all above conclusions to the code, I get the figure 2 from which, we can see that MG/OPT can get to at least the same accuracy as OPT along with its benefit.
- I also tried the constraint in Toint, since its setting of $l_{H,i} = v_{H,i} + max(l_h - v_h)$, since along with the showing of binding variable, the second component of above equation will be zero, so the coarse problem will only supply very tiny step since no more room for the v_H to move, so it fails.

3 Next step:

- Try to implement the code for more than 2 levels, the comlication is I have to fix the constraint in each coarse level corresponding to its right fine level.

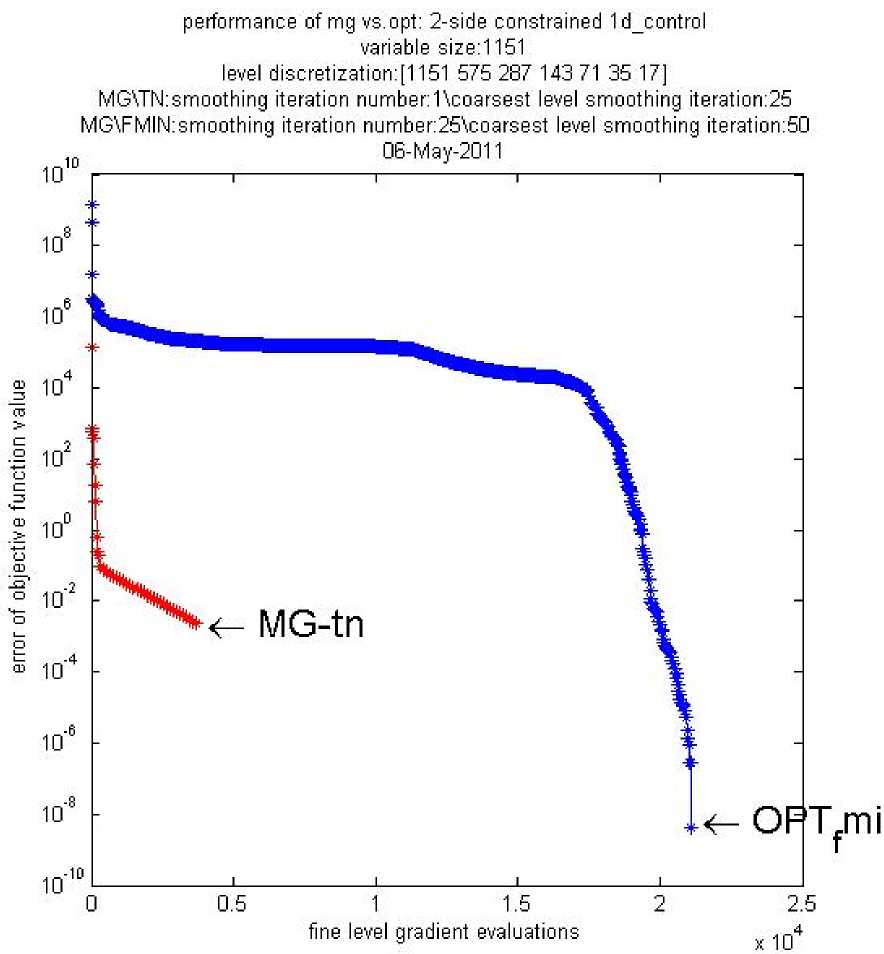

Figure 1: 2-side bounds problem: Improved fine level gradient evaluation according to the modification in 'lmqnbcm'

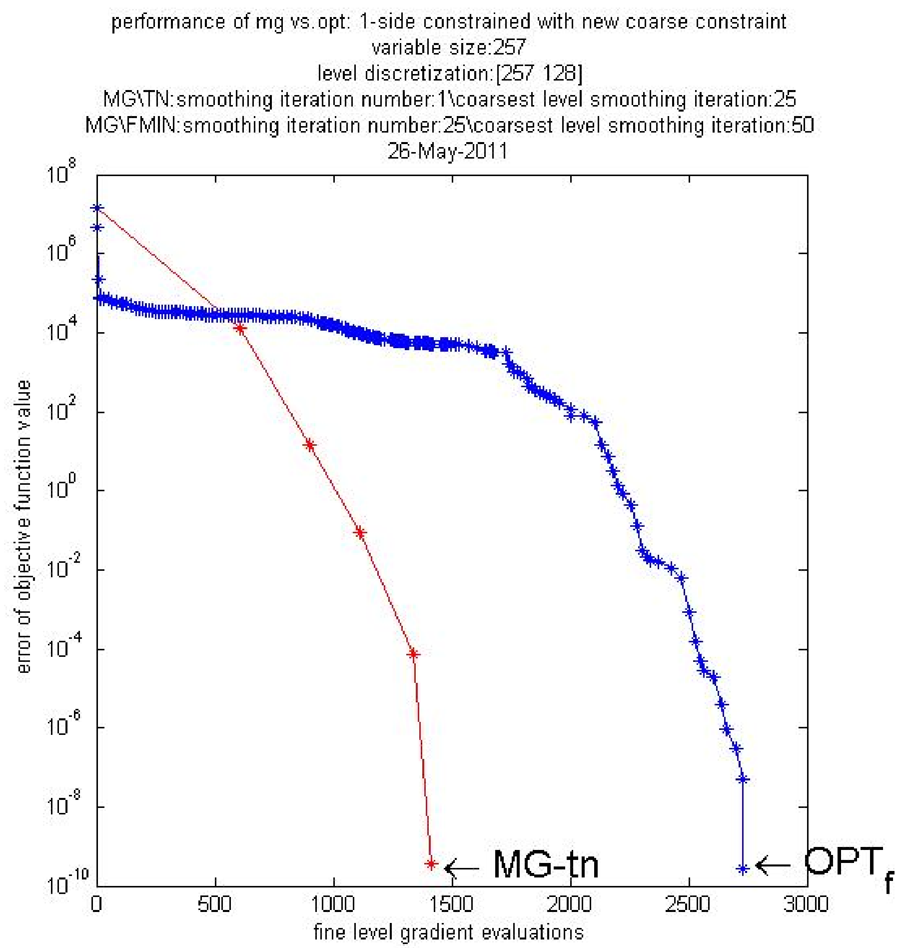

Figure 2: 1-side bounds problem: Improved fine level gradient evaluation according to the modified coarse constraint