(a)

```
void f1(int n)
{
    int i=2;
    while(i < n){
        /* do something that takes O(1) time */
        i = i*i;
    }
}
```

int i=2; ⟶ $\Theta(1)$

/* do something that takes O(1) time */ $\Theta(1)$

i = i*i; (arbitrary k++)

⇓

$i = i^2$

k	1	2	3	arbitrary k	Stop when $k = \log\log n$
i	2^2	2^4	2^8	2^{2^k}	Stop when $i \geq n$

$$T(n) = \Theta(1) + \sum_{k=1}^{\log\log n} \Theta(1)$$

$$= \Theta(\log\log n)$$

(b)

```
void f2(int n)
{
    for(int i=1; i <= n; i++){
        if( (i % (int)sqrt(n)) == 0){
            for(int k=0; k < pow(i,3); k++) {
              /* do something that takes O(1) time */
            }
        }
    }
}
```

m	1	2	3	Arbitrary m	$m = \sqrt{n}$
i	$\sqrt{n}$	$2\sqrt{n}$	$3\sqrt{n}$	$i = m\sqrt{n}$	Stop when $i = n$

$$T(n) = \sum_{i=1}^{n} \Theta(1) + \sum_{m=1}^{\sqrt{n}} \sum_{k=0}^{i^3-1} \Theta(1)$$

$$= \Theta(n) + \Theta\left((1+2^3+3^3+n^{\frac{3}{2}})\, n^{\frac{3}{2}}\right)$$

$$= \Theta(n^{\frac{7}{2}})$$

(c)

```
for(int i=1; i <= n; i++){
  for(int k=1; k <= n; k++){
    if( A[k] == i){
      for(int m=1; m <= n; m=m+m){
        // do something that takes O(1) time
        // Assume the contents of the A[] array are not changed
      }
    }
  }
}
```

q	1	2	3	Arbitrary q	$q = \log_2 n + 1$
m	2^0	2^1	2^2	2^{q-1}	Stop when $m > n$

$$T(n) = \sum_{i=1}^{n} \sum_{k=1}^{n} \theta(1) + n \sum_{n=1}^{\log n} O(1)$$

$$= \theta(n^2) + \theta(n \log n)$$

$$= \theta(n^2)$$

(d)

```
int f (int n)
{
  int *a = new int [10];
  int size = 10;
  for (int i = 0; i < n; i ++)
     {
        if (i == size)
          {
             int newsize = 3*size/2;
             int *b = new int [newsize];
             for (int j = 0; j < size; j ++) b[j] = a[j];
             delete [] a;
             a = b;
             size = newsize;
          }
        a[i] = i*i;
     }
}
```

k	1	2	3	k	$k = \log_{\frac{3}{2}} \frac{n}{10}$
size	$\frac{3}{2}$ size	$(\frac{3}{2})^2$ size	$(\frac{3}{2})^3$ size	$(\frac{3}{2})^k$ size	Stop when $(\frac{3}{2})^k \cdot 10 > n$

$$T(n) = \theta(1) + \theta(1) + \theta(n) + \theta(n) + \sum_{\frac{3}{2}}^{\log_{\frac{3}{2}}(\frac{n}{10})} \sum_{\frac{3}{2}}^{(\frac{3}{2})^{k-1}} \theta(1)$$

$= \theta(n)$

$K=1 \quad J=0$