Non-Relativistic Interacting Fermions in $1+1$ Dimensions

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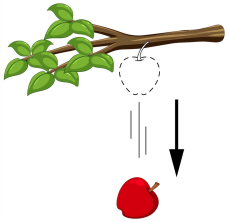
- Using EFT to study three-particle systems consisting of two species in one dimension interacting via contact interactions.
- **Result:** We observe the formation of bound states and universal ratio between three-body and two-body binding energies.

Essences of an Effective Theory

- Degrees of Freedom: Apple and Earth



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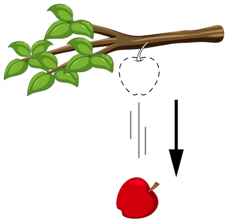

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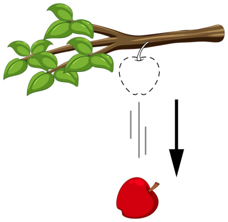

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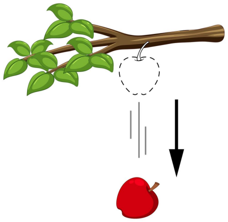

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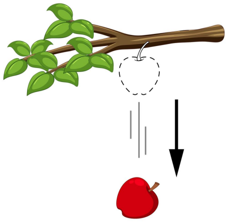

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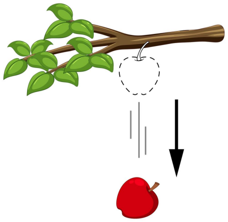

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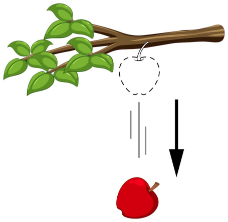
- Theoretical error estimation



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Effective field theory

- Effective field theory techniques for three-body systems.

Yong, DT Son (2018)

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- Symmetry:
  - Parity: $x \to -x$
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- **Degrees of freedom**: ψ_1 and ψ_2.
- Two-body contact potentials include

$$V_\lambda = -\lambda\delta(x)$$
$$V_c = -c\delta'(x)\partial_x$$

Yong, DT Son (2018)

Two-Body Scattering Problem

Feynman diagrams for the full dimer propagator

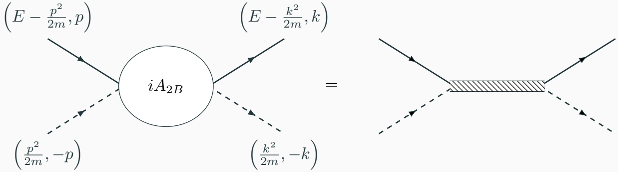

Figure 1: Relationship between two-body scattering amplitude and dressed propagators

Y. Sekino and Y. Nishida (2021)

Two-Body Scattering Amplitude

- Two-body scattering amplitude:

$$A_{2B}(p,k) = \frac{-i\lambda + c\lambda(B_2 + B_0 pk) - icpk}{(1 + i\lambda B_0)(1 + icB_2)}$$

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- Renormalization behavior of the couplings in terms of momentum cutoff:

$$\lambda = \frac{2\gamma}{\Lambda}$$
$$c = -\frac{2\pi}{2\Lambda + \pi\mu}$$

Yong, DT Son (2018)

Three-body scattering amplitude

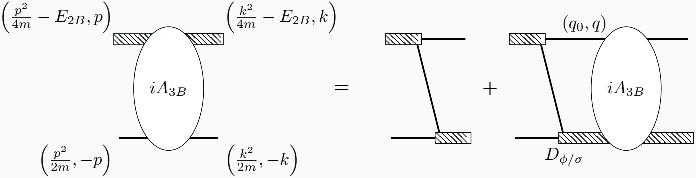
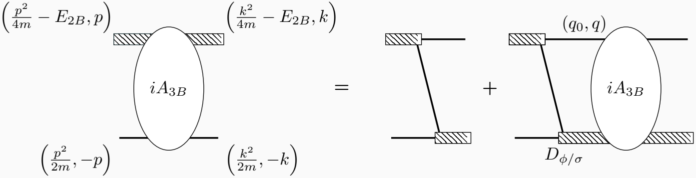


Figure 2: Faddeev equation for scattering amplitude under one interaction

Three-body scattering amplitude



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- The above diagram can be expressed with an integral equation.

$$A_{3B}(p,k) = B(p,k) - \int_{-\Lambda}^{\Lambda} \frac{dq}{2\pi} D_\sigma\left(E - \frac{q^2}{2m}, -q\right) B(p,q) A_{3B}(q,k)$$

Delta Potential Interaction

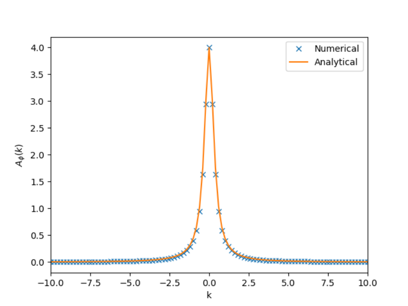

Figure 3: Scattering amplitude as a function of momentum

- The analytical solution is:

$$A_\phi(k) = \frac{m^2\gamma^3}{4k^2 + m^2\gamma^2}$$

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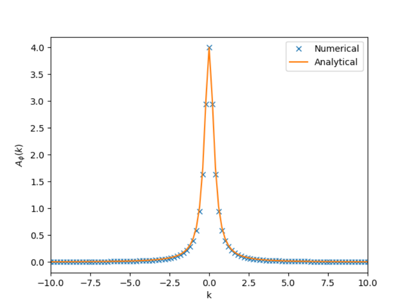

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- We observe the ratio between three-body and two-body binding energies:

$$\frac{E_{3B}}{E_{2B}} = \frac{7}{4}$$

Delta Prime Potential Interaction

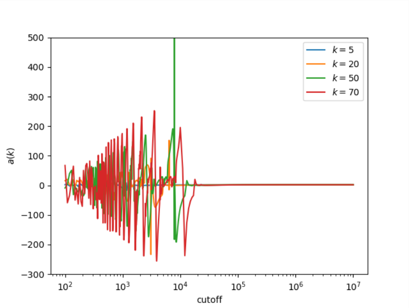

Figure 4: Scattering amplitude as a function of cutoff

- Interaction involving velocity of particles may involve three-body forces.

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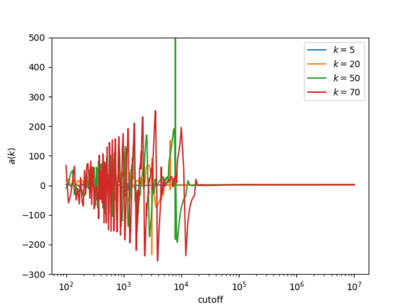

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- Interaction involving velocity of particles may involve three-body forces.
- We don’t need a three-body force in this case as amplitude converges.

Delta Prime Potential Interaction

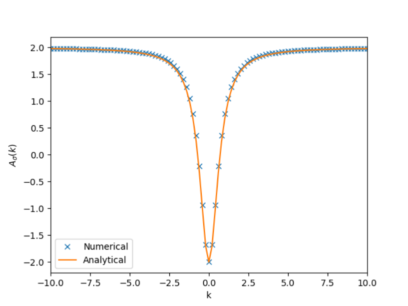

Figure 5: Scattering amplitude as a function of momentum under delta prime potential

- The analytical solution is:

$$A_\sigma(k) = -\frac{8\mu(2k^2 - \mu^2)}{m(2k^2 + \mu^2)}$$

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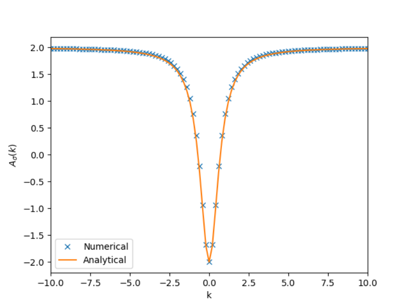

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- We observe:

$$\frac{E_{3B}}{E_{2B}} = \frac{11}{8}$$

Delta and Delta Prime Potential Interaction

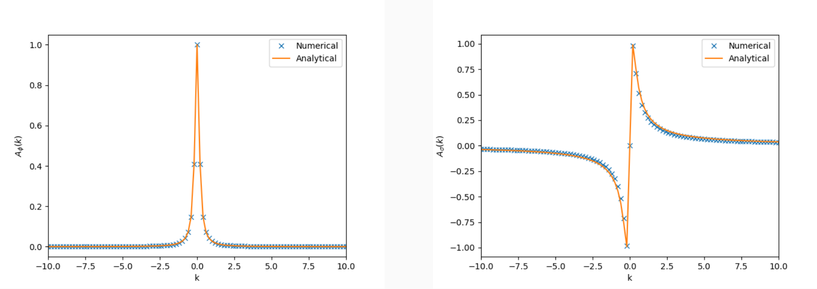
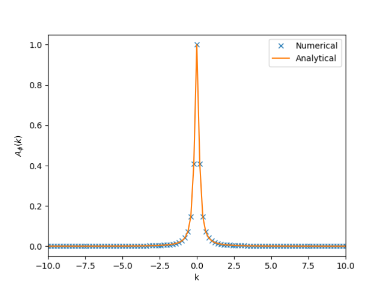


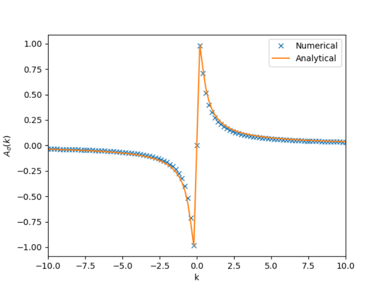
Figure 6: Scattering amplitude as a function of momentum for ϕ and σ field under δ and δ' potentials

- The analytical solutions are:

$$A_\phi(k) = \frac{2m^2\lambda^2}{m^2\lambda^2 + 9k^2} \qquad A_\sigma(k) = \frac{12m^3k}{4\mu(m^2\lambda^2 + 9k^2)}$$

Delta and Delta Prime Potential Interaction





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- We observe:

$$\frac{E_{3B}}{E_{2B}} = \frac{4}{3}$$

Summary

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Summary

- We study a three-body scattering problem in one dimension with the Effective Field Theory approach.
- The system exhibits three-body bound state without the presence of a three-body force
- We observe universal ratios between three-body and two-body binding energies from the results.
- We consider studying four-body systems with the same approach.

Questions?

Two-body Dressed Propagator

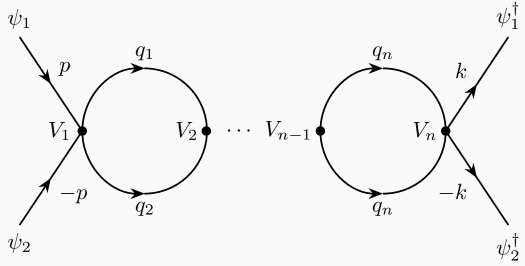

Figure 7: Diagram for two-body scattering problem

Yong, DT Son (2018)

Two-Body Scattering Amplitude

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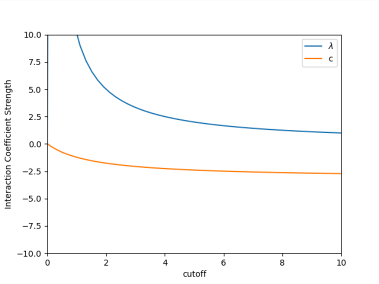

Figure 8: Interaction coefficient strength as function of cutoff

Thank You!