Sec. 7-1 (p.365)

1.	What is the difference between a point estimate and an interval estimate of a parameter? Which is better? Why?

A point estimate of a parameter specifies a particular value, such as μ = 87; an interval estimate specifies a range of values for the parameter, such as 84 < μ < 90.
The advantage of an interval estimate is that a specific confidence level (say 95%) can be selected, and one can be 95% confident that the interval contains the parameter that is being estimated.
區間估計的優勢在於可以選擇特定的信心水平（例如 95%），並且可以有 95% 的信心認為該區間包含正在估計的參數。

4.	What is meant by the 95% confidence interval of the mean?

A 95% confidence interval means that one can be 95% confident that the parameter being estimated will be contained within the limits of the interval.
95% 的信賴區間意味著有 95% 的信心被估計的參數將包含在區間的範圍內

<u>determine the margin of error for each confidence interval</u>

8.	The confidence interval for the proportion is 0.623 < p < 0.626.

$$CI\ of\ p \in (0.623, 0.626)$$

$$CI\ of\ p \in \left(\hat{p} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$$

$$\Rightarrow \left(\hat{p} + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) + \left(\hat{p} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right) = 2\hat{p} = 0.623 + 0.626 = 1.249$$

$$\Rightarrow \hat{p} = 0.6245$$

$$\Rightarrow z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.6245 - 0.623\ or - (0.6245 - 0.626) = 0.0015$$

The margin of error is 0.0015.

12.	The confidence interval for the mean is 74 < u < 84.

$$CI\ of\ \mu \in (74,84)$$

$$CI\ of\ \mu \in \left(\bar{X} - z_{\frac{\alpha}{2}} \cdot \frac{\sqrt{\sigma}}{n}, \bar{X} + z_{\frac{\alpha}{2}} \cdot \frac{\sqrt{\sigma}}{n}\right)$$

$$\Rightarrow \bar{X} = \frac{74+84}{2} = 79$$

$$\Rightarrow CI\ of\ \mu = 79 \pm 5$$

The margin of error is 5.

Sec. 7-2 (p.371)

12. A sociologist (社會學家) found that in a random sample of 50 retired men, the average number of jobs they had during their lifetimes was 7.2. The population standard deviation is 2.1.

a. Find the best point estimate of the population mean.
b. Find the 95% confidence interval of the mean number of jobs.
c. Find the 99% confidence interval of the mean number of jobs.
d. Which is smaller? Explain why.

$$n = 50, \bar{X} = 7.2, \sigma = 2.1$$

a. $\bar{X} = 7.2$ is the point estimate for μ.

$$\bar{X} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

b. $z_{\frac{\alpha}{2}} = 1.96,\ z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 0.582$

$\Rightarrow 7.2 \pm 0.582 = (6.618, 7.782)$
$\Rightarrow 6.62 < \mu < 7.78$

c. $z_{\frac{\alpha}{2}} = 2.58,\ z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 0.766$

$\Rightarrow 7.2 \pm 0.766 = (6.434, 7.966)$
$\Rightarrow 6.43 < \mu < 7.97$

d. The 95% confidence is smaller since there is less of a chance that the mean is contained in the 95% interval as opposed to the 99% confidence interval.

14. First-semester (第一學期) GPAs for a random selection of freshmen (大學新生) at a large university are shown. Estimate the true mean GPA of the freshman class with 99% confidence. Assume $\boldsymbol{\sigma = 0.62}$.

1.9	3.2	2.0	2.9	2.7	3.3
2.8	3.0	3.8	2.7	2.0	1.9
2.5	2.7	2.8	3.2	3.0	3.8
3.1	2.7	3.5	3.8	3.9	2.7
2.0	2.8	1.9	4.0	2.2	2.8
2.1	2.4	3.0	3.4	2.9	2.1

$$z_{\frac{\alpha}{2}} = 2.58, \sigma = 0.62, n = 36$$

$$\bar{X} - z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} < \mu < \bar{X} + z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}}$$

$$\bar{X} = 2.819, z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 0.267$$

$$\Rightarrow 2.819 \pm 0.267 = (2.552, 3.086)$$
$$\Rightarrow 2.55 < \mu < 3.09$$

20.	The growing seasons for a random sample of 35 U.S. cities were recorded, yielding a sample mean of 190.7 days and the population standard deviation of 54.2 days. Estimate for all U.S. cities the true mean of the growing season with 95% confidence.

$$n = 35, \bar{X} = 190.7, \sigma = 54.2$$

$$z_{\frac{\alpha}{2}} = 1.96,\ \ z_{\frac{\alpha}{2}} \cdot \frac{\sigma}{\sqrt{n}} = 17.956$$

$$\Rightarrow 190.7 \pm 17.956 = (172.744, 208.656)$$

$\Rightarrow 172.7 < \mu < 208.7$

24.	A pizza shop owner wishes to find the 95% confidence interval of the true mean cost of a large cheese pizza. How large should the sample be if she wishes to be accurate to within \$0.15? A previous study showed that the standard deviation of the price was \$0.26.

$$E = 0.15, \sigma = 0.26, z_{\frac{\alpha}{2}} = 1.96$$

$n = \left(\frac{z_{\frac{\alpha}{2}} \cdot \sigma}{E}\right)^2 = 11.542$ (無條件進位)

$\Rightarrow n = 12$

26.	It is desired to estimate the mean GPA of each undergraduate (大學部) class at a large university. How large a sample is necessary to estimate the GPA within 0.25 at the 99% confidence level? The population standard deviation is 1.2.

$$E = 0.25, \sigma = 1.2, z_{\frac{\alpha}{2}} = 2.58$$

$$n = \left(\frac{z_{\frac{\alpha}{2}} \cdot \sigma}{E}\right)^2 = 153.36$$

$\Rightarrow n = 154$

Sec. 7-3 (p.379)

6.	The prices (in dollars) for a particular model of digital camera with 18.0 megapixels (百萬像素) and a f/3.5-5.6 <u>zoom lens</u> (變焦鏡頭) are shown here for 10 randomly selected online retailers. Estimate the true mean price for this particular model with 95% confidence. 999 1499 1997 398 591 498 798 849 449 348

$$n = 10, \bar{X} = \frac{\sum X}{n} = 842.6, s = \sqrt{\frac{n\sum X^2 - (\sum X)^2}{n(n-1)}} = 534.297$$

$$95\%, \sigma\ unknown, d.f. = n - 1 = 9 \Rightarrow t_{0.05/2} = 2.262$$

$$CI \Rightarrow \bar{X} \pm t \cdot \frac{s}{\sqrt{n}} = 842.6 \pm 2.262 \cdot \frac{534.297}{\sqrt{10}} = (460.41, 1224.79)$$

10.	The number of students who belong to the <u>dance company</u> (舞蹈團) at each of several randomly selected small universities is shown here. Estimate the true population mean size of a university

	dance company with 99% confidence.						
	25	21	32	22	30	29	30
	28	26	47	35	26	26	28
	28	35	27	32	40		

$n = 19, \bar{X} = 29.842, s = 6.176$

$99\%, \sigma\ unknown, d.f. = n - 1 = 18 \Rightarrow t_{0.01/2} = 2.878$

$$CI \Rightarrow \bar{X} \pm t \cdot \frac{s}{\sqrt{n}} = 29.842 \pm 2.878 \cdot \frac{6.176}{\sqrt{19}} = (25.76, 33.92)$$

16. For a random sample of 24 operating rooms (手術室) taken in the hospital study mentioned in Exercise 19 in Section 7-1, the mean noise level was 41.6 decibels (分貝), and the standard deviation was 7.5. Find the 95% confidence interval of the true mean of the noise levels in the operating rooms.

$n = 24, \bar{X} = 41.6, s = 7.5$

$95\%, \sigma\ unknown, d.f. = n - 1 = 23 \Rightarrow t_{0.05/2} = 2.069$

$$CI \Rightarrow \bar{X} \pm t \cdot \frac{s}{\sqrt{n}} = 41.6 \pm 2.069 \cdot \frac{7.5}{\sqrt{24}} = (38.43, 44.77)$$

20. The number of unhealthy days based on the AQI (Air Quality Index) for a random sample of metropolitan areas (大都會區) is shown. Construct a 98% confidence interval based on the data.

61	12	6	40	27	38	93	5	13	40

$n = 10, \bar{X} = 33.5, s = 27.678$

$98\%, \sigma\ unknown, d.f. = n - 1 = 9 \Rightarrow t_{0.05/2} = 2.821$

$$CI \Rightarrow \bar{X} \pm t \cdot \frac{s}{\sqrt{n}} = 33.5 \pm 2.821 \cdot \frac{27.678}{\sqrt{10}} = (8.81, 58.19)$$

Sec. 7-4 (p.387)

4. In 2014, 6% of the cars sold had a manual transmission (手動變速器). A random sample of college students who owned cars revealed the following: out of 122 cars, 26 had manual transmissions. Estimate the proportion of college students who drive cars with manual transmissions with 90% confidence.

$$n = 122, \hat{p} = \frac{26}{122} = 0.213$$

$90\% \Rightarrow z_{0.1/2} = 1.645$

$$CI \Rightarrow \hat{p} \pm z \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.213 \pm 1.645\sqrt{\frac{0.213(1 - 0.213)}{122}} = (0.152, 0.274)$$

8. A CBS News (哥倫比亞廣播公司新聞)/New York Times (紐約時報) poll found that 329 out of 763 randomly selected adults said they would travel to outer space in their lifetime, given the chance. Estimate the true proportion of adults who would like to travel to outer space with 92% confidence.

$$n = 763, \hat{p} = \frac{329}{763} = 0.431$$

$$92\% \Rightarrow z_{0.08/2} = 1.75$$

$$CI \Rightarrow \hat{p} \pm z \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.431 \pm 1.75 \cdot \sqrt{\frac{0.431(1-0.431)}{763}} = (0.400, 0.462)$$

12. It has been reported that 20.4% of incoming freshmen (新生) indicate that they will major in business or a related field. A random sample of 400 incoming college freshmen was asked their preference, and 95 replied that they were considering business as a major. Estimate the true proportion of freshman business majors with 98% confidence. Does your interval contain 20.4?

$$n = 400, \hat{p} = \frac{95}{400} = 0.2375$$

$$98\% \Rightarrow z_{0.02/2} = 2.33$$

$$CI \Rightarrow \hat{p} \pm z \cdot \sqrt{\frac{\hat{p}\hat{q}}{n}} = 0.2375 \pm 2.33 \cdot \sqrt{\frac{0.2375(1-0.2375)}{400}} = (0.188, 0.287)$$

16. A recent study indicated that 29% of the 100 women over age 55 in the study were widows (寡婦).

a. How large a sample must you take to be 90% confident that the estimate is within 0.05 of the true proportion of women over age 55 who are widows?

b. If no estimate of the sample proportion is available, how large should the sample be?

$\hat{p} = 0.29, n = 100$

$90\% \Rightarrow z_{0.1/2} = 1.645$

a.

$E = 0.05$

$$n = \hat{p}\hat{q}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = 0.29(1-0.29)\left(\frac{1.645}{0.05}\right)^2 = 222.9 \Rightarrow n = 223$$

b.

$\hat{p}$ 未知，用 0.5 代 ($\because$ 這樣變異數最大)

$$n = \hat{p}\hat{q}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = 0.5 \cdot 0.5\left(\frac{1.645}{0.05}\right)^2 = 270.6 \Rightarrow n = 271$$

20.	A federal (聯邦) report indicated that 27% of children ages 2 to 5 years had a good diet — an increase over previous years. How large a sample is needed to estimate the true proportion of children with good diets within 2% with 95% confidence?

$\hat{p} = 0.27, E = 0.02$

$95\% \Rightarrow z_{0.05/2} = 1.96$

$$n = \hat{p}\hat{q}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2 = 0.27(1-0.27)\left(\frac{1.96}{0.02}\right)^2 = 1892.9 \Rightarrow n = 1893$$

Sec. 7-5 (p.396)

5.	The number of carbohydrates (in grams) per 8-ounce serving of yogurt for each of a random selection of brands is listed below. Estimate the true population variance and standard deviation for the number of carbohydrates per 8-ounce serving of yogurt with 95% confidence. Assume the variable is normally distributed. 17 42 41 20 39 41 35 15 43 25 38 33 42 23 17 25 34

- Carbohydrates 碳水化合物; 醣類

$$\frac{(n-1)s^2}{\chi^2_{right}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{left}}$$

$$s^2 = \frac{n\sum X^2 - (\sum X)^2}{n(n-1)} = 102.0294, n = 17$$

$95\%, d.f. = 16, \chi^2_{0.025} = 28.845, \chi^2_{0.975} = 6.908$

$$CI\ of\ variance \Rightarrow \frac{16 \cdot 102.0294}{28.845} < \sigma^2 < \frac{16 \cdot 102.0294}{6.908} = (56.595, 236.316)$$

$$CI\ of\ standard\ deviation \Rightarrow \left(\sqrt{56.595}, \sqrt{236.316}\right) = (7.523, 15.373)$$

8.	Find the 90% confidence interval for the variance and standard deviation of the ages of seniors at Oak Park College if a random sample of 24 students has a standard deviation of 2.3 years. Assume the variable is normally distributed.

$n = 24, s = 2.3 \Rightarrow s^2 = 2.3^2 = 5.29$

$90\%, d.f. = 23, \chi^2_{0.05} = 35.172, \chi^2_{0.95} = 13.091$

$$CI\ of\ variance \Rightarrow \frac{23 \cdot 5.29}{35.172} < \sigma^2 < \frac{23 \cdot 5.29}{13.091} = (3.459, 9.294)$$

$CI\ of\ standard\ deviation \Rightarrow (1.860, 3.049)$

14.	Estimate the variance in mean mathematics SAT scores by state, using the randomly selected scores listed below. Estimate with 99% confidence. Assume the variable is normally distributed.					
	490	502	211	209	499	565
	469	543	572	550	515	500

$n = 12, s^2 = 2.3^2 = 15593.841$

$99\%, d.f. = 11, \chi^2_{0.005} = 26.757, \chi^2_{0.995} = 2.603$

$$CI\ of\ variance \Rightarrow \frac{11 \cdot 15593.841}{26.757} < \sigma^2 < \frac{11 \cdot 15593.841}{2.603} = (6410.74, 65897.91)$$