1. $14^{27} \equiv 4 \pmod 5$

for any integer, $n^5 \equiv n \% 10 \pmod 5$

$\therefore 14^{27} \bmod 5 = 14^2 \cdot 4 \bmod 5 = 4$

some math trick there :)

2. $\sum_{i=1}^{100} i! \bmod 7$

$= 1! + 2! + 3! + 4! + 5! + 6! + \cdots + 100! \bmod 7$

$= 1! + 2! + 3! + 4! + 5! + 6! \bmod 7$

$= 1 + 2 + 6 + 24 + 120 + 720 \bmod 7$

$= 5$

$\therefore \sum_{i=1}^{100} i! \equiv 5 \bmod 7$

3. $8 \cdot 5 = 40 \equiv 1 \bmod 13$

$\therefore$ multiplicative inverse of 8 in Z_{13} is 5

4. $191y + 83x = 1 \bmod 191$ let $a = 191$, $b = 83$

$191 = 83 \times 2 + 25$ $25 = a - 2b$

$83 = 25 \times 3 + 8$ $b - 3a + 6b = 8$

$25 = 3 \times 8 + 1$ $1 = a - 2b + 9a - 21b$

$8 = 8 \times 1 + 0$ $= 10a - 23b$

$\gcd(191, 83) = 1$ $x = -23 \% 191 = 168$

$\therefore$ multiplicative inverse of 83 is 168

5. $58 = 26 \times 2 + 6$

$26 = 4 \times 6 + 2$

$6 = 3 \times 2 + 0$ $\quad\quad\quad\quad$ $2 = 26 - 4(58 - 2 \times 26)$

$Gcd(58, 26) = 2$

$\therefore \quad 58(-4) + 26(9) = 2$

6. let $P = 5n + 1$ $\quad n \in Z^+$

Case ①: n is odd.

$P = 5n + 1$

$P = 5(n-1) + 6$, where $n-1$ is even

$\because P$ is even, it can't be a prime number

conflict

Case ②: n is even

$n = 2k, \quad k \in Z^+$

$\therefore \quad P = 10k + 1$

$\therefore P \equiv 1 \mod 10.$