General Physics I Final Review

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练习 1.

有滑滚动

Q7. (12 points) A uniform ball of radius $R = 0.3\ m$, mass $m = 2.0$kg is kicked with initial speed of its center of mass $v_i = 6.0\ m/s$ and initial angular speed $\omega_i = 0$. The ball first rolls with sliding (non-smooth rolling) along the horizontal ground surface until a moment when its angular speed is increased enough. Then the ball starts roll smoothly. The coefficients of friction between the ball and the ground are $\mu_k = 0.2$ for kinetic friction and $\mu_s = 0.4$ for static friction. Find

纯滚.非纯滚公式

(a) the angular speed ω of the ball when it begins the smooth rolling.

(b) the relative displacement of contact point between ball and ground during non-smooth rolling.

(c) the increase ΔE_{th} in thermal energy of the ball and the ground due to sliding.

f τ

平动减速.转动加速.最终 $V=WR$ 达到纯滚

(a) $f_k = ma$ 受力

$f_k R = I\alpha$ 力矩

$V_0 - at = V$ $\Rightarrow W = 14.3\ s^{-1}$

$w = \alpha t$

$WR = V$ 纯滚

(b) $\mu_k m g s = \frac{1}{2} m V_0^2 - \left(\frac{1}{2} m V^2 + \frac{1}{2} I W^2\right)$

$\Rightarrow S = 2.61\ m$

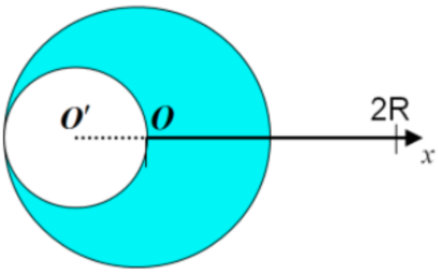
(c) $\Delta E_{th} = \mu_k m g s = 10.23\ J$

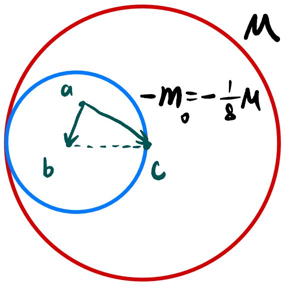
练习 2.

A spherical hollow inside a uniform lead sphere of radius R=8.00cm, the surface of the hollow passes through the center of the sphere and "Touches" the left side of the sphere The mass of the sphere before hollowing was M=24. 34kg. What is the magnitude of the gravitational force due to the hollow sphere on a particle of mass m=0.50kg.

(a)When the particle is located at a distance of 2R from the center of the hollow sphere.

(b)When the particle is located at any point in the cavity of the sphere.





(a) $F = \dfrac{GMm}{(2R)^2} - \dfrac{G m_0 m}{\left(2R + \frac{R}{2}\right)^2}$

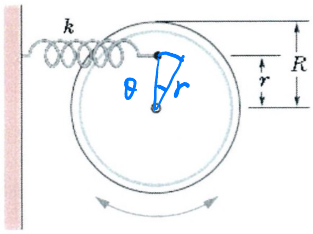
(b) $\vec{F}_{ab} = -\dfrac{G m_0 m}{r_{ab}^2} \cdot \dfrac{r_{ab}^2 \cdot \vec{r}_{ab}}{\left(\frac{R}{2}\right)^3}$

$= -\dfrac{G m_0 m}{\left(\frac{R}{2}\right)^3} \cdot \vec{r}_{ab} = -\dfrac{GMm}{R^3} \cdot \vec{r}_{ab}$

$\vec{F}_{ac} = \dfrac{GMm}{R^3} \vec{r}_{ac}$

$\vec{F}_a = \vec{F}_{ab} + \vec{F}_{ac} = \dfrac{GMm}{R^3} \cdot \vec{r}_{bc}$

7. (10 marks) A wheel is free to rotate about its fixed axle. A spring is attached to one of its spokes a distance r from the axle, as shown in Figure. (a) Assuming that the wheel is a hoop of mass m and radius R, what is the angular frequency ω of small oscillations of this system in terms of m, R, r, and the spring constant k ? What is ω if (b) $r=R$ and (c) $r=0$?



$F = kr\theta$

$\tau = Fr = kr^2\theta = I\alpha$

$I = mR^2$ (hoop)

$\Rightarrow kr^2\theta + mR^2\alpha = 0$

$\omega^2 = \frac{r^2k}{R^2m}$

注意单位

The Fig.3 shows a pattern of resonant oscillation (a standing wave) of a string of mass m = 2.500 g and length $L = 0.800$ m and that is under tension $\tau = 325.0$ N.

(a) What is the frequency f of the standing waves?

(b) What is the maximum magnitude of the transverse velocity of the element oscillating at coordinate $x = 0.480$ m?

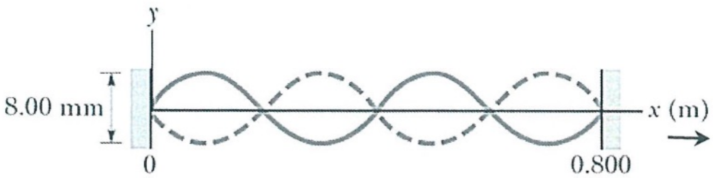


Figure 3

(a) $\rho = \frac{m}{L} = 3.125\times10^{-3}$ kg/m

$v = \sqrt{\frac{\tau}{\rho}} = 322.5$ m/s

$2\lambda = L$

$\Rightarrow f = \frac{v}{\lambda} = 806$ Hz

(b) 由图：$A = 4\times10^{-3}$ m

$y = A\sin\frac{4\pi x}{L}\cos\omega t$ 驻波

$\frac{dy}{dt} = -\omega A\sin\left(\frac{4\pi x}{L}\right)\sin\omega t$

$x = 0.48$ m处求最值即可

$\Rightarrow v = 19.3$ m/s

动能: $E_k = E_{总} - E_{引}$ 或由匀速圆周推导

引力势能: $-\frac{GMm}{r}$ r为到球心距离

总能量: $-\frac{GMm}{2r}$ r为半径/半长轴

练习 3.

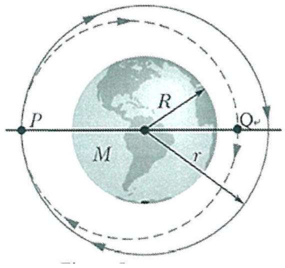
Q7. (12 points)

A small spaceship of mass m = 1500 kg, is in the clockwise circular orbit about earth at an altitude h of 400 km. The mass and radius of the earth are $M=5.98\times10^{24}kg$ and $R=6.37\times10^{6}m$. (a) What is the speed v_0 of that spaceship?

At point P, peter, the commander of the ship, fires a burst in the forward direction. After this burst, he follows a new elliptical orbit with semimajor axis $a=6.65\times10^{6}$ m shown dashed in the figure.

(b) What is the mechanic energy E in peter's new orbit?

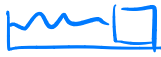
(c) What is the speed of peter when he arrives at point Q?



(a) $\frac{v_0^2}{R+h} = \frac{GM}{(R+h)^2}$

(b) $E = -\frac{GMm}{2a}$

(c) $E = \frac{1}{2}mv^2 - \frac{GMm}{2a-(R+h)}$



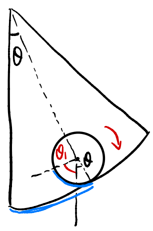
$ma + kx = 0$ $\omega^2 = \sqrt{\frac{k}{m}}$ a与x的线性关系

a与x 可替换成 α与θ

练习 4.

2. (3 points) A solid uniform ball (a sphere) of radius r =15.0 cm rolls in a bowl that has a radius of R =120 cm. Assume that the ball rolls without slipping, find the period of the simple harmonic motion (assuming θ is small).

纯无滑滚动：碗对球有静摩擦力



从底部到此处，球自转角度为θ_1

纯滚：两段蓝色弧长相等

$\Rightarrow r(\theta_1+\theta)=R\theta$

$v=(R-r)\frac{d\theta}{dt}$

$\omega=\frac{d\theta_1}{dt}$

质心平动动能: $\frac{1}{2}m(R-r)^2(\frac{d\theta}{dt})^2$

转动动能: $\frac{1}{2}\cdot(\frac{2}{5}mr^2)\cdot(\frac{d\theta_1}{dt})^2$

重力势能: $mg(R-r)(1-\cos\theta) \approx mg(R-r)\cdot\frac{1}{2}\theta^2$

能量守恒

$\therefore \frac{1}{2}m(R-r)^2(\frac{d\theta}{dt})^2+\frac{1}{2}(\frac{2}{5}mr^2)(\frac{d\theta_1}{dt})^2+mg(R-r)\frac{1}{2}\theta^2 = constant$

又有 $r(\theta_1+\theta)=R\theta$ 弧长相等

$\therefore \frac{1}{2}m(R-r)^2(\frac{d\theta}{dt})^2+\frac{1}{2}\cdot(\frac{2}{5}mr^2)\frac{(R-r)^2}{r^2}(\frac{d\theta}{dt})^2+mg(R-r)\cdot\frac{1}{2}\theta^2 = C$

$\frac{7}{5}(R-r)\omega^2+g\theta^2=C$ 角速度

$\frac{7}{5}\cdot(R-r)\cdot2\omega\cdot\alpha+2g\theta\cdot\omega=0 \Rightarrow \frac{7}{5}(R-r)\alpha+g\theta=0$

$\therefore \omega^2=\frac{g}{\frac{7}{5}(R-r)}$

练习 5.

6) A car of mass (including all passengers inside the car) 1000 kg runs on an expressway(高速公路) that is under construction and has a crack(裂缝) every 15 m. The suspension system of the car has an effective spring constant 2.0×10^5 N/m . At which speed will the passenger feel the strongest oscillation?

A) 85.7m/s B) 67.6 m/s C) 33.8m/s D) 6.66 m/s

谐振

悬挂的 $f=\frac{1}{2\pi}\sqrt{\frac{k}{m}}=2.25\ Hz$

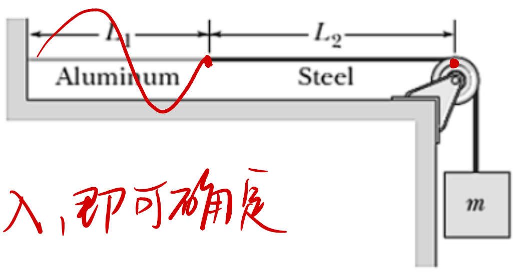
$f\cdot d=v \Rightarrow v=33.8\ m/s$

练习 6. 注意单位

Q7. In figure, an aluminum wire, of length L_1 =60.0 cm, cross-sectional area 1.25×10^{-2} cm, and density 2.60 g/cm^3, is joined to a steel wire, of density 7.80 g/cm^3 and the same cross-sectional area. The compound wire, loaded with a block of mass m =10.0 kg, is arranged so that the distance L_2 from the joint to the supporting pulley is 86.6 cm. Transverse waves are set up on the wire by an external source of variable frequency, a node is located at the pulley, and another node is also located at the left boundary. Find the $\boxed{\text{second}}$ lowest frequency that generates a standing wave having the joint as one of the nodes.

绳上张力相同

L_1 L_2 上频率相同：f 由波源决定



$f=\frac{v_1}{\lambda_1}=\frac{v_2}{\lambda_2}$

$v_1=\frac{\sqrt{mg}}{\sqrt{\rho_1 S}}$ $v_2=\frac{\sqrt{mg}}{\sqrt{\rho_2 S}}$

n_1 确定后，λ_1 即可确定

$n_1=\frac{L_1}{\lambda_1}$ $n_2=\frac{L_2}{\lambda_2}$

$\Rightarrow \frac{n_1}{n_2}=\frac{L_1}{L_2}\times\frac{\lambda_1}{\lambda_2}=\frac{L_1}{L_2}\times\frac{v_1}{v_2}$

$=\frac{L_1}{L_2}\times\sqrt{\frac{\rho_2}{\rho_1}}=\frac{5}{2}$

$n_1=1$, $n_2=2.5$

$n_1=2$, $n_2=5$

9) The temperature of n moles of an ideal monatomic gas is increased by ΔT at constant pressure. The energy Q absorbed as heat, change ΔE_{int} in internal energy, and work W done by the environment are given by:

$\frac{3}{2} nR\Delta T$

$\Delta E = Q + W$

注意 W是谁对谁

A) $Q = (5/2)nR\Delta T$, $\Delta E_{\text{int}} = 0$, $W = -nR\Delta T$

B) $Q = (5/2)nR\Delta T$, $\Delta E_{\text{int}} = (3/2)nR\Delta T$, $W = -nR\Delta T$

C) $Q = (5/2)nR\Delta T$, $\Delta E_{\text{int}} = (5/2)nR\Delta T$, $W = 0$

D) $Q = (3/2)nR\Delta T$, $\Delta E_{\text{int}} = 0$, $W = nR\Delta T$

10) A Carnot heat engine operates between a hot reservoir at absolute temperature T_H and a cold reservoir at absolute temperature T_L. Its efficiency is:

A) T_H / T_L B) T_L / T_H C) $1 - T_H / T_L$ D) $1 - T_L / T_H$ ✓

热学公式:

$PV = nRT$

$E = \frac{3}{2} nRT = Q + W$

$dS = \frac{1}{T} dQ$

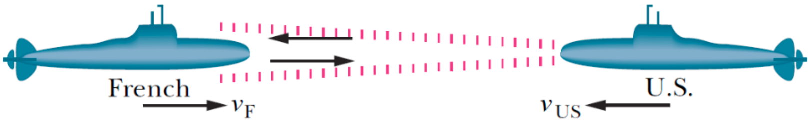
$W = PV$ 可能换成微分形式

练习 7. 多普勒效应

Q8. A French submarine and a U.S. submarine move toward each other during maneuvers in motionless water in the North Atlantic. The French sub moves at speed v_F =50.00 km/h, and the U.S. sub at v_{US}=75.00 km/h. The French sub sends out a sonar signal (sound wave in water) at 1.00×10^3 Hz. Sonar waves travel at 5470 km/h.

(a) What is the signal's frequency as detected by the U.S. sub?

(b) What frequency is detected by the French sub in the signal reflected back to it by the U.S. sub?



耳朵在上方

S	D	f'/f
●→	←●	$\frac{v+v_D}{v-v_s}$
←●	●→	$\frac{v-v_D}{v+v_s}$
●→	●→	$\frac{v-v_D}{v-v_s}$
←●	←●	$\frac{v+v_D}{v+v_s}$

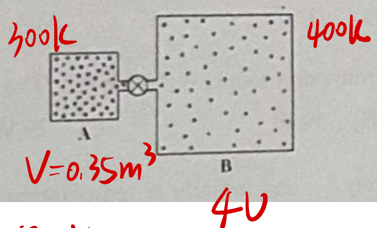
(a) $f' = \frac{V+V_{US}}{V-V_F} f$

(b) U.S. 作为声源，发出频率为 f'。 $f'' = \frac{V+V_F}{V-V_{US}} f'$

热学题记得使用开氏温标！

练习 8.

2. **(3 points)** Container A holds an ideal monatomic gas at a pressure of $\boxed{5.0\times10^5\ \text{Pa}}$ and a temperature of 300 K. It is connected by a thin tube (and a closed valve) to container B, with four times the volume of A. Container B holds the same ideal gas at a pressure of $\boxed{1.0\times10^5\ \text{Pa}}$ and a temperature of 400 K. The valve is opened to allow the pressure to equalize, but the temperature of each container is maintained. If container A holds 0.35 m^3 that gas, what is the net energy transferred as heat to the container during this process?



能量传递由转移的气体造成

$\Delta E = \frac{3}{2} nR\Delta T \longrightarrow$ 求出n即可

计算时注意R的取值

$P_A V_A = n_A R T_A$

$P_B V_B = n_B R T_B$

$P V_A = (n_A - n) R T_A$

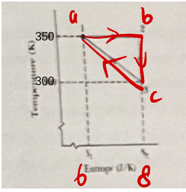
$P V_B = (n_B + n) R T_B$

$\Rightarrow n = n_B$ （先用比值解更简单）

这个条件有用吗?

练习 9.

12) Suppose 2.00 mol of a 双原子的 diatomic gas is taken reversibly around the cycle shown in the T-S diagram of Figure, where $S_1 = 6.00$ J/K and $S_2 = 8.00$ J/K. The molecules do not rotate or oscillate. What is the energy transferred as heat Q for the full cycle?



$$ds = \frac{dQ}{T}$$

$$\Rightarrow dQ = T\,ds.$$

Q 即为 T-S 围成面积

$$\eta = \frac{W \rightarrow \text{面积}}{Q \rightarrow \text{只有AB有}}$$

$$Q = \frac{3}{2}nR(T_B - T_A) + P_0V_0$$

$$2P_0V_0 = nRT_B$$
$$\Rightarrow T_B = \frac{2P_0V_0}{R}$$

$$P_0V_0 = nRT_A$$
$$\Rightarrow T_A = \frac{P_0V_0}{R}$$

$$Q = \cancel{\frac{P_0V_0}{R} + P_0V_0} \quad P_0V_0 + \frac{3}{2}nR\frac{P_0V_0}{R}$$
$$= \frac{5}{2}P_0V_0$$

做功=内能变化.

$$W = \frac{C_v}{R}(P_iV_i - P_fV_f)$$

$$W = P_0V_0 + \frac{C_v}{R}\left(2P_0V_0 - \frac{1}{2}P_0V_0\right) - \frac{C_v}{R}\left(P_0V_0 - \frac{1}{4}P_0V_0\right) - \frac{1}{4}P_0V_0$$

$C_v = \frac{3}{2}R$

$$= P_0V_0 + \frac{3}{2}\times\frac{3}{2}P_0V_0 - \frac{3}{4}P_0V_0$$
$$= \frac{9}{4}P_0V_0$$

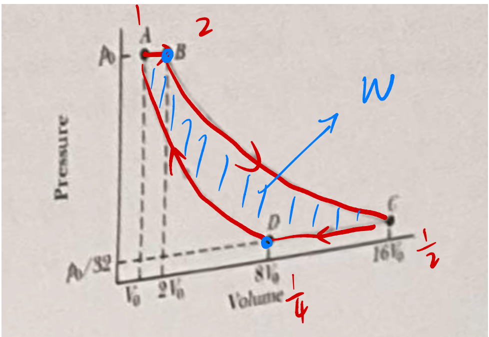
$$\therefore \eta = \frac{W}{Q} = \frac{9}{10}$$

练习 10. 绝热过程常考!

An ideal gas(1.0mol) is the working substance in an engine that operates on the cycle shown in Figure. Processes BC and DA are reversible and adiabatic.

(a) Is the gas monatomic, diatomic, or polyatomic?

(b) What is the engine effiiciency?



(a) $pV^{r} = \text{constant}$

BC: $P_0(2V_0)^{r} = \frac{P_0}{32}(16V_0)^{r}$

$$\Rightarrow r = \frac{5}{3} = \frac{\frac{f}{2}+1}{\frac{f}{2}} \qquad f \text{为自由度}$$

$\Rightarrow f = 3 \qquad \therefore$ 为单原子

单原子 $f=3$. 多原子 $f=5$

(b) $\eta = 1 - \frac{T_{冷}}{T_{热}}$ (卡诺热机成立)

(X) $T_{热}$. $T_{冷}$ 分别表示最热、最冷处的温度

B处 D处

$PV = nRT$ 即可得答案

Expand 1.00 mol of an monatomic gas initially at 8.00 kPa from initial volume $V_i = 1.00\ \text{m}^3$ to final volume $V_f = 2.00\ \text{m}^3$. At any instant during the expansion, the pressure p and volume V of the gas are related by $p = 8.00 \exp[(V_i - V)/a]$, with p in kilopascals, V_i and V in cubic meters, and $a = 1.00\ \text{m}^3$. Assume that the monatomic gas is an ideal gas. 单位！

(a)What is the change of internal energy during that whole process?

(b)How much energy is transferred as heat during the process?

(c)What is the entropy change for the expansion?

(Equation $p = 8.00 \exp[(V_i - V)/a]$ also can be expressed as $p = 8.00e^{\frac{V_i - V}{a}}$.)

(a) $P_f = 8\exp((V_i - V_f)/a) = 2.94\ \text{kPa}$

$\Delta E = \frac{3}{2}nR(T_f - T_i) = -3.17 \times 10^3\ \text{J}$

(b) $W = \int_{V_i}^{V_f} P\,dV = \int_1^2 8\exp(1-V)\,dV \times 10^3 = 5.1 \times 10^3\ \text{J}$

计算器

$\Delta E = Q - W$

(c) $ds = \frac{dQ}{T} = \frac{1}{T}(dE + dw)$

$= \frac{1}{T}\left(\frac{3}{2}nR\,dT + P\,dV\right)$

$\Delta S = \int_{T_i}^{T_f} \frac{3}{2}nR \cdot \frac{1}{T}\,dT + \int_1^2 \frac{1}{T}\, 8\exp(1-V)\,dV$

$PV = nRT$ $\quad\quad \int_1^2 \frac{nR}{PV}\exp(1-V)\,dV$

$= 1.94\ \text{J/K}$