Gradient Descent as Implicit EM in Distance-Based Neural Models

Alan Oursland

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Abstract

Neural networks trained with standard objectives exhibit behaviors characteristic of probabilistic inference: soft clustering, prototype specialization, and Bayesian uncertainty tracking. These phenomena appear across architectures—in attention mechanisms, classification heads, and energy-based models—yet existing explanations rely on loose analogies to mixture models or post-hoc architectural interpretation. We provide a direct derivation. For any objective with log-sum-exp structure over distances or energies, the gradient with respect to each distance is exactly the negative posterior responsibility of the corresponding component: $\partial L/\partial d_j = -r_j$. This is an algebraic identity, not an approximation. The immediate consequence is that gradient descent on such objectives performs expectation-maximization implicitly—responsibilities are not auxiliary variables to be computed but gradients to be applied. No explicit inference algorithm is required because inference is embedded in optimization. This result unifies three regimes of learning under a single mechanism: unsupervised mixture modeling, where responsibilities are fully latent; attention, where responsibilities are conditioned on queries; and cross-entropy classification, where supervision clamps responsibilities to targets. The Bayesian structure recently observed in trained transformers is not an emergent property but a necessary consequence of the objective geometry. Optimization and inference are the same process.

1 Introduction

Neural networks trained with standard objectives repeatedly exhibit behaviors associated with probabilistic inference: soft clustering, prototype specialization, uncertainty tracking, and mixture-model dynamics. These phenomena appear across architectures—in attention mechanisms, classification heads, and energy-based models—yet their origin has remained unclear. Are they emergent properties of scale? Architectural accidents? Or something more fundamental? This paper argues that such behaviors are none of these. They are necessary consequences of the geometry of common objective functions.

1.1 The Puzzle

Consider the range of phenomena that appear, unbidden, in trained neural networks. Attention heads in transformers learn to specialize, each routing information for distinct semantic roles. Classification networks partition representation space into regions that behave like mixture components. Deep networks trained on noisy data exhibit robustness patterns reminiscent of Bayesian inference, down-weighting outliers and tracking uncertainty across inputs. These behaviors emerge without explicit probabilistic modeling, without mixture-model architectures, and without any algorithm resembling expectation-maximization.

The standard explanations are unsatisfying. One view holds that these are emergent properties of scale—that sufficient parameters and data somehow give rise to statistical structure. Another treats

them as architectural coincidences, artifacts of specific design choices like softmax normalization or residual connections. A third offers loose analogies: attention is "like" soft clustering; cross-entropy "resembles" a mixture model. None of these explanations answers the deeper question: *why do these specific behaviors appear, rather than others?* And why do they appear so reliably, across such different architectures and tasks?

1.2 Recent Evidence

Recent work by Aggarwal et al. [2025a,b] has sharpened this puzzle considerably. In controlled experimental settings—"Bayesian wind tunnels" where the true posterior is analytically known—small transformers reproduce Bayesian posteriors with sub-bit precision. This is not approximate Bayesian behavior; it is exact, verifiable, position-by-position agreement with the analytic solution. Capacity-matched MLPs trained under identical conditions fail completely, suggesting that the phenomenon depends on the inductive biases of attention rather than optimization alone.

More striking still is what Aggarwal et al. find in the gradient dynamics. Attention weights stabilize early in training while value vectors continue to refine—a two-timescale structure that mirrors the E-step and M-step of classical expectation-maximization. Values receive updates weighted by attention, precisely as prototypes receive updates weighted by responsibilities in mixture models. The authors provide a complete first-order analysis showing that this structure is not incidental but systematic.

Yet Aggarwal et al. explicitly characterize this EM connection as "structural rather than variational"—an analogy, not a derivation. They observe that attention *behaves like* EM but do not claim that EM is a necessary consequence of the objective. The question of *why* cross-entropy training produces Bayesian geometry remains, in their framing, open.

1.3 This Paper

This paper closes that gap. We show that for objectives with log-sum-exp structure over distances or energies, the gradient of the loss with respect to each distance is exactly the posterior responsibility of the corresponding component. This is not an approximation, not a resemblance, not an analogy. It is an algebraic identity:

$$\frac{\partial L}{\partial d_j} = -r_j \tag{1}$$

The immediate consequence is that gradient descent on such objectives *is* expectation-maximization, performed continuously rather than in discrete alternating steps. The forward pass computes unnormalized likelihoods; normalization yields responsibilities; backpropagation delivers responsibility-weighted updates to parameters. No auxiliary latent variables are introduced. No inference algorithm is invoked. The architecture does not approximate EM—it implements EM, because the gradient *is* the responsibility.

This reframes the relationship between optimization and inference. Inference is not a separate algorithmic layer added on top of learning. It is not a post-hoc interpretation of learned representations. Under the objectives we analyze, inference and optimization are the same computation viewed at different levels of abstraction. The Bayesian structure observed by Aggarwal et al. is not an emergent property that happens to appear; it is forced by the geometry of the loss.

1.4 Contributions

The contributions of this paper are deliberately narrow and can be summarized as three claims at increasing levels of abstraction.

One theorem. For any objective of the form $L = \log \sum_j \exp(-d_j)$, the gradient with respect to each distance satisfies $\partial L / \partial d_j = -r_j$, where r_j is the normalized exponential—the responsibility of component j. This result is derived, not assumed, and requires no conditions beyond differentiability.

One interpretation. This identity implies that gradient descent on distance-based log-sum-exp objectives performs implicit expectation-maximization. The E-step is the forward pass; the M-step is the parameter update; responsibilities are never explicitly computed because they *are* the gradients. EM is not approximated by neural training—it is realized by it.

One unification. The same mechanism manifests across three regimes depending on the constraints imposed. In the *unsupervised* regime, responsibilities are fully latent and prototypes compete freely. In the *conditional* regime—attention—responsibilities are recomputed per query over a shared prototype family. In the *constrained* regime—cross-entropy classification—supervision clamps one responsibility while competition persists among alternatives. These are not three different phenomena. They are one phenomenon under different boundary conditions.

2 The Geometric Substrate

The main result of this paper—that responsibilities emerge as gradients—rests on a specific geometric foundation. This section establishes that foundation. We first summarize the interpretation of neural network outputs as distances or energies rather than confidences, importing this result from prior work. We then define the class of log-sum-exp objectives to which our analysis applies. Finally, we briefly recall the structure of classical expectation-maximization, not because our method requires it, but to establish the reference point against which implicit EM will be compared.

2.1 Distance-Based Representations

The standard interpretation of neural network outputs treats them as confidences or scores indicating the strength of evidence for a hypothesis. A logit is high when the network "believes" in a class; an attention score is high when a query "matches" a key. This interpretation, while intuitive, obscures the geometric structure of what neural networks actually compute.

In prior work [Oursland, 2024], we established a different interpretation: the outputs of standard neural networks are better understood as distances or energies relative to learned prototypes. A linear layer followed by an absolute value or ReLU activation computes a quantity that behaves as a (possibly signed) deviation from a learned reference—a Mahalanobis distance along a principal direction. Concretely, the Mahalanobis distance of a point x from a Gaussian component with mean μ and principal direction v scaled by eigenvalue λ is $|\lambda^{-1/2} v^\top (x - \mu)|$, which has the form $|Wx + b|$ for appropriate W and b. Standard ReLU networks compute this via the identity $|z| = \mathrm{relu}(z) + \mathrm{relu}(-z)$, decomposing signed distance into two half-space detectors. Under this view, a low output indicates proximity to a prototype; a high output indicates deviation. Probabilities are not primitive quantities but derived ones, arising only after exponentiation and normalization transform distances into relative likelihoods.

This interpretation is not a modeling choice imposed for convenience. It is a mathematical property of affine transformations composed with piecewise-linear activations. The weights of a linear layer define a basis; the biases define offsets along that basis; the activation measures deviation. What changes is not the computation but the semantics we assign to it. Throughout this paper, we adopt

the distance-based interpretation and refer to neural outputs as energies or distances interchangeably. The results that follow—particularly the identification of gradients with responsibilities—depend on this geometric framing.

2.2 The Log-Sum-Exp Objective

Given a set of distances or energies $\{d_1, d_2, \ldots, d_K\}$ computed for an input x, we consider objectives of the form:

$$L(x) = \log \sum_{j=1}^{K} \exp(-d_j(x)) \tag{2}$$

This is the log-sum-exp (LSE) objective. It has a natural interpretation: if $\exp(-d_j)$ represents the unnormalized likelihood that component j generated the input, then L is the log marginal likelihood—the log-probability that *some* component generated it. Maximizing L encourages the model to place at least one prototype close to each input. Minimizing the negative of L encourages separation.

We adopt the following notation throughout. Let $P_j = \exp(-d_j)$ denote the unnormalized likelihood of component j. Let $Z = \sum_k P_k = \sum_k \exp(-d_k)$ denote the partition function. Define the responsibility of component j as:

$$r_j = \frac{P_j}{Z} = \frac{\exp(-d_j)}{\sum_k \exp(-d_k)} \tag{3}$$

The responsibilities are non-negative and sum to one. They represent the posterior probability that component j is responsible for the input, under a uniform prior over components.

This structure appears throughout deep learning under various names. In cross-entropy classification, the logits z_j act as negative distances $(d_j = -z_j)$, and the objective becomes $L = -z_y + \log \sum_k \exp(z_k)$, where y is the label. In attention mechanisms, the scores s_{ij} act as negative distances between query i and key j, and softmax normalization produces responsibilities that weight the value vectors. These are not analogies to the LSE objective; they are instances of it.

2.3 Classical EM

Expectation-maximization is the classical algorithm for fitting mixture models with latent assignments. It proceeds in two alternating steps.

In the E-step, responsibilities are computed. Given current parameters, each data point is softly assigned to each component based on relative likelihood:

$$r_j^{(t)} = \frac{P(x \mid \theta_j)}{\sum_k P(x \mid \theta_k)}$$

These responsibilities sum to one and represent the posterior probability that component j generated the observation.

In the M-step, parameters are updated. Each component's parameters are adjusted to better fit the data points assigned to it, weighted by responsibility:

$$\theta_j^{(t+1)} = \arg\max_{\theta_j} \sum_i r_{ij}^{(t)} \log P(x_i \mid \theta_j)$$

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For Gaussian mixtures, this reduces to computing responsibility-weighted means and covariances. The key property is that every data point influences every component, but the influence is gated by how much responsibility that component holds for that point.

Classical EM is discrete and alternating: compute all responsibilities, then update all parameters, then repeat. The E-step and M-step are separate procedures with distinct computational roles. This separation is algorithmic, not fundamental.

3 Main Result: Responsibilities Are Gradients

We now state and derive the central result. The derivation is elementary—a single application of the chain rule—but its implications are significant. We show that for log-sum-exp objectives over distances, the gradient with respect to each distance is exactly the negative responsibility of the corresponding component. This identity requires no approximations and holds for any model that computes distances and optimizes an LSE objective via gradient descent. The derivative itself is a textbook calculation; the contribution is recognizing that the resulting quantity is exactly the posterior responsibility, and that this makes every gradient step a responsibility-weighted update.

3.1 Derivation

Let $d_j(x)$ denote the distance or energy associated with component j for input x. Define unnormalized likelihoods $P_j = \exp(-d_j)$ and the partition function $Z = \sum_k \exp(-d_k)$. Consider the log-sum-exp objective:

$$L = \log \sum_{j=1}^{K} \exp(-d_j) = \log Z \tag{4}$$

We compute the gradient of L with respect to a single distance d_i:

$$\frac{\partial L}{\partial d_i} = \frac{1}{Z} \cdot \frac{\partial}{\partial d_i} \sum_{j=1}^{K} \exp(-d_j)$$

Only the i-th term in the sum depends on d_i:

$$\frac{\partial L}{\partial d_i} = \frac{1}{Z} \cdot \frac{\partial}{\partial d_i} \exp(-d_i) = \frac{1}{Z} \cdot (-1) \cdot \exp(-d_i)$$

Simplifying:

$$\frac{\partial L}{\partial d_i} = -\frac{\exp(-d_i)}{\sum_k \exp(-d_k)}$$

The right-hand side is the negative of the normalized exponential—precisely the responsibility r_i as defined in Section 2.2. This yields the main result.

Theorem 1. *For any objective of the form $L = \log \sum_j \exp(-d_j)$, the gradient with respect to the j-th distance is the negative responsibility of component j:*

$$\frac{\partial L}{\partial d_j} = -r_j \quad where \quad r_j = \frac{\exp(-d_j)}{\sum_k \exp(-d_k)} \tag{5}$$

No approximations have been made. The identity holds exactly whenever the objective has LSE form and the distances are differentiable.

3.2 What This Means

The theorem admits a direct interpretation: responsibilities are not quantities that must be computed and stored. They are gradients. Any system that computes distances, applies a log-sum-exp objective, and updates parameters via gradient descent is already computing responsibilities—not as an intermediate step, but as the learning signal itself.

Note that the identity itself is purely algebraic—it holds for any log-sum-exp objective regardless of how one interprets the quantities involved. The EM interpretation, however, depends on reading the d_j as distances and the r_j as posterior responsibilities. The mathematics forces the gradient structure; the semantics give it meaning.

Consider what happens during training. The forward pass computes distances d_j and evaluates the objective $L = \log \sum_j \exp(-d_j)$. This computation implicitly determines the responsibilities $r_j = \exp(-d_j)/Z$, but they are never explicitly represented. Backpropagation then computes $\partial L/\partial d_j = -r_j$, and the chain rule propagates this gradient to the parameters that produced d_j. The parameters of component j receive gradient signal proportional to r_j—the responsibility of that component for the input.

This is exactly the M-step of expectation-maximization. In classical EM, responsibilities are computed in the E-step and then used to weight parameter updates in the M-step. Here, no E-step is required as a separate operation. The forward pass *is* the E-step; the backward pass *is* the M-step. The two are fused into a single gradient computation.

The consequence is that gradient descent on LSE objectives is exactly EM. It is not an approximation of EM. The discrete alternation of classical EM—compute responsibilities, then update parameters, then repeat—collapses into continuous, simultaneous optimization. Every gradient step is a responsibility-weighted update. Every trained network has been doing EM all along.

Throughout this paper, "implicit EM" refers to the emergence of responsibility-weighted parameter updates under gradient descent on log-sum-exp objectives—not to coordinate-ascent EM or guarantees about convergence.

3.3 Conditions

The result depends on three structural conditions. When all three are satisfied, implicit EM dynamics are not optional—they are forced by the mathematics.

Exponentiation. The distances must be passed through an exponential to form unnormalized likelihoods $P_j = \exp(-d_j)$. This transformation converts additive differences in distance into multiplicative ratios in likelihood. Without exponentiation, the gradient with respect to d_j does not yield a normalized quantity, and responsibilities do not emerge.

Normalization. The likelihoods must be normalized across alternatives, either explicitly through softmax or implicitly through the log-sum-exp structure. Normalization induces competition: increasing the likelihood of one component necessarily decreases the relative likelihood of others. This competition is what makes responsibilities sum to one and what causes gradient signal to be distributed according to relative fit. Without normalization, components operate independently, and there is no assignment structure.

Gradient-based optimization. The parameters must be updated via gradients of the objective. The identity $\partial L/\partial d_j = -r_j$ is a fact about the loss surface; it becomes a fact about learning dynamics only when gradient descent (or a variant) is used. Other optimization methods—evolutionary strategies, random search, discrete updates—would not automatically inherit the responsibility-weighted structure.

When these conditions hold, there is no additional design choice that enables or disables EM-like

behavior. It is a consequence of the objective geometry. Any architecture that computes distances, normalizes via exponentiation, and trains with gradients will exhibit implicit EM.

4   Three Regimes of Implicit Inference

The same mechanism manifests differently under different constraints:

4.1   Unsupervised Regime: Mixture Learning

In the purest case, the log-sum-exp objective operates without supervision. A model computes distances $d_j(x)$ from an input to each of K components, and training minimizes:

$$L = -\log \sum_{j=1}^{K} \exp(-d_j) \tag{6}$$

This is the negative log marginal likelihood—the objective used in classical mixture model fitting. All components compete for every input. No labels constrain which component should win.

By Theorem 1, the gradient with respect to each distance is the negative responsibility: $\partial L/\partial d_j = -r_j$. (The sign flip from Section 3 reflects the switch from maximizing log-likelihood to minimizing its negative; the dynamics are identical.) Components that are close to an input (low d_j, high r_j) receive strong gradient signal to move closer. Components that are far (high d_j, low r_j) receive weak signal and remain largely unchanged. This is precisely the M-step dynamic of Gaussian mixture models: prototypes update toward data points in proportion to their responsibility for those points.

The result is spontaneous specialization. Even with random initialization, components differentiate over training. Each prototype drifts toward a region of input space for which it consistently takes high responsibility, while ceding other regions to competitors. Clustering emerges not because it was specified but because the objective geometry forces responsibility-weighted updates.

This regime corresponds exactly to classical EM on mixture models. The fixed points—and the path toward them—are governed by the same responsibilities.

4.2   Conditional Regime: Attention Mechanisms

Attention mechanisms instantiate the same structure in a conditional setting. For a query q_i and a set of keys $\{k_j\}$, attention scores are computed as:

$$s_{ij} = \frac{q_i^\top k_j}{\sqrt{d_k}}$$

These scores act as negative distances: high scores indicate proximity in the query-key space. Softmax normalization converts scores to attention weights:

$$\alpha_{ij} = \frac{\exp(s_{ij})}{\sum_m \exp(s_{im})}$$

The attention weights satisfy the definition of responsibilities exactly. They are non-negative, sum to one across keys, and represent the degree to which each key "explains" the query. The output is a responsibility-weighted combination of values: $o_i = \sum_j \alpha_{ij} v_j$. This is not an analogy to mixture modeling. It is mixture modeling, conditioned on the query.

A clarification is essential here. In a Gaussian mixture model, the prototypes—the component means—are persistent parameters updated across training. In attention, the value vectors $v_j = W_V x_j$

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are input-derived. They change with every forward pass. This appears to break the analogy: what is being learned if the prototypes are transient?

The resolution is that the prototype is not the value vector itself but the value projection. The matrix W_V defines a learned mapping from inputs to prototype space. Each input indexes into this mapping, producing a transient value, but the mapping itself is persistent. Gradients flow through the attention weights to W_V via the chain rule, and these gradients are responsibility-weighted. The update to W_V from input x_j is scaled by how much responsibility the corresponding value took across queries. This is the M-step, applied not to fixed prototypes but to the parameters of a prototype-generating function.

The EM dynamics thus operate across training, not within a single forward pass. Each forward pass computes responsibilities (attention weights) given current parameters. Each backward pass updates those parameters in proportion to the responsibilities. Over the course of training, W_V specializes: it learns to produce values that are useful precisely to the queries that attend to them.

Attention is conditional mixture inference. The query specifies the context; the keys define candidate components; the values represent what each component contributes. The mixture is re-instantiated for every query, but the underlying parameters—W_Q, W_K, W_V—accumulate responsibility-weighted updates across all queries and all training examples. This is implicit EM in function space rather than data space.

4.3 Constrained Regime: Cross-Entropy Classification

Cross-entropy classification introduces supervision, but it does not escape the implicit EM framework. It constrains it.

In standard classification, the model produces logits z_j for each class. Interpreting logits as negative distances ($d_j = -z_j$), the cross-entropy loss for a sample with label y is:

$$L = -z_y + \log \sum_k \exp(z_k) = d_y + \log \sum_k \exp(-d_k)$$

This is the log-sum-exp objective plus a term that penalizes the distance to the correct class. Computing the gradient with respect to d_j:

$$\frac{\partial L}{\partial d_j} = \frac{\exp(-d_j)}{\sum_k \exp(-d_k)} - \mathbb{1}[j = y] = r_j - \mathbb{1}[j = y]$$

For incorrect classes ($j \neq y$), the gradient is simply r_j: they are pushed away in proportion to their current responsibility. For the correct class ($j = y$), the gradient is $r_y - 1$: it is pulled closer, with strength proportional to how much responsibility it is *missing*.

The label acts as a clamp. In unsupervised mixture learning, responsibilities are entirely latent— the data determines which component wins. In cross-entropy, the label declares that the correct class *should* have responsibility 1. The gradient then corrects any deviation from this target. If the correct class already dominates ($r_y \approx 1$), the gradient is small. If incorrect classes have stolen probability mass ($r_y < 1$), the gradient is large.

Competition among incorrect classes remains intact. When the model misclassifies, the responsibility mass is distributed among wrong answers, and each receives gradient signal proportional to its share. The classes that are "most wrong"—those with highest r_j—are penalized most strongly. This is not uniform repulsion; it is responsibility-weighted correction.

Cross-entropy does not eliminate the EM dynamics; it directs them. The M-step still updates components in proportion to their responsibilities, but supervision biases the process toward a predetermined assignment. This explains why cross-entropy is so effective despite its simplicity: it

inherits the soft competition and automatic weighting of mixture models while channeling those dynamics toward a supervised target. The loss function is doing more work than its familiar form suggests.

4.4 The Taxonomy

The three regimes—unsupervised, conditional, constrained—differ in what is observed and what is latent. But they share a common structure: exponentiation of distances followed by normalization across alternatives. This structure is what produces responsibilities, and responsibilities are what produce implicit EM.

The critical factor is normalization. When outputs are normalized—whether by softmax, by the log-sum-exp partition function, or by any operation that forces a sum-to-one constraint—components compete. An increase in one component's likelihood necessarily decreases the relative likelihood of others. This competition is the source of assignment: each input is probabilistically allocated across components, and gradients distribute accordingly.

Remove normalization, and the structure collapses. Consider objectives based on Gaussian kernels without a partition function, such as maximum correntropy:

$$L = 1 - \exp(-d^2/\sigma^2)$$

Here, each component operates independently. A point far from all prototypes produces vanishing gradients for all of them—not because responsibility has been assigned elsewhere, but because no competition exists to distribute. There is no implicit E-step because there are no responsibilities. The objective gains robustness to outliers (points far from all prototypes are effectively ignored) but loses the assignment structure entirely.

This clarifies the design space. Exponentiation converts distances to likelihoods; normalization converts likelihoods to responsibilities. With both, implicit EM is unavoidable. With exponentiation alone, the model gains robustness but abandons inference. The choice of objective is a choice about whether the model should assign or ignore—and that choice is made at the level of the loss function, not the architecture.

5 Relation to Prior Work

The result derived here does not stand alone. It depends on prior work for its geometric foundation and gains significance from recent empirical findings that it explains. This section positions the contribution relative to three bodies of work: the distance-based interpretation of neural representations, recent evidence of Bayesian structure in transformers, and classical results on energy-based learning and expectation-maximization.

5.1 Prior Work on Distance-Based Representations

In prior work [Oursland, 2024], we established the geometric interpretation on which this paper depends. That work shows that standard neural network layers—affine transformations followed by ReLU or absolute value activations—compute quantities that behave as distances from learned prototypes. Outputs are deviations, not confidences; they measure how far an input lies from a reference structure defined by the weights. This interpretation is not imposed but derived from the mathematics of the operations involved.

The present work takes this geometric substrate as given and asks a different question: what happens when distance-based representations are optimized under standard objectives? Our prior

work is silent on learning dynamics. It characterizes what neural networks represent, not how they learn. The contribution here is to show that log-sum-exp objectives over distances produce responsibility-weighted gradients, and that this induces implicit EM.

The two results are complementary and together form a complete picture. The first says: neural networks compute distances. The second says: optimizing distances with LSE objectives performs inference. Neither subsumes the other. Without the distance interpretation, the identification of gradients with responsibilities is a formal curiosity. Without the gradient identity, the distance interpretation describes static representations with no account of how they emerge. The geometric substrate enables the learning dynamics; the learning dynamics justify attention to the geometric substrate.

5.2   Aggarwal et al. (2025a,b)

Aggarwal et al. [2025a] provide striking empirical evidence that transformers perform exact Bayesian inference. In controlled settings where the true posterior is analytically known, small transformers reproduce Bayesian posteriors with sub-bit precision. This is not approximate or Bayesian-like behavior; it is exact agreement, verified position by position against closed-form solutions. The result establishes that transformers *can* implement Bayesian computation—but does not explain why they *must*.

Aggarwal et al. [2025b] move from statics to dynamics, analyzing the gradients of attention under cross-entropy training. They derive that value vectors receive responsibility-weighted updates and that attention scores adjust according to an advantage-like rule. They observe a two-timescale structure: attention patterns stabilize early while values continue to refine—mirroring the E-step and M-step of classical EM. The analysis is thorough and the EM parallel is explicit.

Yet the authors are careful to characterize this connection as "structural rather than variational." They observe that attention *behaves like* EM but stop short of claiming that EM is a necessary consequence of the objective. The resemblance is documented; the derivation is not attempted.

The present work provides that derivation. Under the distance-based interpretation of neural outputs, the gradient identity $\partial L / \partial d_j = -r_j$ is not a structural analogy but an algebraic fact. The EM-like dynamics Aggarwal et al. observe are not emergent properties that happen to appear in transformers—they are forced by the geometry of the loss function. Any model that computes distances and optimizes a log-sum-exp objective will exhibit the same dynamics, whether or not it resembles a transformer.

The relationship between the two contributions is observation to explanation. Aggarwal et al. discover and document empirical evidence of the phenomenon with precision and rigor. We provide the theoretical mechanism that makes the phenomenon inevitable.

5.3   Other Connections

The energy-based learning framework of LeCun et al. [2006] provides essential conceptual scaffolding. That work reframes learning as the minimization of an energy function, with probabilities derived via exponentiation and normalization. The log-sum-exp objective analyzed here is a special case of the "free energy" formulation in energy-based models. Our contribution is to show that the gradient of this free energy with respect to component energies is exactly the posterior responsibility—a connection implicit in the framework but not, to our knowledge, previously stated as a formal identity.

Dempster et al. [1977] introduced expectation-maximization as an algorithm for maximum likelihood estimation with latent variables. The E-step and M-step are defined as discrete, alternating

operations. The present work shows that for distance-based objectives, these steps collapse into gradient descent: the forward pass computes responsibilities implicitly, and backpropagation applies them. This does not contradict the classical formulation but reveals it as a special case of a more general phenomenon. EM is not merely an algorithm one can choose to apply; it is a property of certain objective geometries under gradient-based optimization.

Vaswani et al. [2017] introduced the transformer architecture with attention as its core mechanism. The original presentation emphasizes attention as a soft retrieval operation—queries attending to keys to aggregate values. The implicit EM perspective reframes attention as conditional mixture inference, with attention weights as responsibilities and value projections as prototype parameters. This interpretation is consistent with the original formulation but provides a probabilistic semantics that the architectural description lacks.

Mixture of Experts models [Jacobs et al., 1991] use explicit gating networks to route inputs to specialized subnetworks. The gating weights are responsibilities by another name. The difference is architectural: in Mixture of Experts, gating is a separate learned function; in standard attention and classification, responsibilities emerge as gradients of the objective without dedicated gating machinery. Implicit EM reveals that the explicit gating of Mixture of Experts is not necessary—any log-sum-exp objective produces responsibility-weighted routing automatically.

6 Limits and Failure Modes

The implicit EM framework is not universal. It applies under specific conditions, and when those conditions fail, so does the analysis. This section delineates the boundaries: when implicit EM does not arise, what pathologies can occur even when it does, and what phenomena lie entirely outside its scope. A clear account of limits strengthens rather than weakens the contribution—it distinguishes precise claims from overreach.

6.1 When Implicit EM Does Not Arise

The identity $\partial L/\partial d_j = -r_j$ depends on the log-sum-exp structure. Remove normalization, and the identity does not hold.

Consider multi-label classification with independent sigmoids. Each class j has its own output $\sigma(z_j)$, and the loss decomposes as a sum of independent binary cross-entropies. There is no partition function, no softmax, no competition across classes. The gradient with respect to z_j depends only on the target for class j, not on the outputs of other classes.

In this setting, responsibilities do not exist. No quantity sums to one across classes; no soft assignment distributes an input among alternatives. Each output channel operates in isolation. A point can be equally close to all prototypes or far from all of them, and the gradients do not redistribute—they simply reflect independent errors.

This is not a failure of the architecture but an absence of the required objective structure. Implicit EM arises from competition, and competition arises from normalization. Systems with independent outputs may learn useful representations, but they do not perform mixture inference and do not exhibit responsibility-weighted dynamics. The explanatory scope of the framework ends where normalization ends.

6.2 Scale and Collapse

A full Gaussian mixture model includes a log-determinant term in the likelihood—a penalty on the volume of each component's covariance. This term prevents collapse: without it, a component could

shrink its covariance to zero, placing infinite density on a single point and achieving unbounded likelihood. The log-determinant diverges as the covariance collapses, counterbalancing the density gain.

Most neural objectives omit this term. Cross-entropy and attention softmax operate on distances or scores without explicit volume penalties. The implicit EM dynamics still hold—gradients are still responsibility-weighted—but nothing prevents the learned metric from degenerating. A network can learn to map all inputs to nearby points, collapsing the distance structure and trivializing the responsibilities.

In practice, collapse is often avoided through implicit mechanisms: weight decay regularizes the scale of projections; layer normalization constrains activation magnitudes; architectural choices like residual connections preserve signal diversity. These interventions work, but they are not derived from the objective—they are heuristics that happen to stabilize geometry.

The implicit EM framework clarifies why collapse is a risk. When components are updated in proportion to their responsibilities, a component that captures slightly more mass receives stronger gradients, captures more mass still, and can dominate entirely. This positive feedback is inherent to EM dynamics and is classically controlled by the volume term. Neural networks remove that control and rely on other mechanisms to fill the gap. The framework does not solve this problem; it explains why the problem exists.

6.3 Supervision Constraints

In the unsupervised regime, responsibilities are entirely latent—the data alone determines which components claim which inputs. Supervision changes this. Labels declare which component *should* take responsibility, overriding what the geometry would otherwise dictate.

This constraint is powerful but rigid. Cross-entropy training forces the correct class toward responsibility 1, regardless of whether the input lies near that class's prototype or far from all prototypes. An input equidistant from all class boundaries still receives a hard label; the model must assign it somewhere. The soft, graded structure of responsibilities persists among incorrect classes, but the correct class is clamped.

A consequence is the closed-world assumption. Softmax normalization guarantees that responsibilities sum to one—some class must take full responsibility for every input. There is no "none of the above," no option for the model to reject an input as foreign to all known categories. An out-of-distribution input, no matter how anomalous, will be assigned to whichever class has the lowest distance, and the model's confidence may be arbitrarily high.

This is a limitation of the objective, not the mechanism. The implicit EM framework describes how responsibilities arise and how they weight learning. It does not claim that the resulting assignments are semantically correct or that the objective captures all desirable behaviors. Cross-entropy with softmax enforces assignment; if non-assignment is needed, the objective must change. Open-set recognition, out-of-distribution detection, and selective prediction require objectives that break the closed-world assumption—either by introducing an explicit reject option or by abandoning normalization entirely. The implicit EM analysis applies to whatever objective is chosen; it does not choose the objective.

6.4 What This Framework Does Not Explain

The implicit EM framework explains one phenomenon: the emergence of responsibility-weighted learning dynamics from distance-based objectives. It does not explain everything neural networks do.

Generalization—why networks perform well on unseen data—is not addressed. The framework describes the dynamics of training, not the inductive biases that enable generalization beyond the training distribution. Scaling laws—the predictable relationship between model size, data, and performance—lie outside the analysis entirely. Nothing in the gradient identity suggests how performance should scale with parameters or compute.

Long-horizon reasoning, planning, and sequential decision-making involve temporal structure that the framework does not capture. Implicit EM describes how a single input is softly assigned to components and how those components update. It does not describe how representations are composed over time, how goals propagate backward through action sequences, or how models learn to search.

Emergent capabilities—the sudden appearance of qualitatively new behaviors at scale—remain unexplained. The framework offers no account of why certain abilities appear discontinuously or why they require specific thresholds of model size. If emergent capabilities arise from implicit EM dynamics, the connection is not obvious; if they arise from other mechanisms, the framework is silent.

These are not failures of the analysis but boundaries of its scope. The contribution is to identify and derive one mechanism precisely, not to explain all of deep learning. Clarity about what is claimed protects against overinterpretation—and leaves room for complementary explanations of phenomena that implicit EM does not reach.

7 Discussion

This section reflects on what the implicit EM framework unifies, what it implies for how we understand neural networks, and what questions it leaves open for future work.

7.1 Unification

The framework reveals that Gaussian mixture models, attention mechanisms, and cross-entropy classification are not three different methods with superficial similarities. They are one mechanism operating under different constraints. In GMMs, responsibilities are fully latent. In attention, responsibilities are conditioned on queries and recomputed per input. In cross-entropy, responsibilities are partially clamped by supervision. The underlying dynamics—exponentiation, normalization, responsibility-weighted updates—are identical.

This suggests a shift in how we interpret neural network training. Probability is often treated as primitive: we define distributions, derive likelihoods, and optimize. The implicit EM perspective inverts this. Distances are primary. Networks compute geometric quantities—deviations from learned structures—and probabilities arise only after exponentiation and normalization. Geometry precedes probability; inference is a consequence of optimization on geometric objectives.

Loss functions, in this view, are not arbitrary choices tuned for performance. They are geometric priors. Cross-entropy encodes an assumption that inputs should be assigned to discrete categories with full responsibility. Log-sum-exp over distances assumes inputs arise from a mixture of latent causes. Correntropy assumes outliers should be ignored. Each objective induces a different geometry of assignment and a different pattern of gradient flow. Choosing a loss function is choosing a theory of how data relates to structure—whether or not that choice is made consciously.

7.2 Implications

For interpretability, the framework offers a direct path from training dynamics to semantic structure. If responsibilities are gradients, then the assignments a network makes are not hidden quantities requiring probes or post-hoc analysis. They are present in the backward pass, computed at every training step. The question "which component is responsible for this input?" has an answer in the gradient itself. This does not solve interpretability—understanding *why* a component takes responsibility requires further analysis—but it locates the assignment structure in a quantity that is already computed, not one that must be extracted.

For objective design, the analysis reframes log-sum-exp not as a numerical convenience but as a structural requirement. Softmax is often introduced to avoid overflow or to produce well-behaved gradients. The implicit EM perspective reveals a deeper role: log-sum-exp is what induces competition, competition is what produces responsibilities, and responsibilities are what make learning behave like inference. If inference-like behavior is desired, LSE structure is not optional. If it is not desired—if independent predictions or robust outlier handling are preferred—then LSE should be deliberately avoided. The choice is not about numerical stability; it is about what kind of learning dynamics the objective will produce.

For theory, the framework dissolves a long-standing separation. Optimization and inference are traditionally distinct: one concerns finding parameters that minimize a loss, the other concerns computing posteriors over latent variables. The implicit EM result shows that under the right objectives, these are the same process viewed at different levels. Gradient descent *is* inference when the gradients *are* responsibilities. This is not a claim that all optimization is inference, but that for a well-defined class of objectives, the distinction disappears. The forward pass computes posteriors; the backward pass applies them. Training is inference, performed continuously over the dataset.

7.3 Open Directions

Several directions remain open. The absence of volume control in neural objectives—the missing log-determinant—leads to collapse risks that are currently managed by heuristics. A principled approach would either derive implicit volume terms from architectural choices or design objectives that include them explicitly. Understanding when normalization layers substitute for volume control, and when they do not, would connect the implicit EM framework to practical stability concerns.

Supervision in real settings is rarely clean. Labels may be noisy, partial, or uncertain. The constrained regime analysis assumes hard labels that clamp responsibilities exactly; a fuller treatment would model soft or probabilistic supervision as partial constraints on the responsibility structure. This could unify semi-supervised learning, label smoothing, and learning from crowds under the implicit EM framework.

Open-set inference requires escaping the closed-world assumption. Current objectives force every input to be assigned; realistic deployment requires the option to reject. Objectives that support non-assignment—an explicit "none of the above" component, or a threshold below which no component takes responsibility—would extend implicit EM to settings where not all inputs belong to known categories.

Finally, diagnostic tools are needed. If trained networks perform implicit EM, it should be possible to measure this: to extract responsibilities from gradients, to track specialization over training, to detect when the mechanism fails or degenerates. Such tools would move the framework from explanatory theory to practical instrument, enabling researchers to verify whether a given model exhibits the dynamics the theory predicts.

8 Conclusion

This paper establishes a single result: for objectives of the form $L = \log \sum_j \exp(-d_j)$, the gradient with respect to each distance is the negative responsibility of the corresponding component. The identity $\partial L / \partial d_j = -r_j$ is exact, requiring no approximations beyond differentiability.

The implication is immediate. Gradient descent on distance-based log-sum-exp objectives performs expectation-maximization implicitly. Responsibilities are not computed as auxiliary quantities; they are the gradients. The forward pass is the E-step; the backward pass is the M-step. No explicit inference algorithm is required because inference is already embedded in optimization.

This mechanism unifies phenomena that have been treated as distinct. Unsupervised mixture learning, attention in transformers, and cross-entropy classification are three regimes of the same underlying process—differing in what is observed and what is latent, but governed by identical dynamics. The Bayesian structure recently observed in trained transformers is not an emergent mystery; it is a necessary consequence of the objectives used to train them.

Optimization and inference are the same process viewed at different scales.

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