Sec. 8-1 (p.419)

14.	For each conjecture, state the null and alternative hypotheses a. The average age of first-year medical school students is at least 27 years. b. The average experience (in seasons) for an NBA player is 4.71. c. The average number of monthly visits/sessions on the Internet by a person at home has increased from 36 in 2009. d. The average cost of a cell phone is \$79.95. e. The average weight loss for a sample of people who exercise 30 minutes per day for 6 weeks is 8.2 pounds.

a.	b.	c.	d.	e.
$H_0: \mu \geq 27$ $H_1: \mu < 27$	$H_0: \mu = 4.71$ $H_1: \mu \neq 4.71$	$H_0: \mu \leq 36$ $H_1: \mu > 36$	$H_0: \mu = 79.95$ $H_1: \mu \neq 79.95$	$H_0: \mu = 8.2$ $H_1: \mu \neq 8.2$

16.	For each conjecture, state the null and alternative hypotheses. a. 90% of people who commit vandalism are male b. More than 45% of people who are committed to psychiatric institutions are schizophrenics c. Less than 25% of households are touched by crime each year

Vandalism 財產破壞; psychiatric 精神病學的; schizophrenics 精神分裂症患者;

a.	b.	c.
$H_0: p = 0.9$ $H_1: p \neq 0.9$	$H_0: p \leq 0.45$ $H_1: p > 0.45$	$H_0: p \geq 0.25$ $H_1: p < 0.25$

18.	For each situation, decide whether the null hypothesis should be rejected a. $\alpha = 0.01, p - value = 0.034$ b. $\alpha = 0.1, p - value = 0.061$ c. $\alpha = 0.05, p - value = 0.027$ d. $\alpha = 0.01, p - value = 0.023$

a.	b.	c.	d.
p-value> α ⇒do not reject H_0	p-value< α ⇒reject H_0	p-value< α ⇒reject H_0	p-value> α ⇒do not reject H_0

Sec. 8-2 (p.427)

2.	Many people believe that the average number of Facebook friends is 338. The population standard deviation is 43.2. A random sample of 50 high school students in a particular county revealed that the average number of Facebook friends was 350. At α = 0.05 , is there sufficient evidence to conclude that the mean number of friends is greater than 338 ?

Step 1. 假設

$$H_0: \mu \leq 338 \;\; and \;\; H_1: \mu > 338 \; (claim)$$

Step 2.　找拒絕域

$\alpha = 0.05$，右尾$\Rightarrow z_{0.05} = 1.645$

拒絕域$R = \{z \geq 1.645\}$

Step 3.　計算檢定統計量

$\sigma = 43.2, n = 50, \bar{X} = 350$

$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{350-338}{43.2/\sqrt{50}} = 1.96$

Step 4.　決策

- 臨界值法

$1.96 \geq 1.645 \Rightarrow$ 拒絕H_0

- P 值法

p-value = $P(Z > z) = P(Z > 1.96) = 0.025$

p-value < $\alpha = 0.05 \Rightarrow$ 拒絕H_0

Step 5.　解釋結果並結論

There is enough evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

4.	The average "moviegoer" sees 8.5 movies a year. A moviegoer is defined as a person who sees at least one movie in a theater in a 12-month period. A random sample of 40 moviegoers from a large university revealed that the average number of movies seen per person was 9.6. The population standard deviation is 3.2 movies. At the 0.05 level of significance, can it be concluded that this represents a difference from the national average?

Moviegoer 影迷

Step 1.　假設

$$H_0: \mu = 8.5 \ \ and\ H_1: \mu \neq 8.5\ (claim)$$

Step 2.　找拒絕域

$\alpha = 0.05$，雙尾$\Rightarrow z_{\frac{0.05}{2}} = z_{0.025} = 1.96$

$R = \{|z| \geq 1.96\}$

Step 3.　計算檢定統計量

$n = 40, \bar{X} = 9.6, \sigma = 3.2$

$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{9.6-8.5}{3.2/\sqrt{40}} = 2.17$

Step 4.　決策

- 臨界值法

$2.17 \geq 1.96 \Rightarrow$ 拒絕H_0

- P 值法

p-value $= 2 \cdot P(Z > z)$

$= 2 \cdot P(Z > 2.17)$

$= 2 \cdot 0.015 = 0.03$

p-value < $\alpha = 0.05 \Rightarrow$ 拒絕H_0

Step 5.　解釋結果並結論

There is enough evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

6.	According to the National Association of Home Builders, the average cost of building a home in the Northeast is \$117.91 per square foot. A random sample of 36 new homes indicated that the mean cost was \$122.57 and the population standard deviation was \$20. Can it be concluded that the mean cost differs from \$117.91, using the 0.10 level of significance?

National Association of Home Builders 全國房屋建築商協會

Step 1.　假設

$$H_0: \mu = 117.91 \ \ and\ H_1: \mu \neq 117.91\ (claim)$$

Step 2.　找拒絕域

$\alpha = 0.1$，雙尾$\Rightarrow z_{\frac{0.1}{2}} = z_{0.05} = 1.645$

$R = \{|z| \geq 1.645\}$

Step 3.　計算檢定統計量

$n = 36, \bar{X} = 122.57, \sigma = 20$

$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{122.57-117.91}{20/\sqrt{36}} = 1.40$

Step 4.　決策

- 臨界值法

$1.4 \ngeq 1.645 \Rightarrow$ 不拒絕H_0

- P 值法

p-value $= 2 \cdot P(Z > z)$

$= 2 \cdot P(Z > 1.4)$

$= 2 \cdot 0.0808 = 0.1616$

p-value $\nless \alpha = 0.05 \Rightarrow$ 不拒絕H_0

Step 5.　解釋結果並結論

There is not enough evidence to support the claim at α = 0.1.

給定顯著水準為 0.1 下，我們沒有足夠的證據支持宣稱

8.	The mean salary of federal government employees on the General Schedule is \$59,593. The average salary of 30 randomly selected state employees who do similar work is \$58,800 with σ = \$1500. At the 0.01 level of significance, can it be concluded that state employees earn on average less than federal employees?

Federal 聯邦

Step 1.　假設

$$H_0: \mu \geq 59593 \ \ and\ \ H_1: \mu < 59593\ (claim)$$

Step 2.　找拒絕域

$\alpha = 0.01$，左尾$\Rightarrow -z_{0.01} = -2.33$

$R = \{z \leq -2.33\}$

Step 3.　計算檢定統計量

$n = 30, \bar{X} = 58800, \sigma = 1500$

$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{58800-59593}{1500/\sqrt{30}} = -2.90$

Step 4.　決策

- 臨界值法

$-2.9 < -2.33 \Rightarrow$ 拒絕H_0

- P 值法

p-value = $P(Z < z) = P(Z < -2.9) = 0.0019$

p-value < $\alpha = 0.05 \Rightarrow$ 拒絕H_0

Step 5.　解釋結果並結論

There is enough evidence to support the claim at α = 0.01.

給定顯著水準為 0.01 下，我們有足夠的證據支持宣稱

18. A store manager hypothesizes that the average number of pages a person copies on the store's copy machine is less than 40. A random sample of 50 customers' orders is selected. At α = 0.01, is there enough evidence to support the claim? Use the P-value hypothesis-testing method. Assume σ = 30.9.

2	2	2	5	32
5	29	8	2	49
21	1	24	72	70
21	85	61	8	42
3	15	27	113	36
37	5	3	58	82
9	2	1	6	9
80	9	51	2	122
21	49	36	43	61
3	17	17	4	1

Step 1.　假設

$$H_0: \mu \geq 40 \ \ and \ \ H_1: \mu < 40(claim)$$

Step 2.　計算檢定統計量

$n = 50, \bar{X} = 29.26, \sigma = 30.9$

$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{29.26-40}{30.9/\sqrt{50}} = -2.46$

Step 3.　找 p-value

p-value = $P(Z < z) = P(Z < -2.46) = 0.0069$

Step 4.　決策

p-value < $\alpha = 0.01 \Rightarrow$ 拒絕H_0

Step 5.　解釋結果並結論

There is enough evidence to support the claim at α = 0.01.

給定顯著水準為 0.01 下，我們有足夠的證據支持宣稱

24. A motorist claims that the South Boro Police issue an average of 60 speeding tickets per day. These data show the number of speeding tickets issued each day for a randomly selected period of 30 days. Assume σ is 13.42. Is there enough evidence to reject the motorist's claim at α = 0.05? Use the P-value method.

72	45	36	68	69	71	57	60	83	26
60	72	58	87	48	59	60	56	64	68
42	57	57	58	63	49	73	75	42	63

motorist 駕駛者; speeding tickets 超速罰單

Step 1.　假設

$$H_0: \mu = 60\ (claim)\ \ and\ H_1: \mu \neq 60$$

Step 2.　計算檢定統計量

$n = 30, \bar{X} = 59.93, \sigma = 13.42$

$z = \frac{\bar{X}-\mu}{\sigma/\sqrt{n}} = \frac{59.93-60}{13.42/\sqrt{30}} = -0.03$

Step 3.　找 p-value

p-value = $2 \cdot P(Z > |z|) = 2 \cdot P(Z > |0.03|) = 2 \cdot 0.4880 = 0.976$

Step 4.　決策

p-value $\nless \alpha = 0.05 \Rightarrow$ 不拒絕H_0

Step 5.　解釋結果並結論

There is not enough evidence to reject the claim at α = 0.05.

給定顯著水準為 0.05 下，我們沒有足夠的證據拒絕宣稱

Sec. 8-3 (p.441)

10.	The National Novel Writing Association states that the average novel is at least 50,000 words. A particularly ambitious writing club at a college-preparatory high school had randomly selected members with works of the following lengths. At α = 0.10, is there sufficient evidence to conclude that the mean length is greater than 50,000 words? 48972 50100 51560 49800 50020 49900 52193

National Novel Writing Association 全國小說寫作協會; club 俱樂部;

Step 1. 假設

$$H_0: \mu \leq 50000 \ \ and\ H_1: \mu > 50000\ (claim)$$

Step 2. 找拒絕域

$\alpha = 0.1$，$d.f. = n - 1 = 6$，單尾$\Rightarrow t_{(\alpha,d.f.)} = t_{(0.1,6)} = 1.440$

$R = \{t > 1.440\}$

Step 3. 計算檢定統計量

$n = 7, \bar{X} = 50363.571, s = 1113.159$

$t = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{50363.571-50000}{1113.159/\sqrt{7}} = 0.864$

Step 4. 決策

- 臨界值法

$0.864 \ngeq 1.440 \Rightarrow$ 不拒絕H_0

- P 值法

p-value $= P(T > t) = P(T > 0.864) > 0.1$

p-value $\nless \alpha = 0.1 \Rightarrow$ 不拒絕H_0

Step 5. 解釋結果並結論

There is not enough evidence to support the claim at α = 0.1.

給定顯著水準為 0.1 下，我們沒有足夠的證據支持宣稱

12.	The average 1-ounce chocolate chip cookie contains 110 calories. A random sample of 15 different brands of 1-ounce chocolate chip cookies resulted in the following calorie amounts. At the α = 0.01 level, is there sufficient evidence that the average calorie content is greater than 110 calories? 100 125 150 160 185 125 155 145 160 100 150 140 135 120 110

Step 1. 假設

$$H_0: \mu \leq 110\ and\ H_1: \mu > 110\ (claim)$$

Step 2. 找拒絕域

$\alpha = 0.01$，$d.f. = n - 1 = 14$，單尾$\Rightarrow t_{(\alpha,d.f.)} = t_{(0.01,14)} = 2.624$

$R = \{t > 2.624\}$

Step 3. 計算檢定統計量

$n = 15, \bar{X} = 137.333, s = 24.118$

$t = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{137.333-110}{24.118/\sqrt{15}} = 4.389$

Step 4. 決策

- 臨界值法

$4.389 \geq 2.624 \Rightarrow$ 拒絕H_0

- P 值法

p-value $= P(T > t) = P(T > 4.389) < 0.005$

p-value $\nless \alpha = 0.01 \Rightarrow$ 拒絕H_0

Step 5. 解釋結果並結論

There is enough evidence to support the claim at α = 0.01.

給定顯著水準為 0.01 下，我們有足夠的證據支持宣稱

16.	A random sample of stipends of teaching assistants in economics is listed. Is there sufficient evidence at the α = 0.05 level to conclude that the average stipend differs from $15,000? The stipends listed (in dollars) are for the academic year.

14000	18000	12000	14356	13185	13419
14000	11981	17604	12283	16338	15000

stipends 津貼

Step 1. 假設

$$H_0: \mu = 15000 \;\; and \; H_1: \mu \neq 15000 \; (claim)$$

Step 2. 找拒絕域

$\alpha = 0.05$，$d.f. = n - 1 = 11$，雙尾$\Rightarrow t_{(\alpha,d.f.)} = t_{(0.05,11)} = 2.201$

$R = \{|t| > 2.201\}$

Step 3. 計算檢定統計量

$n = 12, \bar{X} = 14347.167, s = 2048.541$

$t = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{14347.167-15000}{2048.541/\sqrt{12}} = -1.104$

Step 4. 決策

- 臨界值法

$|-1.104| \ngeq 2.201 \Rightarrow$ 不拒絕H_0

- P 值法

p-value $= 2 \cdot P(T > |t|)$

$= 2 \cdot P(T > |-1.104|) > 2 \cdot 0.1$

p-value $\nless \alpha = 0.05 \Rightarrow$ 不拒絕H_0

Step 5. 解釋結果並結論

There is not enough evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

22.	The U.S. Bureau of Labor and Statistics reported that a person between the ages of 18 and 34 has had an average of 9.2 jobs. To see if this average is correct, a researcher selected a random sample of 8 workers between the ages of 18 and 34 and asked how many different places they had worked. The results were as follows: 8 12 15 6 1 9 13 2 At α = 0.05, can it be concluded that the mean is 9.2? Use the P-value method. Give one reason why the respondents might not have given the exact number of jobs that they have worked.

Bureau of Labor and Statistics 勞工統計局

Step 1. 假設

$$H_0: \mu = 9.2\ (claim)\ \ and\ H_1: \mu \neq 9.2$$

Step 2. 計算檢定統計量

$n = 8, \bar{X} = 8.25, s = 5.064$

$t = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{8.25-9.2}{5.064/\sqrt{8}} = -0.531$

Step 3. 決策

p-value $= 2 \cdot P(T > |t|) = 2 \cdot P(T > \ |-0.531|) > 2 \cdot 0.25$

Step 4. 決策

p-value $\not< \alpha = 0.05 \Rightarrow$ 不拒絕H_0

$(0.5^+ \not< 0.05)$

Step 5. 解釋結果並結論

There is not enough evidence to reject the claim at α = 0.05.

給定顯著水準為 0.05 下，我們沒有足夠的證據拒絕宣稱

Sec. 8-4 (p.449)

6.	It has been found that 50.3% of U.S. households own stocks and mutual funds. A random sample of 300 heads of households indicated that 171 owned some type of stock. At what level of significance would you conclude that this was a significant difference?

mutual funds 共同基金;

Step 1. 假設

$$H_0: p = 0.503\ (claim)\ and\ H_1: p \neq 0.503$$

Step 2. 計算檢定統計量

$n = 300, \hat{p} = \frac{171}{300} = 0.57$

$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.57-0.503}{\sqrt{\frac{0.503\cdot(1-0.503)}{300}}} = 2.321$

Step 3. 找 p-value

p-value = $2 \cdot P(Z > |z|) = 2 \cdot P(Z > |2.321|) = 2 \cdot 0.0102 = 0.0204$

Step 4.　找α，讓結論為顯著差異

當 p-value $\leq \alpha$，則拒絕H_0，表是結果有顯著差異

所以當 $\alpha \geq 0.0204$ 時，結果為有顯著差異

8.	The percentage of physicians who are women is 27.9%. In a survey of physicians employed by a large university health system, 45 of 120 randomly selected physicians were women. Is there sufficient evidence at the 0.05 level of significance to conclude that the proportion of women physicians at the university health system exceeds 27.9%?

physicians 醫生;

Step 1.　假設

$$H_0: p \leq 0.279 \ and \ H_1: p > 0.279 \ (claim)$$

Step 2.　找拒絕域

$\alpha = 0.05$，單尾$\Rightarrow z_{0.05} = 1.645$

$R = \{z \geq 1.645\}$

Step 3.　計算檢定統計量

$$n = 120, \hat{p} = \frac{45}{120} = 0.375$$

$$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.375-0.279}{\sqrt{\frac{0.279\cdot(1-0.279)}{120}}} = 2.345$$

Step 4.　決策

- 臨界值法

$2.345 \geq 1.645 \Rightarrow$ 拒絕H_0

- P 值法

p-value $= P(Z > z) = P(Z > 2.35) = 0.0094$

p-value $< \alpha = 0.05 \Rightarrow$ 拒絕H_0

Step 5.　解釋結果並結論

There is enough evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據支持宣稱

14.	The Energy Information Administration reported that 51.7% of homes in the UnitedStates were heated by natural gas. A random sample of 200 homes found that 115 were heated by natural gas. Does the evidence support the claim, or has the percentage changed? Use α = 0.05 and the P-value method. What could be different if the sample were taken in a different geographic area?

The Energy Information Administration 能源情報署; natural gas 天然氣;

Step 1.　假設

$$H_0: p = 0.517 \ (claim) \ and \ H_1: p \neq 0.517$$

Step 2.　計算檢定統計量

$$n = 200, \hat{p} = \frac{115}{200} = 0.575$$

$$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.575-0.517}{\sqrt{\frac{0.517\cdot(1-0.517)}{200}}} = 1.641$$

Step 3.　找 p-value

p-value = $2 \cdot P(Z > |z|) = 2 \cdot P(Z > |1.64|) = 2 \cdot 0.0505 = 0.1010$

Step 4.　決策

p-value $\nless \alpha = 0.05 \Rightarrow$ 不拒絕H_0

Step 5.　解釋結果並結論

There is not enough evidence to reject the claim at α = 0.05.

給定顯著水準為 0.05 下，我們沒有足夠的證據拒絕宣稱

20. Approximately 70% of the U.S. population recycles. According to a green survey of a random sample of 250 college students, 204 said that they recycled. At α = 0.01, is there sufficient evidence to conclude that the proportion of college students who recycle is greater than 70%?

recycles (v.) 做回收利用;

Step 1.　假設

$$H_0: p \le 0.7 \ and\ H_1: p > 0.7\ (claim)$$

Step 2.　找拒絕域

$\alpha = 0.01$，單尾$\Rightarrow z_{0.01} = 2.33$

$R = \{z \ge 2.33\}$

Step 3.　計算檢定統計量

$$n = 250, \hat{p} = \frac{204}{250} = 0.816$$

$$z = \frac{\hat{p}-p}{\sqrt{\frac{pq}{n}}} = \frac{0.816-0.7}{\sqrt{\frac{0.7\cdot(1-0.7)}{250}}} = 4.002$$

Step 4.　決策

- 臨界值法

$4.002 \ge 2.33 \Rightarrow$ 拒絕H_0

- P 值法

p-value $= P(Z > z) = P(Z > 4.00) \approx 0$

p-value $< \alpha = 0.01 \Rightarrow$ 拒絕H_0

Step 5.　解釋結果並結論

There is enough evidence to support the claim at α = 0.01.

給定顯著水準為 0.01 下，我們有足夠的證據支持宣稱

Sec. 8-5 (p.461)

6.	The number of carbohydrates found in a random sample of fast-food entrees is listed. Is there sufficient evidence to conclude that the variance differs from 100? Use the 0.05 level of significance. 53 46 39 39 30 47 38 73 43 41

carbohydrates 碳水化合物;

Step 1. 假設

$$H_0: \sigma^2 = 100 \ and \ H_1: \sigma^2 \neq 100 \ (claim)$$

Step 2. 找拒絕域

$\alpha = 0.05$，$d.f. = n - 1 = 9$

雙尾$\Rightarrow \chi^2{}_{\left(\frac{0.05}{2},9\right)} = \chi^2{}_{(0.025,9)} = 19.023$ & $\chi^2{}_{\left(1-\frac{0.05}{2},9\right)} = \chi^2{}_{(0.975,9)} = 2.700$

$R = \{\chi^2 \leq 2.7 or \ \chi^2 \geq 19.023\}$

Step 3. 計算檢定統計量

$$n = 8, s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = 135.433$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{9 \cdot 135.433}{100} = 12.189$$

Step 4. 決策

- 臨界值法

$12.189 \ngeq 19.023 \Rightarrow$ 不拒絕H_0

- P 值法

df=9 時，$4.168 < 12.189 < 14.684$

所以 $0.1 < P(\chi^2 > 12.189) < 0.9$

p-value = $2 \cdot P(\chi^2 > 12.189) > 0.2$

p-value $\nless \alpha = 0.05 \Rightarrow$ 不拒絕H_0

Step 5. 解釋結果並結論

There is not enough evidence to support the claim at α = 0.05.

給定顯著水準為 0.05 下，我們沒有足夠的證據支持宣稱

8.	A machine fills 12-ounce bottles with soda. For the machine to function properly, the standard deviation of the population must be less than or equal to 0.03 ounce. A random sample of 8 bottles is selected, and the number of ounces of soda in each bottle is given. At *α* = 0.05 , can we reject the claim that the machine is functioning properly? Use the P-value method. 12.03 12.10 12.02 11.98 12.00 12.05 11.97 11.99

function properly 正常運行;

Step 1. 假設

$$H_0: \sigma \leq 0.03(claim) \ and \ H_1: \sigma > 0.03$$

Step 2. 計算檢定統計量

$$n=8, s^2=\frac{n\sum X^2-(\sum X)^2}{n(n-1)}=0.002$$

$$\chi^2=\frac{(n-1)s^2}{\sigma^2}=\frac{7\cdot 0.002}{0.03^2}=15.556$$

Step 3. 找 p-value

df=7 時，$14.067<15.556<16.013$，所以 $0.025<P(\chi^2>15.556)<0.05$

p-value = $P(\chi^2>15.556)<0.05$

Step 4. 決策

p-value $\le \alpha=0.05 \Rightarrow$ 拒絕H_0

Step 5. 解釋結果並結論

There is enough evidence to reject the claim at α = 0.05.

給定顯著水準為 0.05 下，我們有足夠的證據拒絕宣稱

16. A random sample of second-round golf scores from a major tournament is listed below.At α = 0.10, is there sufficient evidence to conclude that the population variance exceeds 9?

75	67	69	72	70
66	74	69	74	71

major tournament 錦標賽;

Step 1. 假設

$$H_0: \sigma^2 \le 9 \; and \; H_1: \sigma^2 > 9 \; (claim)$$

Step 2. 找拒絕域

$\alpha=0.1$，$d.f.=n-1=9$

右尾$\Rightarrow \chi^2{}_{(0.1,9)}=14.684$

$R=\{\chi^2 \ge 14.684\}$

Step 3. 計算檢定統計量

$$n=10, s^2=\frac{n\sum X^2-(\sum X)^2}{n(n-1)}=9.344$$

$$\chi^2=\frac{(n-1)s^2}{\sigma^2}=\frac{9\cdot 9.344}{9}=9.344$$

Step 4. 決策

- 臨界值法

$9.344 \ngeq 14.684 \Rightarrow$ 不拒絕H_0

- P 值法

df=9 時，$4.168<9.344<14.684$

所以 $0.1<P(\chi^2>9.344)<0.9$

p-value = $P(\chi^2>9.344)>0.1$

p-value $\nleq \alpha=0.1 \Rightarrow$ 不拒絕H_0

Step 5. 解釋結果並結論

There is not enough evidence to support the claim at α = 0.1.

給定顯著水準為 0.1 下，我們沒有足夠的證據支持宣稱

19. Room and board fees for a random sample of independent religious colleges are shown.

7460	7959	7650	8120	7220	8768	7650
8400	7860	6782	8745	7443	9500	9100

Estimate the standard deviation in costs based on $\sigma \approx R/4$. Is there sufficient evidence to conclude that the sample standard deviation differs from this estimated amount? Use $\alpha = 0.05$.

Step 1. 假設

$$\sigma_0 = \frac{R}{4} = \frac{9500-6782}{4} = 679.5$$

$$H_0: \sigma = 679.5 \ and \ H_1: \sigma \neq 679.5 \ (claim)$$

Step 2. 找拒絕域

$\alpha = 0.05$，$d.f. = n - 1 = 13$

雙尾⇒ $\chi^2{}_{\left(\frac{0.05}{2},13\right)} = \chi^2{}_{(0.025,13)} = 24.736$ & $\chi^2{}_{\left(1-\frac{0.05}{2},13\right)} = \chi^2{}_{(0.975,13)} = 5.009$

$R = \{\chi^2 \leq 5.009 \ or \ \chi^2 \geq 24.736\}$

Step 3. 計算檢定統計量

$$n = 14, s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = 593931.648$$

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} = \frac{13 \cdot 593931.648}{9679.5^2} = 16.722$$

Step 4. 決策

- 臨界值法

$16.722 \ngeq 24.736 \Rightarrow$ 不拒絕H_0

- P 值法

df=13 時，$7.042 < 16.722 < 19.812$

所以 $0.1 < P(\chi^2 > 16.722) < 0.9$

p-value = $2 \cdot P(\chi^2 > 9.344) > 0.2$

p-value $\nless \alpha = 0.05 \Rightarrow$ 不拒絕H_0

Step 5. 解釋結果並結論

There is not enough evidence to support the claim at α = 0.1.

給定顯著水準為 0.1 下，我們沒有足夠的證據支持宣稱

Sec. 8-6 (p.466) 課本範例

Example 8-30	Sugar is packed in 5-pound bags. An inspector suspects the bags may not contain 5 pounds. A random sample of 50 bags produces a mean of 4.6 pounds and a standard deviation of 0.7 pounds. Is there enough evidence to conclude that the bags do not contain 5 pounds as stated at α = 0.05? Also, find the 95% confidence interval of the true mean. Assume the variable is normally distributed.

Step 1.　假設

$$H_0: \mu = 5 \ and \ H_1: \mu \neq 5 \ (claim)$$

Step 2.　拒絕域

σ未知，使用 t 檢定，df=n-1=49

$R = \left\{|t| \geq t_{\left(\frac{0.05}{2},45\right)} = 2.014\right\}$ (課本的 t 表有到 df=45)

Step 3.　檢定統計量

$t = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{4.6-5}{0.7/\sqrt{50}} = -4.04$

Step 4.　決策

$|-4.04| \geq 2.014 \Rightarrow$ 拒絕H_0

Step 5.　結論

There is enough evidence to conclude that the the bags do not contain 5 pounds at α=0.05.

給定顯著水準為 0.05 下，有足夠的證據證明一袋糖的重量不為 5 磅

Step 6.　信賴區間

$$\bar{X} - t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$$

$$4.6 - 2.014 \cdot \frac{0.7}{\sqrt{50}} < \mu < 4.6 + 2.014 \cdot \frac{0.7}{\sqrt{50}}$$

$$4.4 < \mu < 4.8$$

95%信賴區間不包含假設的$\mu = 5$，所以假設檢定和信賴區間的結果一致

Example 8-31	A researcher claims that adult hogs fed a special diet will have an average weight of 200 pounds. A random sample of 10 hogs has an average weight of 198.2 pounds and a standard deviation of 3.3 pounds. At α = 0.05, can the claim be rejusted? Also, find the 95% confidence interval of the true mean. Assume the variable is normally distributed.

hogs (供食用的)豬; feed (feed, fed, fed) 餵食

Step 1.　假設

$$H_0: \mu = 200 \ (claim) \ and \ H_1: \mu \neq 200$$

Step 2.　拒絕域

σ未知，使用 t 檢定，df=n-1=9

$$R = \left\{ |t| \geq t_{\left(\frac{0.05}{2},9\right)} = 2.262 \right\}$$

Step 3.　檢定統計量

$$t = \frac{\bar{X}-\mu}{s/\sqrt{n}} = \frac{198.2-200}{3.3/\sqrt{10}} = -1.72$$

Step 4.　決策

$|-1.72| \ngeq 2.262 \Rightarrow$ 不拒絕H_0

Step 5.　結論

There is not enough evidence to conclude that the average weight of adult hogs is not 200 pounds at α=0.05.

給定顯著水準為 0.05 下，沒有足夠的證據證明成年豬的平均體重不為 200 磅

Step 6.　信賴區間

$$\bar{X} - t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}} < \mu < \bar{X} + t_{\frac{\alpha}{2}} \cdot \frac{s}{\sqrt{n}}$$

$$4.6 - 2.262 \cdot \frac{3.3}{\sqrt{10}} < \mu < 4.6 + 2.262 \cdot \frac{3.3}{\sqrt{10}}$$

$$195.8 < \mu < 200.6$$

95%信賴區間包含假設的$\mu = 200$，所以假設檢定和信賴區間的結果一致