**CSU22012 Algorithms and Data Structures II** **Name:** ______________________
**Hilary Term**

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**In-class test**
**20/03/2023**
**Time Limit: 60 Minutes** Student number ______________________

This exam contains 10 pages (including this cover page) and 2 questions.
Total of points is 100.

All the multiple choice questions require only 1 answer – the question will receive a 0 if multiple answers are chosen.

Answer all the questions directly in the exam paper.

No document permitted. Use of Calculators prohibited.

Grade Table (for teacher use only)

| Question | Marks | Score |
|---|---|---|
| 1 | 50 | |
| 2 | 50 | |
| Total: | 100 | |

1. (a) (2 marks) Multiple Choice: Which of the following statements is true about the Bubble Sort algorithm?

   ○ It is the most efficient sorting algorithm.

   ○ It can recognize an already sorted list and thus can perform in $O(n)$ time in the best case.

   ○ It has a time complexity of $O(logn)$ in the worst case.

   ○ It is often used for sorting large datasets.

   (b) (2 marks) What is the worst-case time complexity of Bubble Sort?

   ○ $O(n)$

   ○ $O(nlogn)$

   ○ $O(n^2)$

   ○ $O(logn)$

   (c) (2 marks) For which of the following scenarios would Insertion Sort be most efficient?

   ○ The input list is nearly sorted.

   ○ The input list is completely random.

   ○ The input list is in reverse order.

   ○ The input list is very large.

   (d) (2 marks) What is the average and worst-case time complexity of QuickSort?

   ○ $O(nlogn)$ for both average and worst-case

   ○ $O(nlogn)$ for average and $O(n^2)$ for worst-case

   ○ $O(n^2)$ for both average and worst-case

   ○ $O(n)$ for average and $O(nlogn)$ for worst-case

   (e) (2 marks) Which sorting algorithm can be most efficient for sorting a small number of elements?

   ○ Quick Sort

   ○ 3-way Quicksort Sort

   ○ Insertion Sort

   ○ Merge Sort

   (f) (2 marks) Which sorting algorithm is considered stable?

   ○ Selection Sort

   ○ QuickSort

   ○ Merge Sort

   ○ 3-way Quicksort

(g) (4 marks) Which algorithm would work best to sort data as it arrives, one piece at a time?

- ○ Merge sort
- ○ Selection sort
- ○ Quicksort
- ○ Insertion sort

(h) (6 marks) Describe a scenario where you might prefer to use a sorting algorithm with $O(n^2)$ complexity over a more efficient algorithm with $O(nlogn)$ complexity.

(i) (6 marks) Why does the choice of pivot in QuickSort affect its performance, and how can this be mitigated?

(j) (10 marks) Illustrate the execution of Selection Sort on the following array.

| E | A | S | Y | S | O | R | T | I | N | G |
|---|---|---|---|---|---|---|---|---|---|---|
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(k) (12 marks) Illustrate the execution of QuickSort on the following array. Make sure to show which elements are selected as pivot at every step of the algorithm.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| E | A | S | Y | S | O | R | T | I | N | G |

2. (a) (2 marks) In a directed graph (digraph), edges are:
   - ○ Always weighted
   - ○ Unidirectional
   - ○ Bidirectional
   - ○ Without any direction

   (b) (2 marks) Prim's algorithm is used to find:
   - ○ A cycle in the graph
   - ○ The maximum spanning tree
   - ○ The minimum spanning tree
   - ○ The shortest path between two vertices

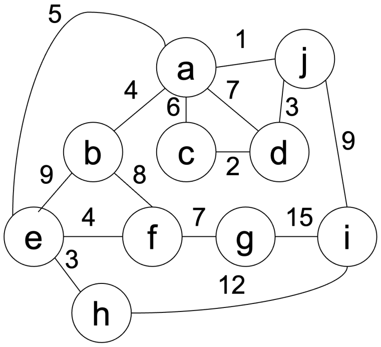
   (c) (2 marks) What does Dijkstra's algorithm compute?
   - ○ Maximum spanning tree
   - ○ Shortest paths from a single source to all other vertices
   - ○ All pairs shortest paths
   - ○ Eulerian path

   (d) (2 marks) A topological sort of a directed acyclic graph (DAG) is:
   - ○ Always unique
   - ○ Not possible if the graph has a cycle
   - ○ Possible only if the graph is complete
   - ○ A sorting of vertices based on their in-degrees

   (e) (6 marks) Why is a graph with negative cycles considered problematic for algorithms like Dijkstra's?

(f) (6 marks) Describe a scenario where topological sorting is necessary.

(g) (10 marks) Represent the graph below using the two representations we have seen in the module (adjacency list and adjacency matrix)

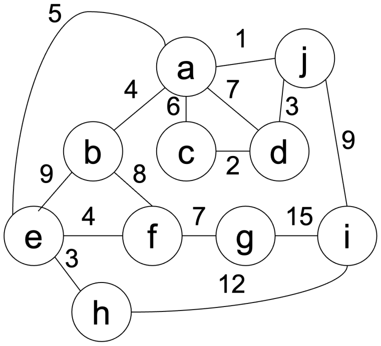


(h) (10 marks) You are given a network of computers in a small company. The network is represented as an undirected graph where computers are vertices, and the edges represent potential direct cable connections between computers with their associated costs. The given graph is as follows: Vertices (Computers): A, B, C, D, E, F, G Edges (Cable connections) with weights (in euros):

- A-B: 2
- A-C: 3
- B-C: 1
- B-D: 5
- B-E: 4
- C-E: 6
- D-E: 2
- D-F: 3
- E-F: 2
- E-G: 3
- F-G: 1

Use one of the algorithms seen in the module to design a new network layout that connects all the computers while minimising the total cost of laying down the cables. Describe how the algorithm works, list the edges included in your design and calculate the total cost of laying down the cables.

(i) (10 marks) Consider the graph below. Run Dijkstra algorithm starting from node/vertex h. In case of a tie, relax the node/vertex that comes first alphabetically. Give the sequence of visited (done) vertices (in the order they are processed) and the shortest distance for every vertex to the source (h).


5
1
a
j
4
7
6
3
9
b
c
d
9
8
2
4
7
15
e
f
g
i
3
12
h