RiordanMultiHermite-Bessel

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In [1]:
```
from sympy import *
from IPython.display import *
init_printing()
var('a:z')
var('A:Z');
```

In [23]:
```
d=2
V=Matrix(d,1,symbols('V:'+str(d)))
v=Matrix(1,d,symbols('v:'+str(d)))
z=Matrix(1,d,symbols('z:'+str(d)))
Z=Matrix(d,d,symbols('Z:'+str(d)+':'+str(d)))
for i in range(d):
    V[i]=z[i]-Rational(1/2)*sum(z[i]**2 for i in range(d))
V,v,z,Z
```

Out[23]:

$$\left(\begin{bmatrix}-\frac{z_0^2}{2}+z_0-\frac{z_1^2}{2}\\ -\frac{z_0^2}{2}-\frac{z_1^2}{2}+z_1\end{bmatrix},\quad \begin{bmatrix}v_0 & v_1\end{bmatrix},\quad \begin{bmatrix}z_0 & z_1\end{bmatrix},\quad \begin{bmatrix}Z_{00} & Z_{01}\\ Z_{10} & Z_{11}\end{bmatrix}\right)$$

In [26]:
```
Z=Matrix(solve([V[i]-v[i] for i in range(d)],[z[i] for i in range(d)])[0])
Z
```

Out[26]:

$$\begin{bmatrix}\frac{v_0}{2}-\frac{v_1}{2}-\frac{\sqrt{-v_0^2+2v_0v_1-2v_0-v_1^2-2v_1+1}}{2}+\frac{1}{2}\\ -\frac{v_0}{2}+\frac{v_1}{2}-\frac{\sqrt{-v_0^2+2v_0v_1-2v_0-v_1^2-2v_1+1}}{2}+\frac{1}{2}\end{bmatrix}$$

In [27]:
```
VPrime=V.jacobian(z)
VPrime
```

Out[27]:

$$\begin{bmatrix}-z_0+1 & -z_1\\ -z_0 & -z_1+1\end{bmatrix}$$

In [28]:
```
W=simplify(VPrime.inv())
W
```

Out[28]:

$$\left[\begin{matrix}\frac{z_1-1}{z_0+z_1-1} & -\frac{z_1}{z_0+z_1-1}\\ -\frac{z_0}{z_0+z_1-1} & \frac{z_0-1}{z_0+z_1-1}\end{matrix}\right]$$

```
In [29]: x=Matrix(1,d,symbols('x:'+str(d)))
         Y=x*W
         Y=[x[i]+simplify(Y[i]-x[i]) for i in range(2)]
         Y
```

Out[29]:

$$\left[x_0 - \frac{z_0\left(x_0 + x_1\right)}{z_0 + z_1 - 1}, \quad x_1 - \frac{z_1\left(x_0 + x_1\right)}{z_0 + z_1 - 1}\right]$$

```
In [30]: def goPolys(f,z,N):
             C=zeros(N)
             for i in range(N):
                 for j in range(N):
                     C[i,j]=(diff(f,z[0],i,z[1],j)).subs([(z[0],0),(z[1],0)]).factor()
             return C
```

```
In [31]: p=goPolys(exp(x*Z),v,4)
         p
```

Out[31]:

$$\left[\begin{matrix}1 & x_1 & \\ x_0 & x_0 x_1 & \\ x_0^2 + x_0 + x_1 & x_0^2 x_1 + x_0 + x_1\left(x_0 + x_1\right) + x_1 & \\ x_0^3 + 3x_0\left(x_0 + x_1\right) + 3x_0 + 3x_1 & x_0^3 x_1 + 3x_0 x_1\left(x_0 + x_1\right) + 3x_0\left(x_0 + x_1\right) + 6x_0 + 3x_1\left(x_0 + x_1\right) + 6x_1 & x_0^3 x_1^2 + x_0^3 (x\end{matrix}\right.$$

```
In [32]: y=Matrix(1,2,symbols('y:2'))
         q=goPolys(exp(y*V),z,4)
         q
```

Out[32]:

$$\left[\begin{matrix}1 & y_1 & -y_0 + y_1^2 - y_1 & \\ y_0 & y_0 y_1 & -y_0\left(y_0 - y_1^2 + y_1\right) & \\ y_0^2 - y_0 - y_1 & -y_1\left(-y_0^2 + y_0 + y_1\right) & y_0^2 y_1^2 - y_0^2\left(y_0 + y_1\right) - y_1^2\left(y_0 + y_1\right) + \left(y_0 + y_1\right)^2 & -y_1 \\ -y_0\left(-y_0^2 + 3y_0 + 3y_1\right) & y_0 y_1\left(y_0^2 - 3y_0 - 3y_1\right) & -y_0\left(-y_0^2 y_1^2 + y_0^2\left(y_0 + y_1\right) + 3y_1^2\left(y_0 + y_1\right) - 3\left(y_0 + y_1\right)^2\right) & y_0 y\end{matrix}\right.$$

In [ ]:

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