## Krav0

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## 1 Generating Functions

$$(1+v)^{N-j}(1-v)^j = \sum_{i=0}^N v^i \, \phi_i(j)$$

## **1.0.1** where the functions $\phi_i$ are polynomials in j

## 1.1 i.e., Krawtchouk polynomials

First, we look at the values of the polynomials.

N=2

$$(v+1)^2$$
$$v^2 + 2v + 1$$

$$(1-v)(v+1)$$
$$1-v^2$$

$$(1-v)^2$$
$$v^2 - 2v + 1$$

N=3

$$(v+1)^3 v^3 + 3v^2 + 3v + 1$$

$$(1-v)(v+1)^2$$
  
 $-v^3-v^2+v+1$ 

$$(1-v)^2 (v+1)$$
  
 $v^3 - v^2 - v + 1$ 

$$(1-v)^3 -v^3 + 3v^2 - 3v + 1$$

N=4

$$(v+1)^4$$
  
 $v^4 + 4v^3 + 6v^2 + 4v + 1$ 

$$(1-v)(v+1)^3$$
  
 $-v^4-2v^3+2v+1$ 

$$(1-v)^2 (v+1)^2$$
  
 $v^4 - 2v^2 + 1$ 

$$(1-v)^3 (v+1)$$
  
 $-v^4 + 2v^3 - 2v + 1$ 

$$(1-v)^4 v^4 - 4v^3 + 6v^2 - 4v + 1$$

N=5

$$(v+1)^5$$
  
 $v^5 + 5v^4 + 10v^3 + 10v^2 + 5v + 1$ 

$$(1-v)(v+1)^4$$
  
 $-v^5 - 3v^4 - 2v^3 + 2v^2 + 3v + 1$ 

$$(1-v)^2 (v+1)^3$$
  
 $v^5 + v^4 - 2v^3 - 2v^2 + v + 1$ 

$$(1-v)^3 (v+1)^2 -v^5 + v^4 + 2v^3 - 2v^2 - v + 1$$

$$(1-v)^4 (v+1)$$
  
 $v^5 - 3v^4 + 2v^3 + 2v^2 - 3v + 1$ 

$$(1-v)^5 -v^5 + 5v^4 - 10v^3 + 10v^2 - 5v + 1$$