Sec.9-1 (p.483)

8.	***state the null and alternative hypothesis for each conjecture*** An automobile engineer believes that the mean number of miles per gallon that a four-wheel drive vehicle gets is less than the mean number of miles per gallon that an all-wheel drive vehicle gets.

$\mu_1 = mean\ number\ of\ miles\ per\ gallon\ that\ a\ four-wheel\ drive\ vehicle\ gets$

$\mu_2 = mean\ number\ of\ miles\ per\ gallon\ that\ an\ all-wheel\ drive\ vehicle\ gets$

$$H_0: \mu_1 \geq \mu_2$$
$$H_1: \mu_1 < \mu_2 (claim)$$

10.	***state the null and alternative hypothesis for each conjecture*** A medical researcher believes that the mean time in minutes that ibuprofen relieves pain is less than the mean time in minutes that aspirin relieves pain.

ibuprofen 布洛芬;

$\mu_1 = mean\ time\ in\ minutes\ that\ ibuprofen\ relieves\ pain$

$\mu_2 = mean\ time\ in\ minutes\ that\ aspirin\ relieves\ pain$

$$H_0: \mu_1 \geq \mu_2$$
$$H_1: \mu_1 < \mu_2 (claim)$$

12.	***write the null and alternate hypotheses for each claim*** An environmentalist believes that the percentage of air pollution produced by trucks is less than the percentage of air pollution produced by automobiles.

environmentalist 環保主義者;

$p_1 = percentage\ of\ air\ pollution\ produced\ by\ trucks$

$p_2 = percentage\ of\ air\ pollution\ produced\ by\ automobiles$

$$H_0: p_1 \geq p_2$$
$$H_1: p_1 < p_2 (claim)$$

16.	***write the null and alternate hypotheses for each claim*** A school superintendent believes that the percentage of senior high school students in her school district who think that the cafeteria food is nutritious is not the same as the percentage of faculty and administrators in the district who do.

school superintendent 學校負責人; faculty 教職員; administrators 管理人員;

$p_1 =$ percentage of senior high school students in her school district who think that the cafeteria food is nutritious

$p_2 =$percentage of faculty and administrators in the district who do

$$H_0: p_1 = p_2$$
$$H_1: p_1 \neq p_2 (claim)$$

Sec. 9-2 (p.488)

6. New York and Massachusetts lead the list of average teacher's salaries. The New York average is \$76,409 while teachers in Massachusetts make an average annual salary of \$73,195. Random samples of 45 teachers from each state yielded the following.

	Massachusetts	**New York**
Sample means	\$73,195	\$76,409
Population standard deviation	8,200	7,800

At α = 0.10, is there a difference in means of the salaries?

Massachusetts 馬薩諸塞州;

Step 1. 假設

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2\ (claim)$$

Step 2. 拒絕域

$$z_{\frac{\alpha}{2}} = 1.645 \Rightarrow R = \{|z| \geq 1.645\}$$

Step 3. 檢定統計量

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(73195 - 76409) - 0}{\sqrt{\frac{8200^2}{45} + \frac{7800^2}{45}}} = -1.905$$

Step 4. 決策

$$|-1.905| \geq 1.645 \Rightarrow \text{拒絕}H_0$$

Step 5. 結論

There is sufficient evidence to support the claim at α = 0.10.

給定顯著水準為 0.10 下，我們有足夠的證據支持宣稱

10. A real estate agent compares the selling prices of randomly selected homes in two municipalities in southwestern Pennsylvania to see if there is a difference. The results of the study are shown. Is there enough evidence to reject the claim that the average cost of a home in both locations is the same? Use α = 0.01.

Scott (斯科特)	**Ligonier (力戈尼耶)**
$\bar{X}_1 = \$93{,}430$	$\bar{X}_2 = \$98{,}043$
$\sigma_1 = \$5{,}602$	$\sigma_2 = \$4{,}731$
$n_1 = 35$	$n_2 = 40$

real estate agent 房地產經紀人; municipalities 直轄市;

southwestern Pennsylvania 賓夕法尼亞州西南部;

Step 1. 假設

$$H_0: \mu_1 = \mu_2\ (claim)$$
$$H_1: \mu_1 \neq \mu_2$$

Step 2. 拒絕域

$$z_{\frac{\alpha}{2}} = 2.575 \Rightarrow R = \{|z| \geq 2.575\}$$

Step 3.　檢定統計量

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(93430 - 98043) - 0}{\sqrt{\frac{5602^2}{35} + \frac{4731^2}{40}}} = -3.823$$

Step 4.　決策

$$|-3.823| \geq 2.575 \Rightarrow \text{拒絕} H_0$$

Step 5.　結論

There is sufficient evidence to reject the claim at α = 0.01.

給定顯著水準為 0.01 下，我們有足夠的證據拒絕宣稱

14. The average monthly Social Security benefit for a specific year for retired workers was $954.90 and for disabled workers was $894.10. Researchers used data from the Social Security records to test the claim that the difference in monthly benefits between the two groups was greater than $30. Based on the following information, can the researchers' claim be supported at the 0.05 level of significance?

	Retired	**Disabled**
Sample size	60	60
Mean benefit	$960.50	$902.89
Population standard deviation	$98	$101

Social Security benefit 社會保障福利; disabled workers 殘疾工人;

Step 1.　假設

$$H_0: \mu_1 - \mu_2 \leq 30$$
$$H_1: \mu_1 - \mu_2 > 30\ (claim)$$

Step 2.　拒絕域

$$z_\alpha = 1.645 \Rightarrow R = \{z \geq 1.645\}$$

Step 3.　檢定統計量

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(960.5 - 902.89) - 30}{\sqrt{\frac{98^2}{60} + \frac{101^2}{60}}} = 1.520$$

Step 4.　決策

$$1.520 \ngeq 1.645 \Rightarrow \text{不拒絕} H_0$$

Step 5.　結論

There is not sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們沒有足夠的證據支持宣稱

18.	Find the 95% confidence interval of the difference in the distance that day students travel to school and the distance evening students travel to school. Two random samples of 40 students are taken, and the data are shown. Find the 95% confidence interval of the difference in the means.

	$\overline{X}$	σ	n
Day students	4.7	1.5	40
Evening Students	6.2	1.7	40

$$(\bar{X}_1 - \bar{X}_2) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$z_{\frac{\alpha}{2}} = 1.96$$

$$\Rightarrow (4.7 - 6.2) \pm 1.96 \cdot \sqrt{\frac{1.5^2}{40} + \frac{1.7^2}{40}}$$

$$\Rightarrow -2.203 < \mu_1 - \mu_2 < -0.797$$

The 95% confidence interval of the difference in the means is (−2.203, −0.797).

20.	In a large hospital, a nursing director selected a random sample of 30 registered nurses and found that the mean of their ages was 30.2. The population standard deviation for the ages is 5.6. She selected a random sample of 40 nursing assistants and found the mean of their ages was 31.7. The population standard deviation of the ages for the assistants is 4.3. Find the 99% confidence interval of the differences in the ages.

nursing director 護理主任; nursing assistants 護理助理;

	$\overline{X}$	σ	n
registered nurses	30.2	5.6	30
nursing assistants	31.7	4.3	40

$$(\bar{X}_1 - \bar{X}_2) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}} < \mu_1 - \mu_2 < (\bar{X}_1 - \bar{X}_2) + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}$$

$$z_{\frac{\alpha}{2}} = 2.575$$

$$\Rightarrow (30.2 - 31.7) \pm 2.575 \cdot \sqrt{\frac{5.6^2}{30} + \frac{4.3^2}{40}}$$

$$\Rightarrow -4.662 < \mu_1 - \mu_2 < 1.662$$

The 99% confidence interval of the differences in the ages is (−4.662,1.662).

24.	According to the almanac, the average sales price of a single-family home in the metropolitan Dallas/Ft. Worth/Irving, Texas, area is \$215,200. The average home price in Orlando, Florida, is \$198,000. The mean of a random sample of 45 homes in the Texas metroplex was \$216,000 with a population standard deviation of \$30,000. In the Orlando, Florida, area a sample of 40 homes had a mean price of \$203,000 with a population standard deviation of \$32,500. At the 0.05 level of significance, can it be concluded that the mean price in Dallas exceeds the mean price in Orlando? Use the P-value method.

single-family home 單戶住宅; metropolitan 大都會區;

Orlando, Florida 佛羅里達州奧蘭多; Dallas/Ft. Worth/Irving, Texas 德克薩斯州達拉斯;

	$\bar{X}$	σ	n
Dallas, Texas	216000	30000	45
Orlando, Florida	203000	32500	40

Step 1. 假設

$$H_0: \mu_1 \leq \mu_2$$
$$H_1: \mu_1 > \mu_2 \ (claim)$$

Step 2. 檢定統計量

$$z = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}}} = \frac{(216000 - 203000) - 0}{\sqrt{\frac{30000^2}{45} + \frac{32500^2}{40}}} = 1.908$$

Step 3. p value

$$p - value = P(Z > z) = P(Z > 1.91) = 0.0281$$

Step 4. 決策

$$p - value = 0.0281 < 0.05 = \alpha \Rightarrow \text{拒絕} H_0$$

Step 5. 結論

There is sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

Sec. 9-3 (p.497)

4.	The mean age of a random sample of 25 people who were playing the slot machines is 48.7 years, and the standard deviation is 6.8 years. The mean age of a random sample of 35 people who were playing roulette is 55.3 with a standard deviation of 3.2 years. Can it be concluded at α = 0.05 that the mean age of those playing the slot machines is less than those playing roulette?

slot machines 老虎機; roulette 輪盤賭

	$\bar{X}$	s	n
slot machines	48.7	6.8	25
roulette	55.3	3.2	35

Step 1. 假設

$$H_0: \mu_1 \geq \mu_2$$
$$H_1: \mu_1 < \mu_2 \ (claim)$$

Step 2.　拒絕域

$$t_{(\alpha,df)} = t_{(0.05,24)} = 1.711 \Rightarrow R = \{t \leq -1.711\}$$

Step 3.　檢定統計量

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(48.7 - 55.3) - 0}{\sqrt{\frac{6.8^2}{25} + \frac{3.2^2}{35}}} = -4.509$$

Step 4.　決策

$$-4.509 \leq -1.711 \Rightarrow \text{拒絕} H_0$$

Step 5.　結論

There is sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

10. Find the 95% confidence interval for the difference of the means in Exercise 6 of this section.

Hard body types				**Soft body types**			
21	17	17	20	24	13	11	13
16	17	15	20	12	15		
23	16	17	17				
13	15	16	18				
18							

	$\bar{X}$	s	n
hard	17.412	2.451	17
soft	14.667	4.761	6

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$t_{\left(\frac{\alpha}{2},df\right)} = t_{(0.025,5)} = 2.571$$

$$\Rightarrow (17.412 - 14.667) \pm 2.571 \cdot \sqrt{\frac{2.451^2}{17} + \frac{4.761^2}{6}}$$

$$\Rightarrow -2.481 < \mu_1 - \mu_2 < 7.971$$

The 95% confidence interval for the difference of the means is (−2.481,7.971).

14. The number of points held by random samples of the NHL's highest scorers for both the Eastern Conference and the Western Conference is shown. At $\alpha = 0.05$, can it be concluded that there is a difference in means based on these data?

Eastern Conference				Western Conference			
83	60	75	58	77	59	72	58
78	59	70	58	37	57	66	55
62	61	59		61			

Eastern Conference 東部聯盟; Western Conference 西部聯盟;

	$\bar{X}$	s	n
Eastern	65.727	9.122	11
Western	60.222	11.388	9

Step 1. 假設

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2 \ (claim)$$

Step 2. 拒絕域

$$t_{(\alpha,df)} = t_{(0.025,8)} = 2.306 \Rightarrow R = \{|t| \geq 2.306\}$$

Step 3. 檢定統計量

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(65.727 - 60.222) - 0}{\sqrt{\frac{9.122^2}{11} + \frac{11.388^2}{9}}} = 1.174$$

Step 4. 決策

$$|1.174| \ngeq 2.306 \Rightarrow \text{不拒絕} H_0$$

Step 5. 結論

There is not sufficient evidence to support the claim at $\alpha = 0.05$.

給定顯著水準為 0.05 下，我們沒有足夠的證據支持宣稱

18. The out-of-state tuitions (in dollars) for random samples of both public and private 4-year colleges in a New England state are listed. Find the 95% confidence interval for the difference in the means.

Private		Public	
13,600	13,495	7,050	9,000
16,590	17,300	6,450	9,758
23,400	12,500	7,050	7,871
		16,100	

out-of-state tuitions 非居民學費

	$\bar{X}$	s	n
private	16147.5	4023.744	6
public	9039.857	3325.547	7

$$(\bar{X}_1 - \bar{X}_2) \pm t_{\frac{\alpha}{2}} \cdot \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$$

$$t_{\left(\frac{\alpha}{2}, df\right)} = t_{(0.025,5)} = 2.571$$

$$\Rightarrow (16147.5 - 9039.857) \pm 2.571 \cdot \sqrt{\frac{4023.744^2}{6} + \frac{3325.547^2}{7}}$$

$$\Rightarrow 1789.765 < \mu_1 - \mu_2 < 12425.521$$

The 95% confidence interval for the difference of the means is (1789.765, 12425.521).

20. A large group of friends went miniature golfing together at a par 54 course and decided to play on two teams. A random sample of scores from each of the two teams is shown. At α = 0.05, is there a difference in mean scores between the two teams? Use the P-value method.

Team 1	61	44	52	47	56	63	62	55
Team 2	56	40	42	58	48	52	51	

miniature golf 小型高爾夫; par (高爾夫球)標準桿數;

	$\bar{X}$	s	n
Team 1	55	7.010	8
Team 2	49.571	6.729	7

Step 1. 假設

$$H_0: \mu_1 = \mu_2$$
$$H_1: \mu_1 \neq \mu_2 \ (claim)$$

Step 2. 檢定統計量

$$t = \frac{(\bar{X}_1 - \bar{X}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} = \frac{(55 - 49.571) - 0}{\sqrt{\frac{7.010^2}{8} + \frac{6.729^2}{7}}} = 1.529$$

Step 3. p value

$$d.f. = 6$$
$$1.440 < 1.529 < 1.943 \Rightarrow 0.1 < p - value < 0.2$$

Step 6. 決策

$$p - value \nless 0.05 = \alpha \Rightarrow \text{不拒絕} H_0$$

Step 4. 結論

There is not sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們沒有足夠的證據支持宣稱

Sec. 9-4 (p.509)

1. Classify each as independent or dependent samples.
   a. Heights of identical twins (同卵雙胞胎).
   b. Test scores of the same students in English and psychology (心理學).
   c. The effectiveness of two different brands of aspirin on two different groups of people.
   d. Effects of a drug on reaction time of two different groups of people, measured by a before-and-after test.
   e. The effectiveness of two different diets on two different groups of individuals.

a.	b.	c.	d.	e.
dependent	dependent	independent	dependent	independent

4. An obstacle course was set up on a campus, and 8 randomly selected volunteers were given a chance to complete it while they were being timed. They then sampled a new energy drink and were given the opportunity to run the course again. The "before" and "after" times in seconds are shown. Is there sufficient evidence at α = 0.05 to conclude that the students did better the second time? Discuss possible reasons for your results.

Student	1	2	3	4	5	6	7	8
Before	67	72	80	70	78	82	69	75
After	68	70	76	65	75	78	65	68

obstacle course 障礙訓練場;

Step 1. 假設

$$D = before - after$$
$$H_0: \mu_D \leq 0$$
$$H_1: \mu_D > 0\ (claim)$$

Step 2. 拒絕域

$$t_{(\alpha,df)} = t_{(0.05,7)} = 1.895 \Rightarrow R = \{t \geq 1.895\}$$

Step 3. 檢定統計量

Before	67	72	80	70	78	82	69	75	
After	68	70	76	65	75	78	65	68	
$\boldsymbol{D}$	-1	2	4	5	3	4	4	7	$\sum D = 28$
$\boldsymbol{D^2}$	1	4	16	25	9	16	16	49	$\sum D^2 = 136$

$$\bar{D} = \frac{\sum D}{n} = \frac{28}{8} = 3.5, \qquad s_D = \sqrt{\frac{n\sum D^2 - (\sum D)^2}{n(n-1)}} = 2.330$$

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} = \frac{3.5 - 0}{2.33/\sqrt{8}} = 4.249$$

Step 4. 決策

$$4.249 > 1.895 \Rightarrow \text{拒絕} H_0$$

Step 5. 結論

There is sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

6. At a recent PGA tournament the following scores were posted for eight randomly selected golfers for two consecutive days. At α = 0.05 , is there evidence of a difference in mean scores for the 2 days?

Golfer	1	2	3	4	5	6	7	8
Thursday	67	65	68	68	68	70	69	70
Friday	68	70	69	71	72	69	70	70

tournament 錦標賽;

Step 1. 假設

$$H_0: \mu_D = 0$$
$$H_1: \mu_D \neq 0 \ (claim)$$

Step 2. 拒絕域

$$t_{(\alpha,df)} = t_{(0.05,7)} = 2.365 \Rightarrow R = \{|t| \geq 2.365\}$$

Step 3. 檢定統計量

Thursday	67	65	68	68	68	70	69	70	
Friday	68	70	69	71	72	69	70	70	
$\boldsymbol{D = Thu.-Fri.}$	-1	-5	-1	-3	-4	1	-1	0	$\sum D = -14$
$\boldsymbol{D^2}$	1	25	1	9	16	1	1	0	$\sum D^2 = 54$

$$\bar{D} = \frac{\sum D}{n} = \frac{-14}{8} = -1.75, \qquad s_D = \sqrt{\frac{n\sum D^2 - (\sum D)^2}{n(n-1)}} = 2.053$$

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} = \frac{-1.75 - 0}{2.053/\sqrt{8}} = -2.411$$

Step 4. 決策

$$|-2.411| > 2.365 \Rightarrow \text{拒絕} H_0$$

Step 5. 結論

There is sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

10. An educational researcher devised a wooden toy assembly project to test learning in 6-year-olds. The time in seconds to assemble the project was noted, and the toy was disassembled out of the child's sight. Then the child was given the task to repeat. The researcher would conclude that learning occurred if the mean of the **second** assembly times was **less than** the mean of the **first** assembly times. At α = 0.01 , can it be concluded that learning took place? Use the P-value method, and find the 99% confidence interval of the difference in means.

Child	1	2	3	4	5	6	7
Trial 1	100	150	150	110	130	120	118
Trial 2	90	130	150	90	105	110	120

wooden toy assembly project 木製玩具組裝項目

Step 1. 假設

$$D = Trial\ 1 - Trial\ 2$$
$$H_0: \mu_D \leq 0$$
$$H_1: \mu_D > 0 \ (claim)$$

Step 2.　檢定統計量

Trial 1	100	150	150	110	130	120	118	
Trial 2	90	130	150	90	105	110	120	
D	10	20	0	20	25	10	-2	$\sum D = 83$
D^2	100	400	0	400	625	100	4	$\sum D^2 = 1629$

$$\bar{D} = \frac{\sum D}{n} = \frac{83}{7} = 11.857, \qquad s_D = \sqrt{\frac{n\sum D^2 - (\sum D)^2}{n(n-1)}} = 10.367$$

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} = \frac{11.857 - 0}{10.367/\sqrt{7}} = 3.026$$

Step 3.　p-value

$$At\ d.f = 6$$

$$2.447 < 3.026 < 3.143 \Rightarrow 0.01 < p - value < 0.025$$

Step 4.　決策

$$p - value \nless 0.01 = \alpha \Rightarrow \text{不拒絕} H_0$$

Step 5.　結論

There is not sufficient evidence to support the claim at α = 0.01.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

Step 6.　99% C.I.

$$\bar{D} - t_{\frac{\alpha}{2}} \cdot \frac{s_D}{\sqrt{n}} < \mu_D < \bar{D} + t_{\frac{\alpha}{2}} \cdot \frac{s_D}{\sqrt{n}}$$

$$t_{(0.005,6)} = 3.707$$

$$\Rightarrow 11.857 \pm 3.707 \cdot \frac{10.367}{\sqrt{7}}$$

$$\Rightarrow -2.668 < \mu_D < 26.382$$

The 99% confidence interval for the difference of the means is (-2.668, 26.382).

12. A random sample of six music students played a short song, and the number of mistakes in music each student made was recorded. After they practiced the song 5 times, the number of mistakes each student made was recorded. The data are shown. At α = 0.05, can it be concluded that there was a decrease in the mean number of mistakes?

Student	A	B	C	D	E	F
Before	10	6	8	8	13	8
After	4	2	2	7	8	9

Step 1.　假設

$$D = before - after$$

$$H_0: \mu_D \leq 0$$

$$H_1: \mu_D > 0\ (claim)$$

Step 2.　拒絕域

$$t_{(\alpha,df)} = t_{(0.05,5)} = 2.015 \Rightarrow R = \{t \geq 2.015\}$$

Step 3.　檢定統計量

Before	10	6	8	8	13	8	
After	4	2	2	7	8	9	
$\boldsymbol{D}$	6	4	6	1	5	-1	$\sum D = 21$
$\boldsymbol{D^2}$	36	16	36	1	25	1	$\sum D^2 = 115$

$$\bar{D} = \frac{\sum D}{n} = \frac{21}{6} = 3.5, \qquad s_D = \sqrt{\frac{n\sum D^2 - (\sum D)^2}{n(n-1)}} = 2.881$$

$$t = \frac{\bar{D} - \mu_D}{s_D/\sqrt{n}} = \frac{3.5 - 0}{2.881/\sqrt{6}} = 2.976$$

Step 4.　決策

$$2.976 > 2.015 \Rightarrow \text{ 拒絕}H_0$$

Step 5.　結論

There is sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

Sec. 9-5 (p.518)

8. In a sample of 150 men, 132 said that they had less leisure time today than they had 10 years ago. In a random sample of 250 women, 240 women said that they had less leisure time than they had 10 years ago. At α = 0.10, is there a difference in the proportions? Find the 90% confidence interval for the difference of the two proportions. Does the confidence interval contain 0? Give a reason why this information would be of interest to a researcher.

leisure time 閒暇時間;

Step 1.　假設

$$H_0: p_1 = p_2$$
$$H_1: p_1 \neq p_2\ (claim)$$

Step 2.　拒絕域

$$z_{\frac{\alpha}{2}} = z_{0.05} = 1.645 \Rightarrow R = \{|z| \geq 1.645\}$$

Step 3.　檢定統計量

$$\hat{p}_1 = \frac{132}{150} = 0.88, \qquad \hat{p}_2 = \frac{240}{250} = 0.96$$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{132 + 240}{150 + 250} = 0.93$$

$$z = \frac{(0.88 - 0.96) - 0}{\sqrt{0.93 \cdot 0.07\left(\frac{1}{150} + \frac{1}{250}\right)}} = -3.036$$

Step 4.　決策

$$|-3.036| > 1.645 \Rightarrow \text{ 拒絕}H_0$$

Step 5.　結論

There is sufficient evidence to support the claim at α = 0.1.

給定顯著水準為 0.1 下，我們有足夠的證據支持宣稱

Step 6. 90% C.I.

$$(\hat{p}_1 - \hat{p}_2) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$z_{0.05} = 1.645$$

$$\Rightarrow -0.08 \pm 1.645 \cdot 0.029$$

$$\Rightarrow -0.128 < p_1 - p_2 < -0.032$$

Since the confidence interval does not contain zero, the decision is to reject H_0.

10. In Cleveland, a random sample of 73 mail carriers showed that 10 had been bitten by an animal during one week. In Philadelphia, in a random sample of 80 mail carriers, 16 had received animal bites. Is there a significant difference in the proportions? Use α = 0.05. Find the 95% confidence interval for the difference of the two proportions.

Cleveland 克利夫蘭; mail carriers 郵遞員; Philadelphia 費城;

Step 1. 假設

$$H_0: p_1 = p_2$$

$$H_1: p_1 \neq p_2\ (claim)$$

Step 2. 拒絕域

$$z_{\frac{\alpha}{2}} = z_{0.025} = 1.96 \Rightarrow R = \{|z| \geq 1.96\}$$

Step 3. 檢定統計量

$$\hat{p}_1 = \frac{10}{73} = 0.137, \qquad \hat{p}_2 = \frac{16}{80} = 0.2$$

$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{26}{153} = 0.170$$

$$z = \frac{(0.137 - 0.2) - 0}{\sqrt{0.17 \cdot 0.83 \left(\frac{1}{73} + \frac{1}{80}\right)}} = -1.036$$

Step 4. 決策

$$|-1.036| \ngeq 1.96 \Rightarrow \text{不拒絕} H_0$$

Step 5. 結論

There is not sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們沒有足夠的證據支持宣稱

Step 7. 95% C.I.

$$(\hat{p}_1 - \hat{p}_2) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}} < p_1 - p_2 < (\hat{p}_1 - \hat{p}_2) - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}_1\hat{q}_1}{n_1} + \frac{\hat{p}_2\hat{q}_2}{n_2}}$$

$$z_{0.05} = 1.96$$

$$\Rightarrow -0.063 \pm 1.96 \cdot 0.060$$

$$\Rightarrow -0.181 < p_1 - p_2 < 0.055$$

14. It has been found that 26% of men 20 years and older suffer from hypertension (high blood pressure) and 31.5% of women are hypertensive. A random sample of 150 of each gender was selected from recent hospital records, and the following results were obtained. Can you conclude that a higher percentage of women have high blood pressure? Use α = 0.05.

Men	43 patients had high blood pressure
Women	52 patients had high blood pressure

Step 1. 假設

$$p_1 = men, \qquad p_2 = women$$
$$H_0: p_1 \geq p_2$$
$$H_1: p_1 < p_2 \ (claim)$$

Step 2. 拒絕域

$$z_\alpha = z_{0.05} = 1.645 \Rightarrow R = \{z < -1.645\}$$

Step 3. 檢定統計量

$$\hat{p}_1 = \frac{43}{150} = 0.287, \qquad \hat{p}_2 = \frac{52}{150} = 0.347$$
$$\bar{p} = \frac{X_1 + X_2}{n_1 + n_2} = \frac{95}{300} = 0.317$$
$$z = \frac{(0.287 - 0.347) - 0}{\sqrt{0.317 \cdot 0.683 \left(\frac{1}{150} + \frac{1}{150}\right)}} = -1.117$$

Step 4. 決策

$$-1.117 \not< -1.645 \Rightarrow \text{不拒絕} H_0$$

Step 5. 結論

There is not sufficient evidence to support the claim at α = 0.05.
給定顯著水準為 0.05 下，我們沒有足夠的證據支持宣稱

20. In a specific year 53.7% of men in the United States were married and 50.3% of women were married. Two independent random samples of 300 men and 300 women found that 178 men and 139 women were married (not to each other). At the 0.05 level of significance, can it be concluded that the proportion of men who were married is greater than the proportion of women who were married?

Step 1. 假設

$$p_1 = men, \qquad p_2 = women$$
$$H_0: p_1 \leq p_2$$
$$H_1: p_1 > p_2 \ (claim)$$

Step 2. 拒絕域

$$z_\alpha = z_{0.05} = 1.645 \Rightarrow R = \{z > 1.645\}$$

Step 3. 檢定統計量

$$\hat{p}_1 = \frac{178}{300} = 0.593, \qquad \hat{p}_2 = \frac{139}{300} = 0.463$$

$$\bar{p}=\frac{X_1+X_2}{n_1+n_2}=\frac{317}{600}=0.528$$

$$z=\frac{(0.593-0.463)-0}{\sqrt{0.528\cdot 0.472\left(\frac{1}{300}+\frac{1}{300}\right)}}=3.189$$

Step 4. 決策

$$3.189>1.645\Rightarrow 拒絕H_0$$

Step 5. 結論

There is sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

24. According to the U.S. Bureau of Labor Statistics, approximately equal numbers of men and women are engaged in sales and related occupations. Although that may be true for total numbers, perhaps the proportions differ by industry. A random sample of 200 salespersons from the industrial sector indicated that 114 were men, and in the medical supply sector, 80 of 200 were men. At the 0.05 level of significance, can we conclude that the proportion of men in industrial sales differs from the proportion of men in medical supply sales?

Bureau of Labor Statistics 勞工統計局;

Step 1. 假設

$$p_1=industrial,\qquad p_2=medical\ supply$$
$$H_0: p_1=p_2$$
$$H_1: p_1\neq p_2\ (claim)$$

Step 2. 拒絕域

$$z_\alpha=z_{0.025}=1.96\Rightarrow R=\{|z|>1.96\}$$

Step 3. 檢定統計量

$$\hat{p}_1=\frac{114}{200}=0.57,\qquad \hat{p}_2=\frac{80}{200}=0.4$$

$$\bar{p}=\frac{X_1+X_2}{n_1+n_2}=\frac{194}{400}=0.485$$

$$z=\frac{(0.57-0.4)-0}{\sqrt{0.485\cdot 0.515\left(\frac{1}{200}+\frac{1}{200}\right)}}=3.402$$

Step 4. 決策

$$|3.402|>1.96\Rightarrow 拒絕H_0$$

Step 5. 結論

There is sufficient evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱