Sec. 4-1 (p.206)

12. Classify each statement as an example of classical probability, empirical probability, or subjective probability.

a. The probability that a student will get a C or better in a statistics course is about 70%.
b. The probability that a new fast-food restaurant will be a success in Chicago is 35%.
c. The probability that interest rates will rise in the next 6 months is 0.50.
d. The probability that the unemployment rate will fall next month is 0.03.

a. Empirical probability
(根據以往學生在統計學這門課的學習成績，觀察學生在統計學課程獲得 C 成績或是更高的次數，來推測說學生獲得 C 成績或是更高成績的機率約為 70%)

b. Empirical probability
(根據以前的餐廳在芝加哥的發展，透過觀察餐廳在芝加哥營業的成功次數，來推測說這間即將新開的餐廳其成功率為 35%)

c. Subjective probability
(「接下來 6 個月的利率將會上升的機率為 0.50」，並沒有任何證據可以得知真正會上升多少，憑藉個人經驗或直覺來判定事件發生的機率)

d. Subjective probability
(「下個月的失業率下降的機率為 0.03」，，並沒有任何證據可以得知真 正會下降多少，憑藉個人經驗或直覺來判定事件發生的機率)

16. **Rolling Two Dice**
If two dice are rolled one time, find the probability of getting these results:

a. A sum less than 9
b. A sum greater than or equal to 10
c. A 3 on one die or on both dice

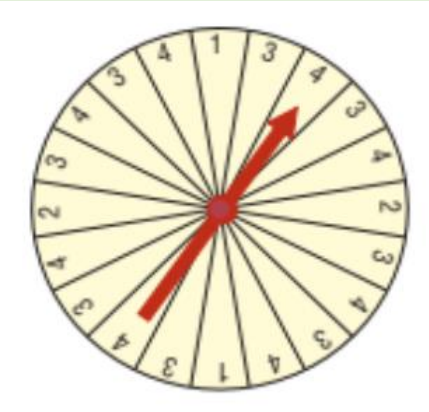
擲兩顆骰子，所有可能的結果個數為 $6^2 = 36$

a. $P(\text{sum} < 9) = 1 - P(\text{sum} \geq 9) = 1 - \frac{10}{36} = \frac{26}{36} = \frac{13}{18}$	9: {6,3}, {5,4}, {4,5}, {3,6} 10: {6,4}, {5,5}, {4,6} 11: {6,5}, {5,6} 12: {6,6}
b. $P(\text{sum} \geq 10) = \frac{6}{36} = \frac{1}{6}$	
c. $P(3\ occur \geq 1) = 1 - P(3\ occur = 0) = 1 - \frac{25}{36} = \frac{11}{36}$	3 沒有出現的可能結果個數為 $5^2 = 25$

19.

Shopping Mall Promotion

A shopping mall has set up a promotion as follows. With any mall purchase of $50 or more, the customer gets to spin the wheel shown here. If a number 1 comes up, the customer wins $10. If the number 2 comes up, the customer wins $5; and if the number 3 or 4 comes up, the customer wins a discount coupon. Find the following probabilities.



a. The customer wins $10.
b. The customer wins money.
c. The customer wins a coupon.

Number	1	2	3	4	Total
個數	2	2	8	8	20

a. $P(win\ \$10) = P(number\ 1) = \frac{2}{20} = \frac{1}{10} = 0.1$

b. $P(win\ money) = P(number\ 1\ or\ 2) = \frac{4}{20} = \frac{1}{5} = 0.2$

c. $P(win\ coupon) = P(number\ 3\ or\ 4) = \frac{16}{20} = \frac{4}{5} = 0.8$

24.

Rural Speed Limits

Rural speed limits for all 50 states are indicated below.

60 mph	65 mph	70 mph	75 mph
1 (HI)	18	18	13

Choose one state at random. Find the probability that its speed limit is

a. 60 or 70 miles per hour
b. Greater than 65 miles per hour
c. 70 miles per hour or less

a. $P(speed\ limits = 60\ or\ 70) = \frac{1+18}{50} = \frac{19}{50} = 0.38$

b. $P(speed\ limits > 65) = \frac{18+13}{50} = \frac{31}{50} = 0.62$

c. $P(speed\ limits \leq 70) = \frac{1+18+18}{50} = \frac{37}{50} = 0.74$

25. **Gender of Children**

A couple has 4 children. Find each probability.

a. All girls

b. Exactly two girls and two boys

c. At least one child who is a girl

d. At least one child of each gender

所有可能的結果個數為 $2^4 = 16$

a. $P(all\ girls) = \frac{1}{16} = 0.0625$

b. $P(2\ girls\ 2\ boys) = \frac{6}{16} = 0.375$ (BBGG 的排列組合$= \frac{4!}{2!2!} = \frac{4 \cdot 3 \cdot 2 \cdot 1}{2 \cdot 2} = 6$)

c. $P(at\ least\ 1\ girl) = 1 - P(no\ girl) = 1 - \frac{1}{16} = \frac{15}{16} = 0.9375$

d. $P(at\ least\ 1\ girl\ and\ 1\ boy) = 1 - P(all\ boy) - P(all\ girl) = 1 - \frac{1}{16} - \frac{1}{16} = \frac{14}{16} = 0.875$

31. **Crimes Committed**

The numbers show the number of crimes committed in a large city. If a crime is selected at random, find the probability that it is a motor vehicle theft. What is the probability that it is not an assault?

Theft (竊盜)	1375
Burglary of home or office (家庭或辦公室入室盜竊)	500
Motor vehicle theft (機動車竊盜)	275
Assault (攻擊)	200
Robbery (搶劫)	125
Rape or homicide (強姦或殺人)	25

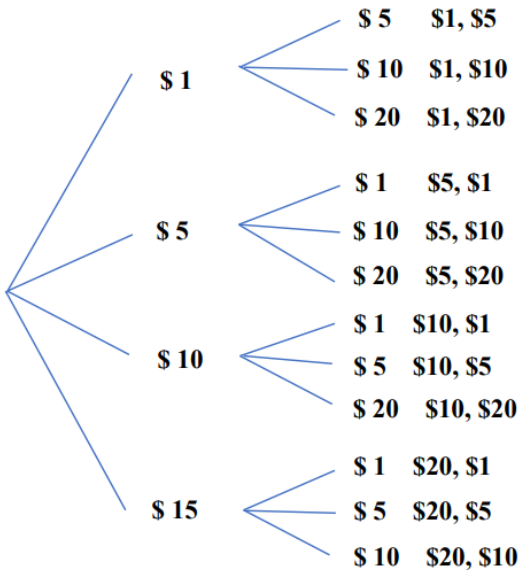
$$n = 1375 + \cdots + 25 = 2500$$

1. $P(\text{motor vehicle theft}) = \frac{275}{2500} = 0.11$

2. $P(\text{not an assault}) = 1 - P(\text{assault}) = 1 - \frac{200}{2500} = \frac{2300}{2500} = 0.92$

35. **Selecting a Bill**

A box contains a $1 bill, a $5 bill, a $10 bill, and a $20 bill. A bill is selected at random, and it is not replaced; then a second bill is selected at random. Draw a tree diagram and determine the sample space.



Sec. 4-2 (p.214)

3. **Cards, Dice, and Students**

Determine whether these events are mutually exclusive:

a. Draw a card: get a spade and get a 6
b. Roll a die: get a prime number (質數) (2, 3, 5)
c. Roll two dice: get a sum of 7 or get a sum that is an even number
d. Select a student at random in your class: get a male or get a sophomore (大二)

a. Not mutually exclusive
b. Mutually exclusive
c. Mutually exclusive
d. Not mutually exclusive

8. **U.S. Population**

The data show the U.S. population by age.

Under 20 years	27.0%
20 years and over	73.0
65 years and over	13.1

Choose one person from the United States at random. Find the probability that the person is

a. From 20 years to 64 years
b. Under 20 or 65 and over
c. Not 65 and over

a. $P(20 \le age \le 64) = 1 - P(age < 20) - P(age > 64) = 1 - 0.27 - 0.131 = 0.599$
or $P(20 \le age \le 64) = P(age \ge 20) - P(age \ge 65) = 0.73 - 0.131$

b. $P(age < 20 \text{ or } age \ge 65) = P(age < 20) + P(age \ge 65) = 0.27 + 0.131 = 0.401$

c. $P(age < 65) = 1 - P(age \ge 65) = 1 - 0.131 = 0.869$

10. **Selecting a Movie**

A media rental store (媒體租賃店) rented the following number of movie titles in each of these categories: 170 horror, 230 drama, 120 mystery, 310 romance, and 150 comedy. If a person selects a movie to rent, find the probability that it is a romance or a comedy. Is this event likely or unlikely to occur? Explain your answer.

Horror (恐怖)	170
Drama (劇情)	230
Mystery (懸疑)	120
Romance (愛情)	310
Comedy (喜劇)	150
Total	980

$$P(romance \text{ or } comedy) = P(romance) + P(comedy)$$
$$= \frac{310 + 150}{980} = 0.469$$

The probability of the event is slightly less than 0.5, which makes it about equally likely to occur or not to occur.

14. **Endangered Species**

The chart below shows the numbers of endangered and threatened species both here in the United States and abroad.

	Endangered		Threatened		Total
	United States	Foreign	United States	Foreign	
Mammal	68	251	10	20	349
Birds	77	175	13	6	271
Reptiles (爬蟲類)	14	64	22	16	116
Amphibians (兩棲類)	11	8	10	1	30
Total	170	498	55	43	766

Choose one species at random. Find the probability that it is

a. Threatened and in the United States

b. An endangered foreign bird

c. A mammal or a threatened foreign species

a. $P(threatened \text{ and } US) = \frac{10+13+22+10}{766} = 0.0718 \approx 0.072$

b. $P(endangered\ foreign\ bird) = \frac{175}{766} = 0.2284 \approx 0.228$

c. $P(mammal \text{ or } threatened\ foreign)$
$= P(mammal) + P(treatened\ foreign) - P(mammal \text{ and } threatened\ foreign)$
$= \frac{349 + 43 - 20}{766} = 0.4856 \approx 0.486$

18. **Mail Delivery**

A local postal carrier distributes firstclass letters, advertisements, and magazines. For a certain day, she distributed the following numbers of each type of item.

Delivered to	First-class letters 一類郵件(最快的)	Ads	Magazines	Total
Home	325	406	203	934
Business	732	1021	97	1850
Total	1057	1427	300	2784

If an item of mail is selected at random, find these probabilities.

a. The item went to a home.

b. The item was an ad, or it went to a business.

c. The item was a first-class letter, or it went to a home.

a. $P(home) = \frac{934}{2784} = 0.3354 \approx 0.335$

b. $P(ad\ or\ business) = P(ad) + P(business) - P(ad\ and\ business)$

$= \frac{1427+1850-1021}{2784} = 0.8103 \approx 0.810$

c. $P(first-class\ or\ home) = \frac{1057+934-325}{2784} = 0.5984 \approx 0.598$

24. **Rolling Die**

Two dice are rolled. Find the probability of getting

a. A sum of 8, 9, or 10

b. Doubles or a sum of 7

c. A sum greater than 9 or less than 4

d. Based on the answers to a, b, and c, which is least likely to occur?

擲兩顆骰子，所有可能的結果個數為 $6^2 = 36$

a. $P(sum = 8\ or\ 9\ or\ 10)$ $= P(sum = 8) + P(sum = 9) + P(sum = 10)$ $= \frac{5+4+3}{36} = \frac{12}{36} = \frac{1}{3}$	1: none 2: {1,1} 3: {1,2}, {2,1}
b. $P(\text{double or sum} = 7) = P(double) + P(sum = 7)$ $= \frac{6+6}{36} = \frac{12}{36} = \frac{1}{3}$	7: {6,1}, {5,2}, {4,3}, {3,4}, {2,5}, {1,6} 8: {6,2}, {5,3}, {4,4}, {3,5}, {2,6} 9: {6,3}, {5,4}, {4,5}, {3,6}
c. $P(sum > 9\ or\ sum < 4) = P(sum > 9) + P(sum < 4)$ $= \frac{(3+2+1)+(1+2)}{36} = \frac{9}{36} = \frac{1}{4}$	10: {6,4}, {5,5}, {4,6} 11: {6,5}, {5,6} 12: {6,6}
d. The event in part c is least likely to occur since it has the lowest probability.	

Sec. 4-3 (p.230)

2. State which events are independent, and which are dependent.

a. Having a large shoe size and having a high IQ

b. A father being left-handed and a daughter being left-handed

c. Smoking excessively and having lung cancer

d. Eating an excessive (過量的) amount of ice cream and smoking an excessive amount of cigarettes

a. Independent

b. Dependent

c. Dependent

d. Independent

7. **Government Employees**

In 2013 about 66% of full-time law enforcement workers (執法人員) were sworn officers (宣誓官), and of those, 88.4% were male. Females however make up 60.7% of civilian employees (文職僱員).

Choose one law enforcement worker at random and find the following.

a. The probability that she is a female sworn officer

b. The probability that he is a male civilian employee

c. The probability that he or she is male or a civilian employee

a. $P(female\ in\ sworn\ officer) = 1 - 0.884 = 0.116$

$\Rightarrow P(female\ sworn\ officer) = 0.66 \cdot 0.116 = 0.07656$

b. $P(civilian\ employees) = 1 - 0.66 = 0.34,\ P(male\ in\ civilian\ employees) = 1 - 0.607 = 0.393$

$\Rightarrow P(male\ civilian\ employee) = 0.34 \cdot 0.393 = 0.13362$

c. $P(male\ or\ civilian\ employee) = P(male) + P(civilian) - P(both)$

$= (0.66 \cdot 0.884 + 0.34 \cdot 0.393) + 0.34 - 0.34 \cdot 0.393$

$= 0.71706 + 0.34 - 0.13362 = 0.92344$

10. A bag contains 9 red marbles (彈珠), 8 white marbles, and 6 blue marbles. Randomly choose two marbles, one at a time, and without replacement.

Find the following.

a. The probability that the first marble is red and the second is white

b. The probability that both are the same color

c. The probability that the second marble is blue

9 R, 8 W, 6 B, 總共 23 顆，抽兩次，取後不放回

a. $P(RW) = \frac{9}{23} \cdot \frac{8}{22} = 0.1422 \approx 0.142$

b. $P(same\ color) = P(RR) + P(WW) + P(BB) = \frac{9}{23} \cdot \frac{8}{22} + \frac{8}{23} \cdot \frac{7}{22} + \frac{6}{23} \cdot \frac{5}{22} = 0.3122 \approx 0.312$

c. $P(2nd\ is\ B) = P(RB) + P(WB) + P(BB) = \frac{9}{23} \cdot \frac{6}{22} + \frac{8}{23} \cdot \frac{6}{22} + \frac{6}{23} \cdot \frac{5}{22} = 0.2608 \approx 0.261$

15. **Drawing Cards**

If two cards are selected from a standard deck of 52 cards and are not replaced after each draw, find these probabilities.

a. Both are 9s.

b. Both cards are the same suit.

c. Both cards are spades.

a. $P(both\ 9) = \frac{4}{52} \cdot \frac{3}{51} = \frac{1}{221}$

b. $P(same\ suit) = P(\spadesuit\spadesuit) + P(\clubsuit\clubsuit) + P(\diamondsuit\diamondsuit) + P(\heartsuit\heartsuit)$

$= \left(\frac{13}{52} \cdot \frac{12}{51}\right) + \left(\frac{13}{52} \cdot \frac{12}{51}\right) + \left(\frac{13}{52} \cdot \frac{12}{51}\right) + \left(\frac{13}{52} \cdot \frac{12}{51}\right) = \frac{4}{17}$

c. $P(\spadesuit\spadesuit) = \left(\frac{13}{52} \cdot \frac{12}{51}\right) = \frac{1}{17}$

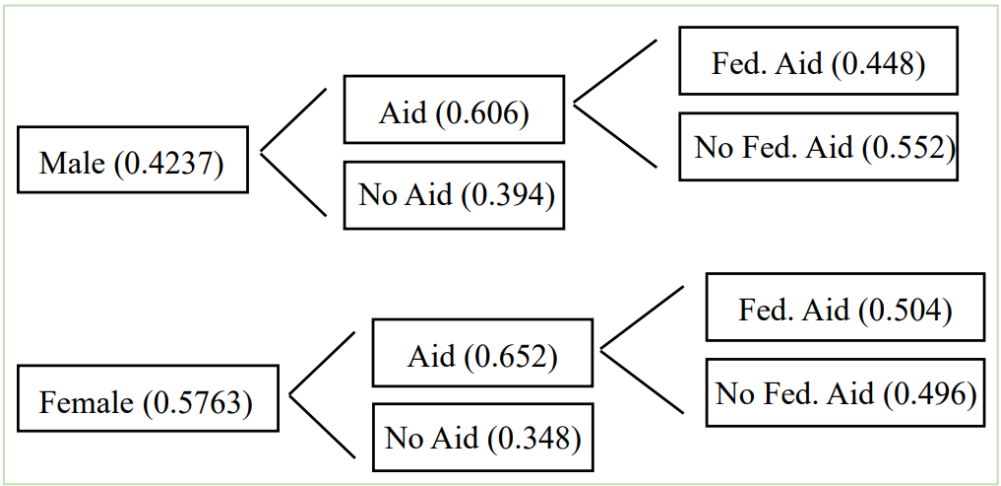
19. **Student Financial Aid**

In a recent year 8,073,000 male students and 10,980,000 female students were enrolled as undergraduates (大學生). Receiving aid (助學金) were 60.6% of the male students and 65.2% of the female students. Of those receiving aid, 44.8% of the males got federal aid and 50.4% of the females got federal aid (聯邦援助). Choose 1 student at random. (Hint: Make a tree diagram.) Find the probability that the student is

a. A male student without aid

b. A male student, given that the student has aid

c. A female student or a student who receives federal aid



a. $P(male\ without\ aid) = 0.4237 \cdot 0.394 = 0.1669 \approx 0.167$

b. $P(male|aid) = \frac{P(male\ and\ aid)}{P(aid)} = \frac{0.427 \cdot 0.606}{0.427 \cdot 0.606 + 0.5763 \cdot 0.652} = \frac{0.256762}{0.6325098} = 0.4058 \approx 0.406$

c. $P(female\ or\ fed.aid) = P(female) + P(fed.aid) - P(both)$

$= 0.5763 + (0.4237 \cdot 0.606 \cdot 0.448 + 0.5763 \cdot 0.652 \cdot 0.504) - 0.5763 \cdot 0.652 \cdot 0.504$

$= 0.6913 \approx 0.691$

28.

Congressional Tearms

Below is given the summary from the 112th Congress of Senators (參議員大會) whose terms end in 2013, 2015, or 2017.

	2013	2015	2017	Total
Democrat (民主黨)	21	20	1	42
Republican (共和黨)	8	15	13	36
Total	29	35	14	78

Choose one of these Senators at random and find

a. $P(Democrat\ and\ term\ expires\ in\ 2015)$

b. $P(Republican\ or\ term\ expires\ in\ 2013)$

c. $P(Republican\ given\ term\ expires\ in\ 2017)$

Are the events "Republican" and "term expires in 2015" independent? Explain.

(term, 期限) (expire, 到期;屆滿)

a. $P(Democrat\ and\ term\ expires\ in\ 2015) = \frac{20}{78} = 0.2564 \approx 0.256$

b. $P(Republican\ or\ term\ expires\ in\ 2013) = \frac{36}{78} + \frac{29}{78} - \frac{8}{78} = 0.7307 \approx 0.731$

c. $P(Republican|term\ expires\ in\ 2017) = \frac{13/78}{14/78} = 0.9285 \approx 0.929$

32.

Doctor Specialties

Below are listed the numbers of doctors in various specialties by gender.

	Pathology (病理學)	Pediatrics (小兒科)	Psychiatry (精神科)	Total
Male	12,575	33,020	27,803	73,398
Female	5,604	33,351	12,292	51,247
Total	18,179	66,371	40,095	124,645

Choose one doctor at random.

a. Find $P(male|pediatrician)$.

b. Find $P(pathologist|female)$.

c. Are the characteristics "female" and "pathologist" independent? Explain.

a. $P(male|pediatrician) = \frac{P(male\ \&\ pediatrician)}{P(pediatrician)} = \frac{33020}{66371} \approx 0.498$

b. $P(pathologist|female) = \frac{P(pathologist\ \&\ female)}{P(female)} = \frac{5604}{51247} \approx 0.109$

c. No. $\because\ P(pathologist|female) \neq P(pathologist)$

40.

On-Time Flights

A flight from Pittsburgh (匹茲堡) to Charlotte (夏洛特) has a 90% on-time record. From Charlotte to Jacksonville (傑克遜維爾), the flight is on time 80% of the time. The return flight from Jacksonville to Charlotte is on time 50% of the time and from Charlotte to Pittsburgh, 90% of the

time. Consider a round trip from Pittsburgh to Jacksonville on these flights. Assume the flights are independent.

a. What is the probability that all 4 flights are on time?
b. What is the probability that at least 1 flight is not on time?
c. What is the probability that at least 1 flight is on time?
d. Which events are complementary?

a. $P(all\ flights\ on\ time) = 0.9 \cdot 0.9 \cdot 0.8 \cdot 0.5 = 0.324$
b. $P(at\ least\ 1\ flight\ not\ on\ time) = 1 - P(all\ flights\ on\ time)$
$= 1 - 0.324 = 0.676$
c. $P(at\ least\ 1\ flight\ on\ time) = 1 - P(all\ flights\ not\ on\ time)$
$= 1 - 0.1 \cdot 0.1 \cdot 0.2 \cdot 0.5 = 1 - 0.001 = 0.999$
d. The events in parts a and b are complementary.

50. **U.S. Organ Transplants (器官移植)**

As of June 2015, 81.4% of patients were waiting on a kidney (腎臟), 11.7% were waiting on a liver (肝臟), and 3.1% were waiting on a heart. Choose 6 patients on the transplant waiting list at random in 2015. Find the probability that

a. All were waiting for a kidney.
b. None were waiting for a kidney.
c. At least 1 was waiting for a kidney

a. $P(all\ kidney) = (0.814)^6 \approx 0.291$
b. $P(no\ kidney) = (1 - 0.814)^6 = (0.186)^6 \approx 0.00004$
c. $P(at\ least\ 1\ kidney) = 1 - P(none\ kidney) = 1 - 0.00004 = 0.99996$

52. **Selecting a Flower**

In a large vase, there are 8 roses, 5 daisies (雛菊), 12 lilies (百合花), and 9 orchids (蘭花). If 4 flowers are selected at random, and not replaced, find the probability that at least 1 of the flowers is a rose. Would you consider this event likely to occur? Explain your answer.

$$P(at\ least\ 1\ rose) = 1 - P(no\ rose) = 1 - \left(\frac{26}{34} \cdot \frac{25}{33} \cdot \frac{24}{32} \cdot \frac{23}{31}\right) \approx 0.678$$

Yes; the event is a little more likely to occur than not since the probability is about 68%.

Sec. 4-4 (p.243)

6. **Quinto Lottery (抽獎)**

A lottery game called Quinto is played by choosing five numbers each, from 0 through 9. How many numbers are possible? Although repeats are allowed, how many numbers are possible if repeats are not allowed?

✦ Allow repeats

$$10^5 = 100000$$

✦ Not allow repeats

$$10 \cdot 9 \cdot 8 \cdot 7 \cdot 6 = 30240$$

14. Evaluate each expression

a. 6!	b. 7!	c. 2!	d. 10!
e. ${}_9P_6$	f. ${}_{11}P_4$	g. ${}_8P_0$	h. ${}_{10}P_2$

a. $6! = 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 720$

b. $7! = 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 5040$

c. $2! = 2 \cdot 1 = 2$

d. $10! = 10 \cdot 9 \cdots 1 = 3628800$

e. ${}_9P_6 = \frac{9!}{(9-6)!} = 9 \cdot 8 \cdots 4 = 60480$

f. ${}_{11}P_4 = \frac{11!}{(11-4)!} = 11 \cdot 10 \cdots 8 = 7920$

g. ${}_8P_0 = \frac{8!}{(8-0)!} = 1$

h. ${}_{10}P_2 = \frac{10!}{(10-2)!} = 10 \cdot 9 = 90$

18. **Inspecting Restaurants**

How many different ways can a city health department inspector (城市衛生部門檢查員) visit 5 restaurants in a city with 10 restaurants?

一次只能去一間餐廳 ∴ 考慮順序

${}_{10}P_5 = \frac{10!}{(10-5)!} = 10 \cdot 9 \cdots 6 = 30240$

22. **Free-Sample Requests**

An online coupon service has 13 offers (優惠) for free samples. How many different requests are possible if a customer must request exactly 3 free samples? How many are possible if the customer may request up to 3 free samples?

拿到優惠的順序不重要 ∴ 不考慮順序

✦ 恰好 3 個

$$_{13}C_3 = \frac{13!}{(13-3)!\,3!} = \frac{13\cdot 12\cdot 11}{3\cdot 2\cdot 1} = 286$$

✦ 至多 3 個

$$\begin{aligned}
&{}_{13}C_0 + {}_{13}C_1 + {}_{13}C_2 + {}_{13}C_3 \\
&= \frac{13!}{(13-0)!\,0!} + \frac{13!}{(13-1)!\,1!} + \frac{13!}{(13-2)!\,2!} + \frac{13!}{(13-3)!\,3!} \\
&= 1 + \frac{13}{1} + \frac{13\cdot 12}{2\cdot 1} + \frac{13\cdot 12\cdot 11}{3\cdot 2\cdot 1} \\
&= 1 + 13 + 78 + 286 = 378
\end{aligned}$$

28. **Word Permutations**

How many permutations can be made using all the letters in the word *MASSACHUSETTS*?

MASSACHUSETTS:

1M, 2A, 4S, 1C, 1H, 1U, 1E, 2T ⇒ 13 words

$$\frac{13!}{2!\,4!\,2!} = \frac{13\cdot 12\cdots 1}{(2\cdot 1)(4\cdot 3\cdot 2\cdot 1)(2\cdot 1)} = 64864800$$

36. **Selecting Players**

How many ways can 4 baseball players and 3 basketball players be selected from 12 baseball players and 9 basketball players?

不考慮順序

$$_{12}C_4 \cdot {}_9C_3 = \frac{12!}{(12-4)!\,4!} \cdot \frac{9!}{(9-3)!\,3!} = 495\cdot 84 = 41580$$

40. **Bridge (橋牌) Foursomes (四人賽)**

How many different tables of 4 can you make from 16 potential bridge players?

(16 位橋牌玩家能組合出多少張不同的 4 人桌)

How many different tables if 4 of the players insist on playing together?

(如果有 4 個人堅持一起玩)

順序的排列不重要 ∴ 不考慮順序

1. $_{16}C_4 = \frac{16!}{(16-4)!4!} = \frac{16\cdot 15\cdot 14\cdot 13}{4\cdot 3\cdot 2} = 1820$

2. $_{12}C_4 = \frac{12!}{(12-4)!4!} = \frac{12\cdot 11\cdot 10\cdot 9}{4\cdot 3\cdot 2} = 495$

51.	**Automobile Selection** An automobile dealer (汽車經銷商) has 12 small automobiles, 8 mid-size automobiles, and 6 large automobiles on his lot. How many ways can two of each type of automobile be selected from his inventory?

幾種選擇方法，順序不重要 ∴ 不考慮順序

$$_{12}C_2 \cdot {}_8C_2 \cdot {}_6C_2 = \frac{12 \cdot 11}{2} \cdot \frac{8 \cdot 7}{2} \cdot \frac{6 \cdot 5}{2} = 66 \cdot 28 \cdot 15 = 27720$$

66.	**Selecting Council (理事會) Members** The presidents (校長), vice presidents (副校長), and secretary-treasurers (財務秘書) from each of four classes are eligible (有資格的) for an all-school council. How many ways can four officers be chosen from these representatives (代表)? How many ways can they be chosen if the president must be selected from the sitting presidents, the vice president from the sitting vice presidents, the secretary from the sitting secretary-treasurers, and the treasurer (司庫;財務主管) from everybody who's left? (若是校長只能從原本是校長的人員中選出，副校長只能從原本是副校長的人員中選出，秘書只能從原本是財務秘書的人員中選出，司庫從剩餘的人員中選出)

4 個班級 3 個職位都有資格參加理事會，共 12 人 ∴ 考慮順序

1. $_{12}P_4 = \frac{12!}{(12-4)!} = 12 \cdot 11 \cdots 9 = 11880$
2. $_4P_1 \cdot {}_4P_1 \cdot {}_4P_1 \cdot {}_{(12-3)}P_1 = 4 \cdot 4 \cdot 4 \cdot 9 = 576$

Sec. 4-5 (p.253)

4.	**Senate (參議院) Partisanship (黨派性)** The composition (布局) of the Senate of the 114th Congress is 54 Republicans (共和黨) 2 Independent (獨立黨) 44 Democrats (民主黨) A new committee (委員會) is being formed (成立) to study ways to benefit the arts in education. If 3 Senators are selected at random to form a new committee, What is the probability that they will all be Republicans? What is the probability that they will all be Democrats? What is the probability that there will be 1 from each party, including the Independent?

順序不重要

所有可能: $_{100}C_3 = \frac{100 \cdot 99 \cdot 98}{3 \cdot 2 \cdot 1} = 161700$

1. All Republicans: $_{54}C_3 = \frac{54 \cdot 53 \cdot 52}{3 \cdot 2 \cdot 1} = 24804 \Rightarrow P(all\ Republicans) = \frac{{}_{54}C_3}{{}_{100}C_3} \approx 0.153$
2. All Democrats: $_{44}C_3 = \frac{44 \cdot 43 \cdot 42}{3 \cdot 2 \cdot 1} = 13244 \Rightarrow P(all\ Democrats) = \frac{{}_{44}C_3}{{}_{100}C_3} \approx 0.082$
3. 1 from each: $_{54}C_1 \cdot {}_{44}C_1 \cdot {}_2C_1 = 54 \cdot 44 \cdot 2 = 4752 \Rightarrow P(1\ from\ each) = \frac{4752}{161700} \approx 0.029$

9. **World-Class Orchestras (世界級管弦樂團)**

A list of 20 World Class Orchestras includes the following from the United States: Boston Symphony (交響樂) Orchestra, Chicago Symphony Orchestra, Cleveland (克里夫蘭) Orchestra, Los Angeles (洛杉磯) Philharmonic (愛樂樂團), New York Philharmonic, the Metropolitan (大都會) Opera (歌劇) Orchestra, and the San Francisco (舊金山) Symphony. Choose 5 at random from the list of 20 for a benefit CD.

What is the probability that the collection will include at least one group from the United States?

At least 2 from the United States?

That all 5 will be from the United States?

美國的樂團有 7 個

1. $P(at\ least\ 1\ from\ US) = 1 - P(no\ US) = 1 - \frac{{}_7C_0 \cdot {}_{13}C_5}{{}_{20}C_5} = 1 - \frac{1 \cdot 1287}{15504} \approx 0.917$

2. $P(at\ least\ 2\ from\ US) = 1 - P(no\ US) - P(1\ from\ US) = 1 - \frac{{}_7C_0 \cdot {}_{13}C_5}{{}_{20}C_5} - \frac{{}_7C_1 \cdot {}_{13}C_4}{{}_{20}C_5}$

   $= 0.917 - 0.323 = 0.594$

3. $P(all\ US) = \frac{{}_7C_5}{{}_{20}C_5} = \frac{21}{15504} \approx 0.001$

13. **Rolling the Dice**

If three dice are rolled, find the probability of getting a sum of 6.

所有可能: $6^3 = 216$

6: {4,1,1}, {3,2,1}, {3,1,2}, {2,3,1}, {2,2,2}, {2,1,3}, {1,4,1}, {1,3,2}, {1,2,3}, {1,1,4}

$$P(sum = 6) = \frac{10}{216} \approx 0.046$$

17. **Plant Selection**

All holly plants (冬青植物) are dioecious (雌雄異體) – a male plant must be planted within 30 to 40 feet of the female plants in order to yield (產生) berries. A home improvement (居家裝修) store has 12 unmarked holly plants for sale, 8 of which are female. If a homeowner buys 3 plants at random, what is the probability that berries will be produced

至少一雌一雄才能產出漿果，順序不重要

Method 1:

$$P(at\ least\ 1F\ 1M) = 1 - P(all\ F) - P(all\ M) = 1 - \frac{{}_8C_3 \cdot {}_4C_0}{{}_{12}C_3} - \frac{{}_8C_0 \cdot {}_4C_3}{{}_{12}C_3} = 1 - \frac{56}{220} - \frac{4}{220} = \frac{160}{220}$$

Method 2:

$$P(1M\ or\ 2M) = \frac{{}_8C_2 \cdot {}_4C_1}{{}_{12}C_3} + \frac{{}_8C_1 \cdot {}_4C_2}{{}_{12}C_3} = \frac{28 \cdot 4}{220} + \frac{8 \cdot 6}{220} = \frac{160}{220} = 0.\overline{72} \approx 0.727$$