Lattice Regularization of Non-Relativistic Interacting Fermions in 1+1 Dimensions

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Introduction

We study a three-body system composed of two different species of particles in one spatial dimension using effective field theory[1, 2]. Particles of the same type do not interact with each other, but particles of different types can interact via pairwise local interactions, which can depend on the relative velocity between them. We compute the binding energy of three-body bound states and analyze their dependence on the binding energy of the two-body subsystem using exact diagonalization method. Our research focuses on comparing renormalization schemes with published literature as well.

The Hamiltonian of our model is:

$$\mathcal{H} = \sum_{\sigma=1}^{2} -\psi_\sigma^\dagger \frac{\partial_x^2}{2m_a}\psi_\sigma + \lambda\psi_1^\dagger\psi_2^\dagger\psi_2\psi_1 - c\left(\psi_1^\dagger i\overleftrightarrow{\partial_x}\psi_2^\dagger\right)\left(\psi_2 i\overleftrightarrow{\partial_x}\psi_1\right)$$

- ψ_1 and ψ_2 are two non-relativistic fermion particles.
- λ is an on-site interaction parameter.
- c is a pairwise local velocity-dependent.
- The system is invariant under parity and Galilean boost.

System with Contact Interaction ($c = 0$)

- With only contact interaction, the Hamiltonian is:

$$\mathcal{H}_1 = \mathcal{H}_0 + \mathcal{H}_{int}^{(\lambda)}$$

- $\mathcal{H}_0 = \sum_x \varepsilon(a)(\psi_{\sigma,x}^\dagger\psi_{\sigma,x+\alpha} - 2\psi_{\sigma,x}^\dagger\psi_{\sigma,x} + \psi_{\sigma,x+\alpha}^\dagger\psi_{\sigma,x})$
- $\mathcal{H}_{int}^{(\lambda)} = \varepsilon(a)\lambda(a)\psi_{1,x}^\dagger\psi_{1,x}\psi_{2,x}^\dagger\psi_{2,x}$
- At continuum limit, we can show that $\lim_{a\to 0} U_{lat} = a\varepsilon(a)\lambda(a) = -0.140966$ [3,4].
- We compute the energy of a two-body system's first two excited states, which agrees with a calculation using Finite Volume method [5], as shown in Fig. 1.

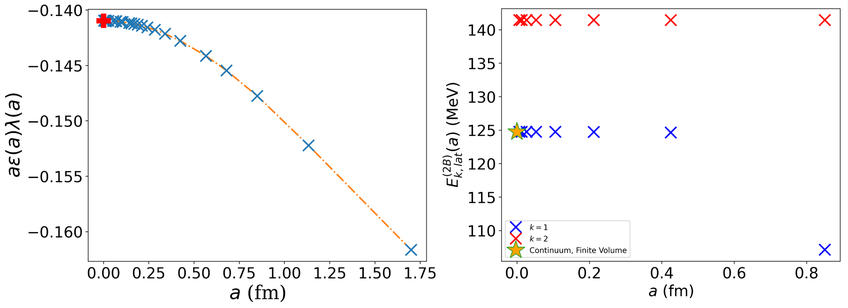


Fig. 1. Left is the plot of interaction coefficient, U_{lat}, as a function of lattice spacing, a. U_{lat} is observed to be constant at the continuum limit. Right is the plot comparing energy levels generate by our method and finite volume method calculation.

- No three-body force is needed . The lattice theory has a well-defined continuum limit in three- and four-body sectors [3], as shown in Fig. 2.
- To extrapolate our results to infinite volume, we fix the lattice spacing a small enough and increase the lattice sites. Our calculation matches reduce to the energy levels of an infinite square well model, as presented in Fig. 3.
- This shows our method's potential to extend to three- or four-body calculation. However, this infinite volume extrapolation method works well for approximately $a \leq 0.1$ fm.

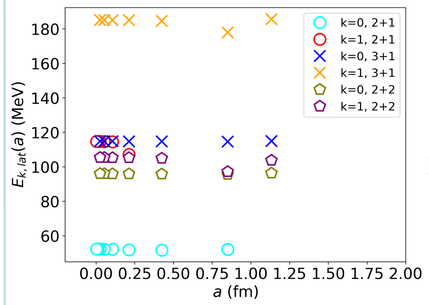


Fig. 2. Three- and four-body ground state $k = 0$ and first excited state $k = 1$ energy as functions of lattice spacing. The energy converge without the presence of a three-body force.

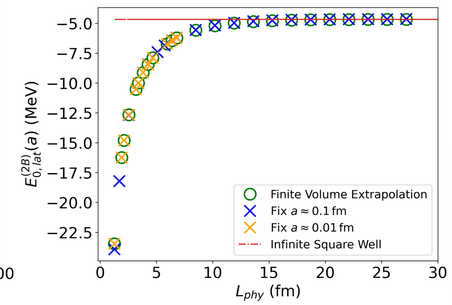


Fig. 3. Infinite volume extrapolation of a two-body system. We fix the lattice spacing small and increase lattice size. It produces the same result as the continuum theory.

System with Momentum-Dependent Interaction ($\lambda = 0$)

- With only the pairwise momentum-dependent local interaction, the Hamiltonian is:

$$\mathcal{H}_2 = \mathcal{H}_0 + \mathcal{H}_{int}^{(c)}$$

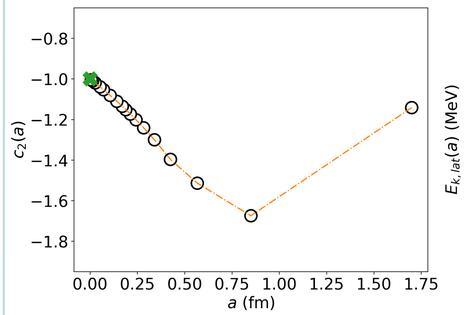
- $\mathcal{H}_0 = \sum_x \varepsilon(a)(\psi_{\sigma,x}^\dagger\psi_{\sigma,x+\alpha} - 2\psi_{\sigma,x}^\dagger\psi_{\sigma,x} + \psi_{\sigma,x+\alpha}^\dagger\psi_{\sigma,x})$
- $\mathcal{H}_{int}^{(c)} = \varepsilon(a)c(a)\sum_x\left(\psi_{2,x+\alpha}\psi_{1,x} - \psi_{2,x}\psi_{1,x+\alpha}\right)^\dagger\left(\psi_{2,x+\alpha}\psi_{1,x} - \psi_{2,x}\psi_{1,x+\alpha}\right)$
- To solve analytically, we reduce the two-body system to a one-body system by introducing a relative momentum, q , and relative position, δ[3]:

$$|q,\delta\rangle = \frac{1}{L}\sum_x \psi_{2,x}^\dagger\psi_{1,x+\delta}^\dagger e^{i\left(\frac{2\pi}{L}\right)qx}|0\rangle$$

- Define $I_{lat}^{(\lambda)}(E,a) = 2\epsilon(a)\langle\delta = 0|(E - H_0)^{-1}|\delta = 0\rangle$ that appears in the denominator of the first-order expansion of Green function.
- At pole,

$$\lim_{a\to 0} c(E,a) = \lim_{a\to 0} 2/I_{lat}^{(\lambda)}(E,a) = -2/\int_0^{2\pi}\frac{dp}{2\pi}\frac{(1-\cos 2p)}{(1-\cos p)} = -\frac{2}{2} = -1$$

- This suggests that, at continuum limit, $c(E,a) \to -1$.
- Fix the two-body ground state energy, we track the strength of dimensionless interaction coefficient and perform infinite volume extrapolation with our method, as in Fig. 4.
- We perform similar infinite volume extrapolation method in Fig. 5.
- At the continuum limit, $E_{3B}/E_{2B} = 1.375$, according to three-body EFT calculation



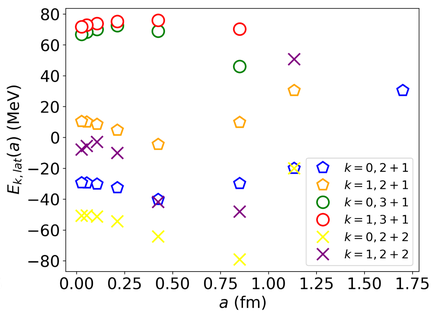
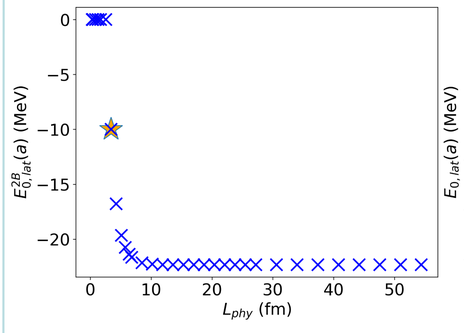


Fig. 4. Left: Plot of dimensionless interaction strength c as a function of lattice spacing, a. Right: Three- and four-body ground state and first excited state energy.



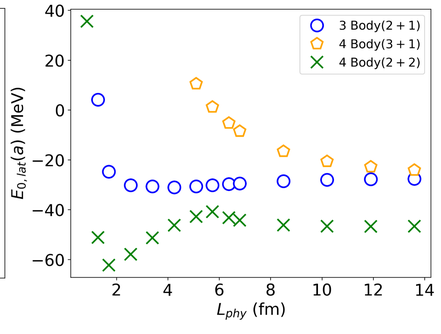


Fig. 5. Left: Two-body Infinite volume extrapolation result with ground state energy fixed at -10 MeV. Right: Three- and four-body infinite volume extrapolation at $a \approx 0.1$ fm. $E_{3B}/E_{2B} \approx 1.23$.

Outlook

- Further study the momentum-dependent interaction on a lattice, including the regularization of a three-body force.
- Develop our method to one-dimensional general mass-spin imbalance cases, as well as two- and three-dimensional methods.
- Investigate EFT on a lattice via Quantum Monte Carlo algorithms.

References

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