WARP Shoe Project

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Abstract

This report outlines a comprehensive analysis conducted to optimize WARP Shoe Company's production plan amid a market shift triggered by a competitor's bankruptcy in 2006. Our goal is to maximize profits within constraints of a $10 million raw material budget, machine operation limits, hourly worker wages, and raw material quantities. Our approach integrates AMPL and Excel tools. The resulting optimization model delivers an optimal profit of $14,275,952, exploring factors of changing constraint values and analyzing characteristics within mathematical program – notably in the discussion. The report covers diverse aspects, including demand estimation methodologies, variable constraints, and economic considerations like warehouse space. Findings on binding constraints, shadow prices, and reduced costs equip WARP with actionable insights for effective decision-making and profitability in February 2006.

Introduction

With a competitor's bankruptcy in 2006, the WARP shoe company found doubling shoe demand in February. To capitalize, WARP aims to maximize profits while navigating a $10 million raw material budget, machine operation limits, and hourly worker wages and raw material limits. This project aims to produce a production plan to maximize profit for the WARP shoe company. In short, this plan yields an optimal profit of $14,275,952. With a focus on practicality, we explore demand estimation, variable constraints, and economic considerations like warehouse space. This analysis uses AMPL and Excel as our main tools to help us gain insights to meet this demand efficiently and profitably in the dynamic market landscape of February 2006.

Methodology

Definitions

The following parameters will be used throughout the rest of the document.

P_i = Price of shoe of product type i ∀ i=1...557

O_j = Operating costs for each machine j per min ∀ j=1...72

R_{ik} = Raw material k used for shoe of product type i ∀ i=1...557, ∀ k=1...165

H_{ij} = Operation time (s) on machine j to produce shoe of product type i ∀ i=1...557, ∀ j=1...72

D_i = Demand for show of product type i ∀ i=1...557

C_k = Cost of raw material k ∀ k=1...165

W = Warehouse Capacity

Q_k = Raw Material k Upper Bound ∀ k=1...165

Assumptions

1. We assumed that there would always be one worker for each machine. Thus, adding in an additional variable based on the concepts of fixed-costs would be redundant.
2. We assumed that there is no requirement for which type of shoe goes to which warehouse; thus, we can simply look at the warehouse capacity as the total of all the warehouse capacity from *Warehouse_Master* Table combined. Additionally, we are looking at the warehouse capacity for only one month.

Math Model

<u>Decision Variables</u>

$$x_i = Number\ of\ shoe\ produced\ of\ shoe\ type\ i\ \forall i = 1 \dots 557$$

<u>Objective Function</u>

$$z^{\cdot} = \sum_{i=1}^{557} P_i\, x_i - \sum_{k=1}^{165} \sum_{i=1}^{557} C_k (R_{ik} x_i) - \sum_{j=i}^{72} \sum_{i=1}^{557} \frac{O_j}{60} x_i - \sum_{j=i}^{72} \sum_{i=1}^{557} \frac{25}{3600} H_{ij}\, x_i - \sum_{i=1}^{557} 10(x_i - D_i)$$

Constraints

(1) Raw Materials Constraint in Dollars:

$$\sum_{i=1}^{557} \sum_{k=1}^{165} C_k (R_{ik} x_i) \leq 10{,}000{,}000$$

(2) Time Limit in seconds:

$$\sum_{i=1}^{557} H_{ij}\, x_i \leq 1{,}209{,}600\ \forall j = 1 \dots 72$$

(3) Warehouse Capacity:

$$\sum_{i=1}^{557} (x_i - D_i) \leq 140{,}000$$

(4) Raw Material Limit:

$$\sum_{i=1}^{557} R_{ik}\, x_i \leq Q_k \qquad \forall k = 1 \dots 165$$

Solved Model Explanation & Suggested Implementation Strategy

Optimal Profit

Our optimal objective value, which can be interpreted as the optimal maximum profit, has the value of: $14,275,952.

Shoe Production Plan

We suggest the production of the types of shoes and the quantity presented in the Objective Solution section. As the company reads this report, here's how they can interpret the results.

```
SH005    104   165.974
```

Given these outputs of *x* we can interpret that the company should produce 104 pairs of Shoe Product type 5.

```
:         x       x.rc
SH001      0    103.011
```

On the other hand, the reduced costs of the decision variables (*x.rc)* defines how much the decision variable must increase such that it would be considered into the basis – in other words, how much the decision variable benefit must increase until it's profitable to produce. In this example, SH001 must increase its benefit by 103.11 before it can be produced.

Shadow Prices

The shadow prices can be found using the dual and represent the amount of increase within the objective function optimal value given an increase in the right-hand side of the constraint associated with the dual. For our solution, all the constraints had shadow prices equal to zero, meaning the increase (relaxation) would not benefit the profit. In the real world, this could be that machines may not have been fully used to begin with – meaning there would be no use in paying for more machines with the intent of producing more shoes and increasing profit.

Slack

The slack values represent whether the constraint is a binding constraint – meaning the left–hand side of the constraint is at its upper bound. This can inform us of the possible constraints that could be overall limiting the production of shoes. For example, if Raw Material k constraint is a binding constraint, we may not be able to produce more shoes that use Raw Material k despite increasing the budget of Raw Materials since there is not enough material to begin with.

Discussion

1. How should you estimate the demand for the month of February?

We first extracted the table "Product_Demand" from the provided database into Excel. Then, we sorted the data into two columns. The first one being the product number from 1 to 557, the second being the average demand of each product for the month of February calculated from the formula shown in the "Avg_Demand" column above in the Excel screenshot. It illustrates the total demand for February of 1997-2003 of each product divided by product of the number of years and the number of months recorded in the dataset (6 years * 10 months = 60). Then, we exported the Excel file in the format of .mdb and read the database in AMPL.

Demand Estimation

	A	B	C	D	E	F	G	H	I	J	K	L
1	Product_N	Year	Month	Demand			Product_Num	Avg_Demand				
2	SH001	1997	1	9			SH001	= ROUNDDOWN((SUMIFS(D:D,A:A,G2,C:C,2) / 60),0)				
3	SH001	1997	1	7			SH002	26				
4	SH001	1997	1	18			SH003	27				
5	SH001	1997	1	19			SH004	24				
6	SH001	1997	1	11			SH005	27				
7	SH001	1997	1	11			SH006	24				
8	SH001	1997	1	39			SH007	25				
9	SH001	1997	1	28			SH008	25				
10	SH001	1997	1	33			SH009	25				
11	SH001	1997	1	48			SH010	23				
12	SH001	1997	2	13			SH011	25				
13	SH001	1997	2	10			SH012	25				
14	SH001	1997	2	21			SH013	24				
15	SH001	1997	2	18			SH014	25				
16	SH001	1997	2	12			SH015	22				
17	SH001	1997	2	11			SH016	27				
18	SH001	1997	2	41			SH017	24				
19	SH001	1997	2	27			SH018	29				
20	SH001	1997	2	48			SH019	24				

2. How many variables and constraints do you have?

Our decision variable x_i is the number of shoes of type i produced. According to the database, we have 557 different types of shoes. Thus, there are 557 variables. We also generated 4 different constraints which can be found in our "Math Model" section.

3. If you had to relax your integer program to an LP, how many constraints were violated after rounding the LP solution to the closest integer solution?

After relaxing our program to an LP, we got $14,278,300.91 as the optimal solution. To see if any of the constraints are violated, we first extracted the optimal decision variable values from the LP into Excel and using the ROUND function we rounded them up to the nearest integer [*Q3:A* in Appendix]. Then, we exported our excel table in the format of .mdb so we can populate a new parameter called LPDV which takes "ProductNum" as input [*Q3:B* in Appendix] Then, using LPDV, we plugged it into each of the constraint equations to see if they violate any constraints [*Q3:C* in Appendix]. The results can be found in D of the appendix. As you can see, only some iterations (79 in total) of constraint 4 are violated, in other words, the maximum quantity of several types of raw material exceeded their maximum quantity limit.

4. Which constraints are binding, and what is the real-world interpretation of those binding constraints?

To identify the binding constraints, we look at the slack values for each constraint. When the slack value is zero, it means that the constraint is exactly satisfied; the resources on the right-hand side are fully utilized, and there is no surplus or slack in the constraint. Based on this, we have identified that only some iterations of constraint 4 (Raw Material Constraint) are binding, namely the Raw Material constraints listed in the Appendix Section *Q4: Raw Materials with Binding Constraints*.

The real-world interpretation of these binding constraints in the context of this project is that all the Raw Material (k) are fully used where the Raw Material constraint (k) has a slack value of zero. Additionally, these Raw Material constraints limit the rest of the model. For example, since other constraints aren't binding, the profit is limited by the number of raw materials we have rather than the machine time limit or warehouse capacity.

5. Assume that some additional warehouse space is available at the price of $10/box of shoes. Is it economical to buy it? What is the optimal amount of space to buy in this situation?

To approach this question, we looked at the shadow price of constraint 3 (Warehouse Constraint). The shadow price tells us how much we can improve the objective function (profit) when we increase the right-hand side by one unit. In this case, since the time limit constraint has a shadow price of zero, modifying the right-hand side (in this case, buying more storage) would not contribute to increasing our profit. Thus, we believe it is not economical to buy warehouse space.

6. Imagine that machines were available for only 8 hours per day. How would your solution

change? Which constraints are binding now? Does the new solution seem realistic to you?

To modify our formulation, we change the right-hand side of the Time Limit Constraint (2) to the new value of:

(28days (about 4 weeks) * 8h * 60mins * 60seconds) = 806400seconds (about 1 and a half weeks)

With the new formulation of the constraint in the form:

```
subject to TimeLimit{j in MachineNum}: sum{i in ProductNum} AvgDur[i,j]*x[i] <= 806400; #Time Limit
```

As a result, our new optimal profit value becomes $14275567. There is a decrease in the profit which can be accounted for by the reduced number of hours of machines working that could be assumed to be proportional to the total number of shoes produced and sold. Whether the degree of decrease is proportional to the number of hours could be further investigated; however, the overall relationship between decreasing the time constraint leading to a decrease in the objective value seems realistic.

Now, for the binding solution, the new list can be found in the Appendix in section *Q6-Binding Constraints.* Overall, we see that all constraints are not binding except for some iterations of constraint 4 (Raw Materials). There is a change in the Raw Material number found in the Appendix in section *Q6: New Raw Material Binding Constraint Numbers.*

7. If in addition there were a $7,000,000 budget available to buy raw materials, what would you do? Change your formulation and solve again.

First, we would change the right-hand side of Raw Material Budget Constraint (1) to be $7 000 000. Here's the new constraint formulation:

```
subject to Budget: sum{i in ProductNum, k in RMNum} RMCost[k] * MaterialsQuantity[i, k] * x[i] <= 7000000; #Budget
```

Next, we solve the formulation and give us the optimal profit of $14,275,598. The decrease can be accounted for by the decrease in the budget for the raw materials; thus, limiting the number of shoes that could be produced, decreasing the profit we can make.

Conclusion

With the goal of optimizing shoe production during changes in demand, our analysis unveils key insights for WARP shoe company. With an optimal profit of $14,275,952, the model guides decisions on shoe types, raw materials, and machine hours. Constraints, notably in raw material supply, emerge as a limiting factor. The economic viability of additional warehouse space is deemed impractical, while adjusting machine hours does not provide much benefit to production and profit. In this report, we also explored the different meanings behind optimization concepts such as binding constraints, shadow prices and reduced costs. We hope the results of this bring insight into how the WARP shoe company can navigate their February production plan with the use of demand projections and optimization.

Appendices

Q3: LP

A:

Not Rounded	Rounded	Shoe Num
0	=ROUND(A2:A558,0)	
0	0	SH002
0	0	SH003
0	0	SH004
103.79	104	SH005
0	0	SH006
0	0	SH007
0	0	SH008
0	0	SH009
0	0	SH010
464.23	464	SH011
0	0	SH012
716.93	717	SH013
0	0	SH014
59.11	59	SH015
0	0	SH016
0	0	SH017
0	0	SH018
109.64	110	SH019

B:

```
### Read data for the rounded decision variables
table LPRounded IN "ODBC" "W:\MIE262\MIE262_WARP\Book1 (2).mdb" "LPRounded":
    ProductNum <- [C], LPDV ~ B;
read table LPRounded;
```

C:

```
option solver gurobi;
solve;
display profit;
display sum{i in ProductNum, k in RMNum} RMCost[k] * MaterialsQuantity[i, k] * LPDV[i];
display sum{i in ProductNum} ((LPDV[i]-PredictedDemand[i]));

for {j in MachineNum} {
    if (sum{i in ProductNum} AvgDur[i,j] * LPDV[i]) > 1209600
    then printf "Time limit exceeded at %d \n",j;
    }

for {k in RMNum} {
    if (sum{i in ProductNum} MaterialsQuantity[i,k] * LPDV[i]) > RMQuant[k]
    then printf "Raw material quantity exceeded at %d \n",k;
    }
```

D:

```
sum{i in ProductNum, k in RMNum} RMCost[k]*MaterialsQuantity[i,k]*LPDV[i] =
4606280

sum{i in ProductNum} (LPDV[i] - PredictedDemand[i]) = 66577

Raw material quantity exceeded at 3
Raw material quantity exceeded at 5
Raw material quantity exceeded at 6
Raw material quantity exceeded at 7
Raw material quantity exceeded at 10
Raw material quantity exceeded at 12
Raw material quantity exceeded at 13
Raw material quantity exceeded at 14
Raw material quantity exceeded at 15
Raw material quantity exceeded at 17
Raw material quantity exceeded at 20
Raw material quantity exceeded at 21
Raw material quantity exceeded at 23
Raw material quantity exceeded at 25
Raw material quantity exceeded at 26
Raw material quantity exceeded at 30
Raw material quantity exceeded at 34
Raw material quantity exceeded at 38
Raw material quantity exceeded at 42
Raw material quantity exceeded at 50
Raw material quantity exceeded at 52
Raw material quantity exceeded at 54
Raw material quantity exceeded at 55
Raw material quantity exceeded at 57
Raw material quantity exceeded at 58
Raw material quantity exceeded at 59
Raw material quantity exceeded at 63
Raw material quantity exceeded at 64
Raw material quantity exceeded at 66
Raw material quantity exceeded at 68
Raw material quantity exceeded at 70
Raw material quantity exceeded at 72
Raw material quantity exceeded at 73
```

Q4: Raw Materials with Binding Constraints:

Constraints of the following Raw Materials numbers are binding:

13 15 16 17 19 21 22 27 32 33 34 36 38 41 44 45 46 48 50 51 54 60 67 76 88 89 90 92 94 96 103 104 107 111 113 117 118 119 122 123 130 131 137 138 152 156 157 160 161

Q6: Binding Constraints

```
TimeLimit.slack [*] :=
 1 727826      16 665226      31 775335      46 207035      61 478005
 2 486650      17 729757      32 708936      47 487643      62 338885
 3 404135      18 672959      33 718243      48 219160      63    307.11
 4 635493      19 701629      34 721911      49 119645      64 266944
 5 370516      20 679465      35 724714      50 471966      65 137680
 6 510057      21 794534      36 722617      51 763440      66 314394
 7 441440      22 780131      37 675601      52 677943      67 128559
 8 343904      23 741366      38 685078      53 679163      68  87427.7
 9 567804      24 771377      39 751112      54 565960      69    136.99
10 647963      25 756734      40 782566      55 586780      70 486853
11 683238      26 769917      41 404708      56 710908      71 468825
12 656926      27 742330      42 328203      57 660491      72 482681
13 594464      28 774632      43 430207      58 706833
14 643799      29 756666      44 395047      59 662954
15 590810      30 776139      45 420178      60 702650
;

Budget.slack = 5394610

TimeLimit.slack [*] :=
 1 727826      16 665226      31 775335      46 207035      61 478005
 2 486650      17 729757      32 708936      47 487643      62 338885
 3 404135      18 672959      33 718243      48 219160      63    307.11
 4 635493      19 701629      34 721911      49 119645      64 266944
 5 370516      20 679465      35 724714      50 471966      65 137680
 6 510057      21 794534      36 722617      51 763440      66 314394
 7 441440      22 780131      37 675601      52 677943      67 128559
 8 343904      23 741366      38 685078      53 679163      68  87427.7
 9 567804      24 771377      39 751112      54 565960      69    136.99
10 647963      25 756734      40 782566      55 586780      70 486853
11 683238      26 769917      41 404708      56 710908      71 468825
12 656926      27 742330      42 328203      57 660491      72 482681
13 594464      28 774632      43 430207      58 706833
14 643799      29 756666      44 395047      59 662954
15 590810      30 776139      45 420178      60 702650
;

Warehouse.slack = 73443

RawMaterial.slack [*] :=
  1  2    20  8    39  0    58  0    77 12    96  2   115  1   134  1   153  8
  2 19    21  0    40  0    59  5    78  1    97  0   116  0   135  3   154  1
  3  3    22  7    41  0    60  2    79  1    98  3   117  2   136  1   155  2
  4  0    23 20    42  2    61  2    80  8    99  0   118  0   137  0   156  1
  5  2    24  3    43  0    62  1    81  1   100  2   119  3   138  1   157  0
  6  3    25  7    44  0    63  1    82  8   101  1   120  0   139  0   158  1
  7  1    26  2    45  0    64  3    83  0   102  4   121  2   140  0   159  1
  8  3    27  0    46  4    65  0    84  9   103  4   122  3   141  3   160  1
  9  5    28  5    47  4    66  5    85  4   104  1   123  2   142  0   161  2
 10  1    29 11    48  1    67  6    86  0   105  2   124  0   143  1   162  0
 11  1    30  2    49  2    68  2    87  3   106  3   125  2   144  0   163  1
 12  0    31  0    50  0    69  8    88  1   107  1   126  2   145  2   164 11
 13  2    32  0    51  5    70  2    89  4   108  6   127  4   146  2   165  7
 14  2    33  0    52  8    71  0    90  1   109  5   128  5   147  1
 15  2    34  0    53  0    72  2    91  9   110  0   129  3   148  0
 16  2    35  2    54  0    73  0    92  0   111  5   130  2   149  2
 17  0    36  0    55  2    74  0    93  0   112  6   131  0   150  1
 18  5    37  0    56  1    75  2    94  3   113  0   132  1   151  1
 19  0    38  0    57  4    76  3    95  1   114  9   133  1   152  1
;
```

Q6: New Raw Material Binding Constraint Numbers.

Constraints of the following Raw Materials numbers are binding:

4, 12, 13, 17, 19, 21, 27, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 43, 44, 45, 50, 53, 54, 58, 65, 68, 71, 73, 74, 83, 86, 92, 93, 97, 99, 106, 113, 116, 118, 120, 124, 131, 137, 139, 140, 142, 144, 148, 157, 162