Geophysical joint inversion using joint minimum entropy constraints

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1. Motivation

Geophysical imaging techniques are widely used in numerous fields of research. However, the **solution** to the inverse problem is **inherently non-unique**, meaning that several geophysical models explain the data. It is beneficial to acquire several geophysical datasets to decrease interpretational uncertainties, however separate inversions might produce inconsistent models. **Joint inversion schemes** can be used to face this issue by jointly inverting several multi-physical datasets with structural coupling that **enforce structural similarity in the models**.

Zhdanov et al. (2022) introduce a new approach that includes **joint minimum entropy formulations in a structurally-coupled cooperative inversion (SCCI)** scheme with very promising results on gravity and magnetic field data for mineral exploration purposes. A proof of concept for other near-surface geophysical methods, namely electrical resistivity tomography (ERT) and seismic refraction tomography (SRT), and its extension to 3D, is yet to be presented.

2. Theory of joint minimum entropy stabilizers

The **objective function** ϕ_J of the joint inversion of M geophysical methods can be expressed as:

$$\phi_J\left(\mathbf{m}^{(1)}, \mathbf{m}^{(2)}, \dots, \mathbf{m}^{(i)}\right) = \sum_{i=1}^{M} \phi_d{}^{(i)} + \lambda\, S_J\left(\mathbf{m}^{(1)}, \mathbf{m}^{(2)}, \dots, \mathbf{m}^{(i)}\right),$$

where $\phi_d{}^{(i)}$ describes the data misfit term of the method i, $\mathbf{m}^{(i)}$ the corresponding model parameters, λ the regularization parameter and S_J the **joint stabilizer**. In this study the joint stabilizer is subject to a joint entropy formulation and, therefore, it controls the entropy of the models. After Zhdanov et al. (2022) it is defined as follows:

$$S_J = \sum_{i=1}^{M} \left(\mathbf{m}^{(i)} - \mathbf{m}_{ref}^{(i)}\right)^T W_e^{(i)T} W_e^{(i)} \left(\mathbf{m}^{(i)} - \mathbf{m}_{ref}^{(i)}\right).$$

$W_e^{(i)}$ represents a **diagonal matrix** that holds method-specific model weights ω_{qJME}. Therefore, the regularization can be interpreted as **cell-specific damping** towards some reference model $\mathbf{m}_{ref}$. The weights are **re-calculated after each iteration** as:

$$\omega_{qJME} = \left[\frac{\left|\mathbf{m}^{(i)} - \mathbf{m}_{ref}^{(i)}\right|^q + \beta}{Q_{qJME}\left(\left|\mathbf{m}^{(i)} - \mathbf{m}_{ref}^{(i)}\right|^2 + \beta\right)} \ln \frac{Q_{qJME}}{\sum_{j=1}^{M}\left|\mathbf{m}^{(j)} - \mathbf{m}_{ref}^{(j)}\right|^q + \beta}\right]^{\frac{1}{2}}.$$

Here, q is the order, β a numerical stabilizer and Q_{qJME} is a normalization factor.

3. Implementation

The approach is implemented using the **open-source library pyGIMLi** (Rücker et al., 2017) as in **Figure 1**. The original formulation was extended to include additional smoothing and method weighting factors. The **governing parameters** that influence the joint inversion are:

- Order q
- Regularization parameter λ
- Smoothing factor a
- Method weighting factor mw

Exemplary model weights of simple 2D models are shown **in Figure 2**.

- **Strong model contrast → less damping →** promote changes
  → especially in overlapping region
- **Close to background → stronger damping** towards background
- Method weighting factor controlling presence of methods in weights

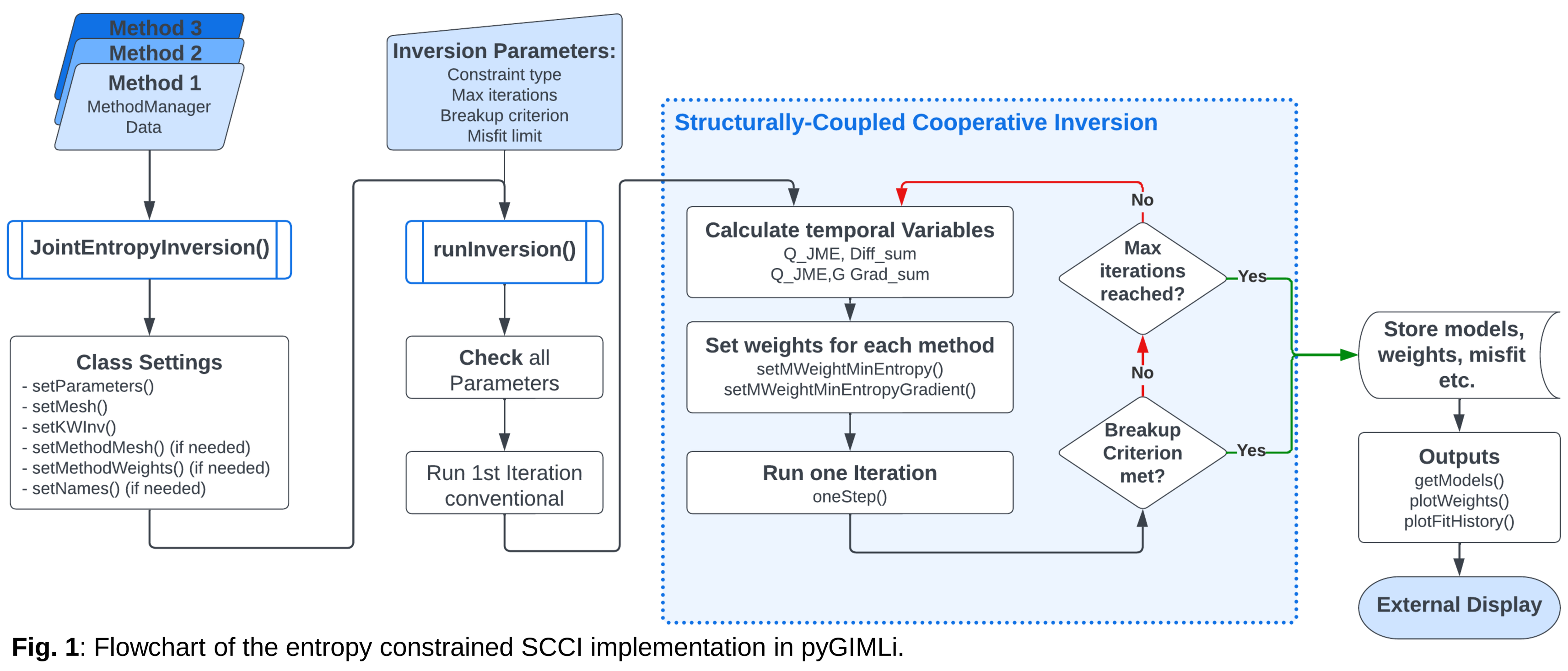

Fig. 1: Flowchart of the entropy constrained SCCI implementation in pyGIMLi.

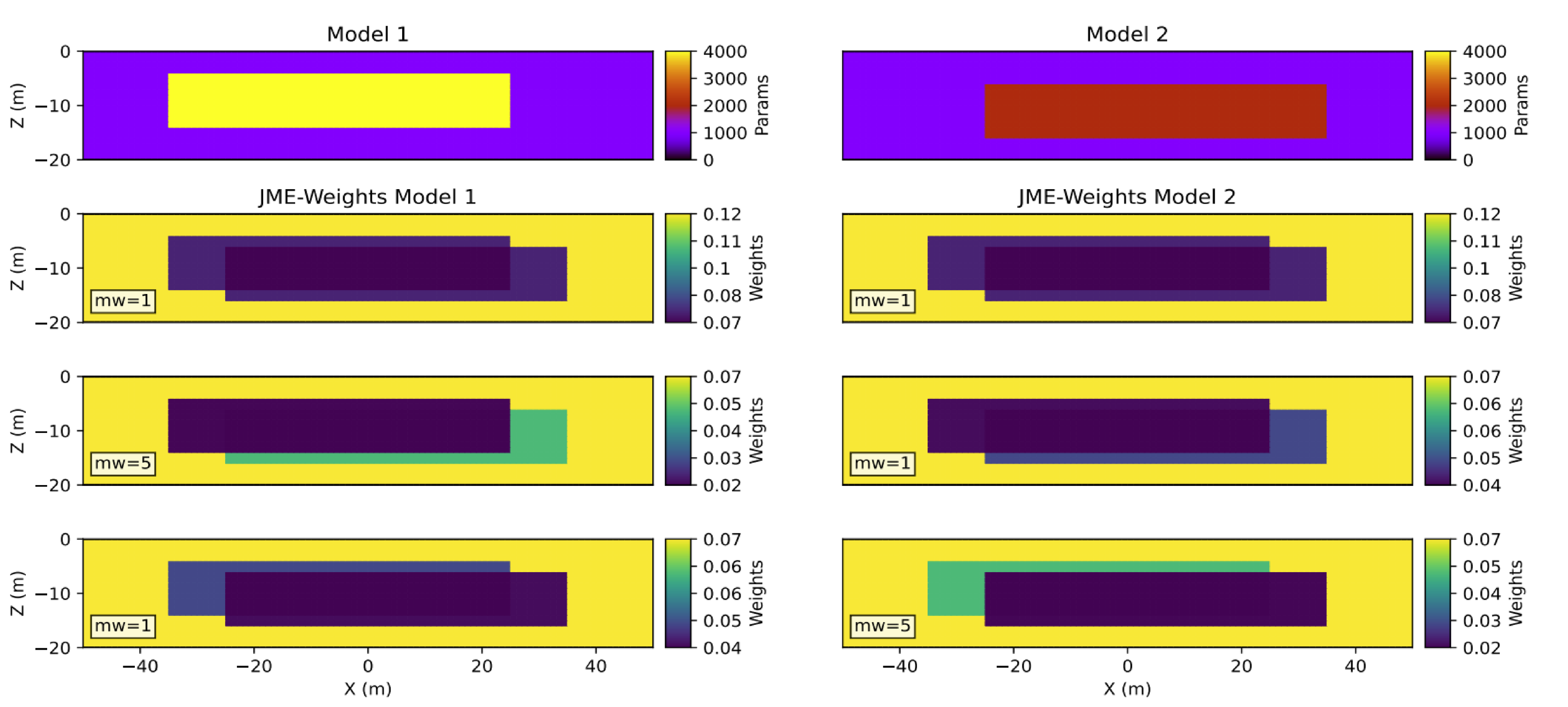

Fig. 2: Example of joint minimum for simple 2D models. The reference model is a homogeneous model. The rows correspond to related weights. The method weighting factor for both models are indicated by the text box in bottom left corner of each model weights plot. Note that following parameters were fixed: $q = 1,\ \beta = 1e-5$.

4. Synthetic Models

Synthetic ERT and SRT data sets were generated using the 2D seismic velocity and electrical resistivity models shown in **Figure 3**.

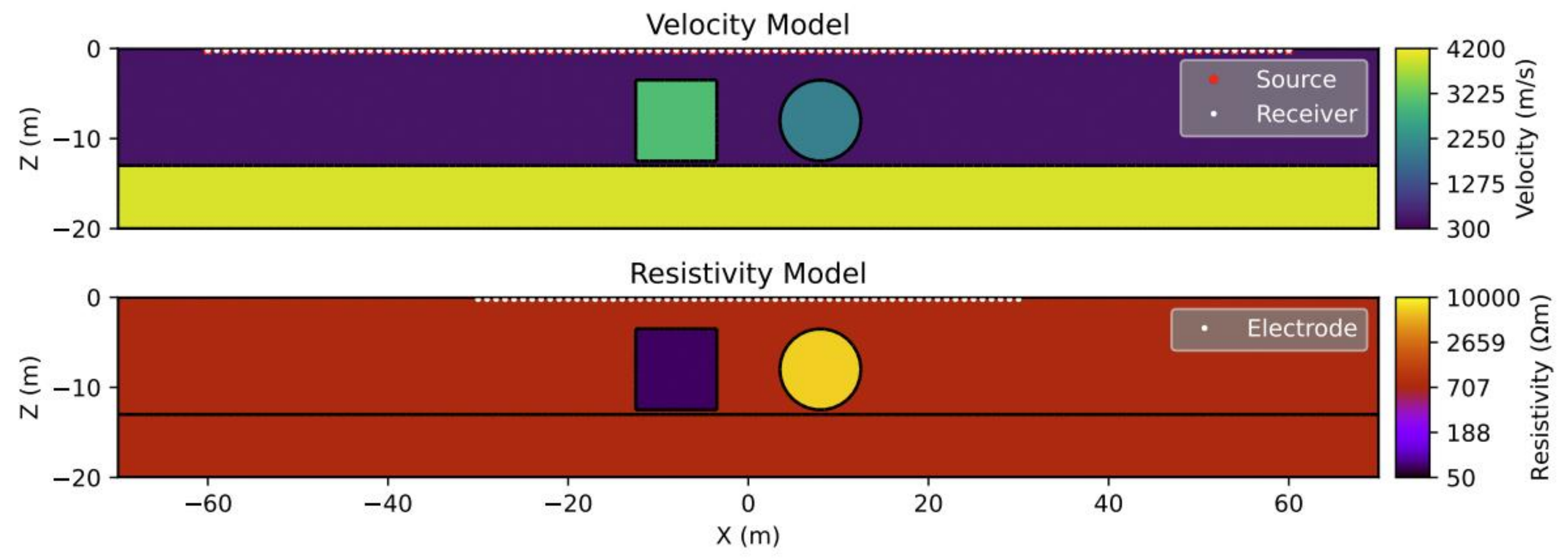

Fig. 3: Synthetic velocity model (top) and resistivity model (bottom) used for the synthetic SRT and ERT data study, respectively.

5. Results

The results are presented in **Figure 4**. Note that the fourth row indicates a joint inversion with stronger weighting of the ERT method as the conventional results look more promising for that method than for the SRT method. The **main findings** are:

- Second order beneficial for ERT method but not for the SRT method as the entropy constraint causes collapse of high-velocity zones
  → second-order not suitable for joint inversion
- First-order **joint minimum entropy inversion leads to improvements** in misfit and imaging
  → right velocity anomaly and bottom high-velocity layer are improved
  → left velocity anomaly is more homogeneous
  → left resistivity anomaly develops two zones due to entropy constraint
- An increased **method weighting** on ERT shows **no significant effects**
- The presented approach **does not enforce bottom layer** of the SRT method on the resistivity model
- The approach is **computationally cheap** in comparison to other structurally-coupled joint inversion approaches, like cross-gradients, as no model gradients need to be calculated (Wagner et al., 2021).

This study confirms that the **joint minimum entropy inversion** proposed by Zhdanov et al. (2022) **can be generalized** for other geophysical methods. Further field data studies will reveal its full potential and the impact it can make for near-surface geophysical imaging.

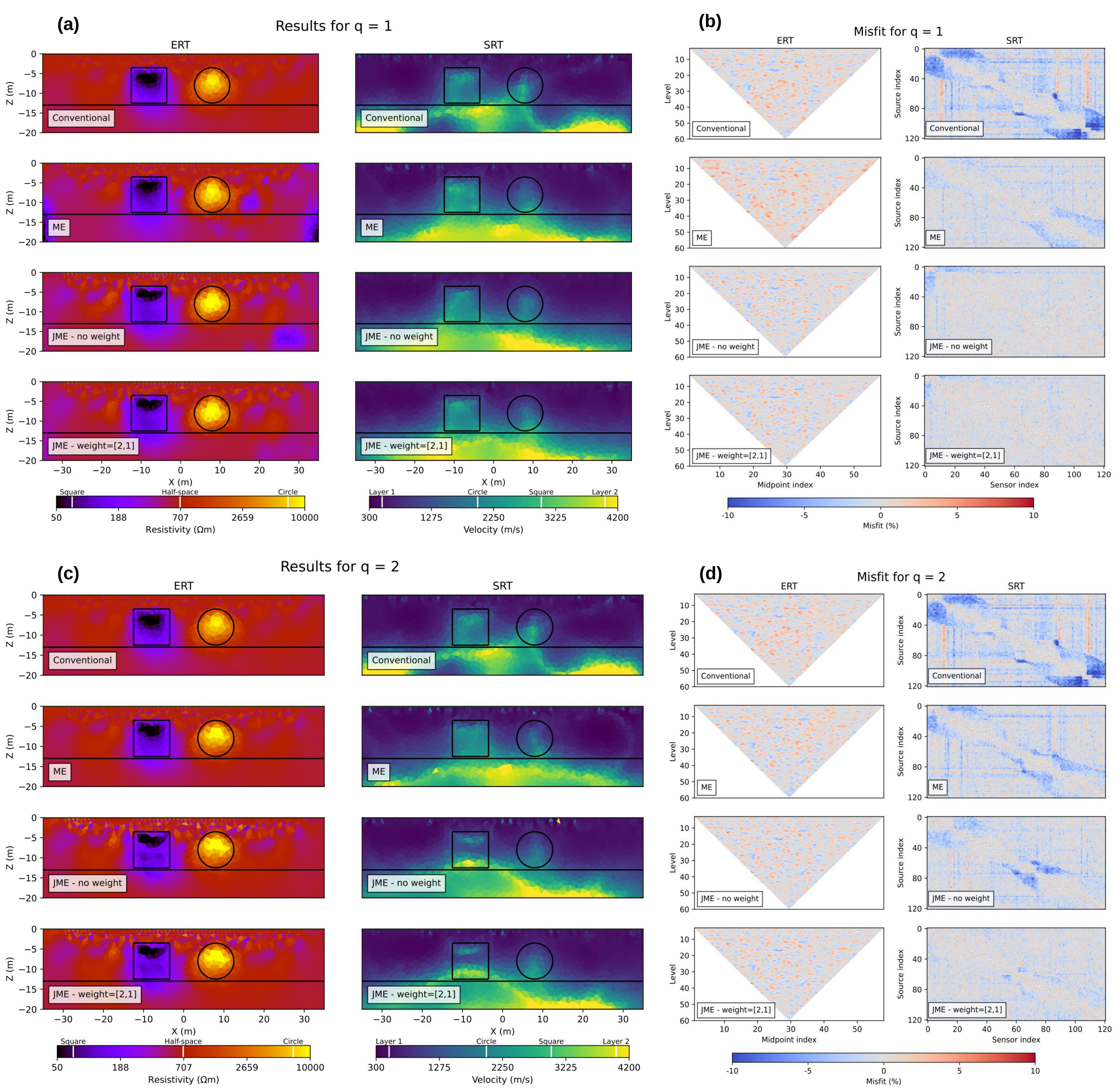

Fig. 4: Comparison of conventional inversion, standalone ME inversion as well as unweighted and weighted JME inversion results of ERT and SRT data. Shown are the estimated model parameters of the last iteration (a), (c) and the corresponding data misfit (b), (d) for first and second-order constraints, respectively.
The entropy inversions have the following fixed parameters: $\beta = 1e-5, \lambda_{ERT} = 0.003, \lambda_{SRT} = 0.01, a_{ERT} = 15, a_{SRT} = 90$.

References

Rücker, C., Günther, T., and Wagner, F. M.: pyGIMLi: An open-source library for modelling and inversion in geophysics, Computers & Geosciences, 109, 106–123, 2017. DOI: 10.1016/j.cageo.2017.07.011
Wagner, F. M. and Uhlemann, S.: An overview of multimethod imaging approaches in environmental geophysics, Advances in Geophysics, 62, 1–72, 2021. DOI: 10.1016/bs.agph.2021.06.001
Zhdanov, M. S., Tu, X., and Cuma, M.: Cooperative inversion of multiphysics data using joint minimum entropy constraints, Near Surface Geophysics, 20, 623–636, 2022. DOI: 10.1002/nsg.12203

GitHub repository with additional material

www.gim.rwth-aachen.de

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