A Quantum Computing Algorithm for an Ising Model in One and Two Dimensions

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Introduction

Quantum Computing is a fast-growing emerging technology with important applications in medicine, finance, physics, and engineering. Quantum computers have the ability to solve complex problems that cannot be solved by classical computers. The goal of this project is to introduce quantum computing methods to physics students by applying these methods to the Ising model – a fundamental model in statistical physics. We developed an algorithm for solving a one-dimensional Ising model with no external field in Qiskit, a quantum computing platform developed by IBM Quantum. We also designed a more efficient algorithm for the Ising model in two and three dimensions.

Model Description

- The Ising model describes the evolution of a system of spins under the influence of an external magnetic field and the spin neighbor-neighbor interactions.

- The evolution of the system is described by the Hamiltonian:

$$H = -J\sum_{i=1}^{n} s_i s_{i+1} - B\sum_{i=1}^{n} s_i \qquad (1)$$

- S is the spin number for each spin. It is +1 for spin-up, and -1 for the spin-down.

- J stands for the coupling constant, indicating the strength of interaction.

- B is the strength of the magnetic field, considered to be zero here.

- For both algorithms discussed in the paper we use periodic boundary conditions, where the last spin is connected with the first spin.

- The possibility to flip a spin is determined by the energy change after the flip.

$$\begin{cases} p = 1 & \Delta E \leq 0 \\ p = e^{-\frac{\Delta E}{4T}} & \Delta E > 0 \end{cases} \qquad (2)$$

Algorithm Description

One-Dimensional Ising Model without External Magnetic Field

- For the one-dimensional Ising model, the energy change of a spin flip is only correlated with its two neighbors.

$$\Delta E_m = -J(-2s_{m-1}s_m - 2s_m s_{m+1}) \qquad (3)$$

- With three spins, there are only 8 situations. We list them out with their respective energy change (Figure I).

ASB	$\lvert S'\rangle$	S'_{cl}	p_d
↓↓↓	$\sqrt{P}\lvert\uparrow\rangle + \sqrt{1-P}\lvert\downarrow\rangle$	↓	$1-P$
		↑	P
↓↓↓	$\lvert\uparrow\rangle$	↑	1
↓↑↓	$\lvert\downarrow\rangle$	↓	1
↓↑↑	$\lvert\downarrow\rangle$	↓	1
↑↓↓	$\lvert\uparrow\rangle$	↑	1
↑↓↑	$\lvert\uparrow\rangle$	↑	1
↑↑↓	$\lvert\downarrow\rangle$	↓	1
↑↑↑	$\sqrt{P}\lvert\downarrow\rangle + \sqrt{1-P}\lvert\uparrow\rangle$	↑	$1-P$
		↓	P

Note: $P = e^{-\frac{4J}{T}}$

Figure I. A table of all eight possible situations

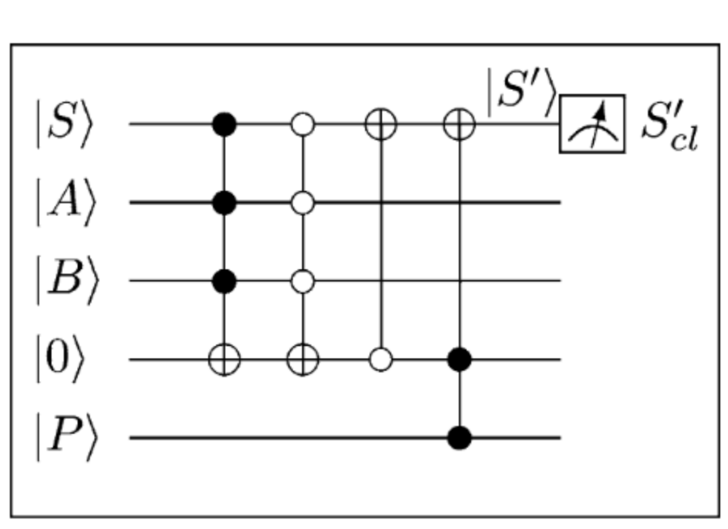

Figure II. Quantum circuit for three qubits

- When constructing a circuit, we use 3 qubits to represent 3 spins. We identify different situations with CNOT gates and store this information and possibility with 2 more qubits (Figure II).

- The circuit is only designed for three spins. The whole spins are updated through classic channels (lists) and the same process is repeated on the next three qubits.

Two-Dimensional Ising Model without External Magnetic Field

- For this case we use a different approach. We can divide the two-dimensional Ising model into two one-dimensional Ising models.

$$\Delta E_m = -J(s_{mn}(s_{m-1n} + s_{m+1n} + s_{mn-1} + s_{mn+1})) \qquad (4)$$

$$\Delta E_m = -J(s_{mn}s_{m-1n} + s_{mn}s_{m+1n}) - J(s_{mn}s_{mn-1} + s_{mn}s_{mn+1}) \qquad (5)$$

- There are two special cases. From the point of view of one-dimensional Ising models, they should both bring positive energy changes, but for a two-dimensional Ising model, they cancel each other, so they should be excluded from the circuit.

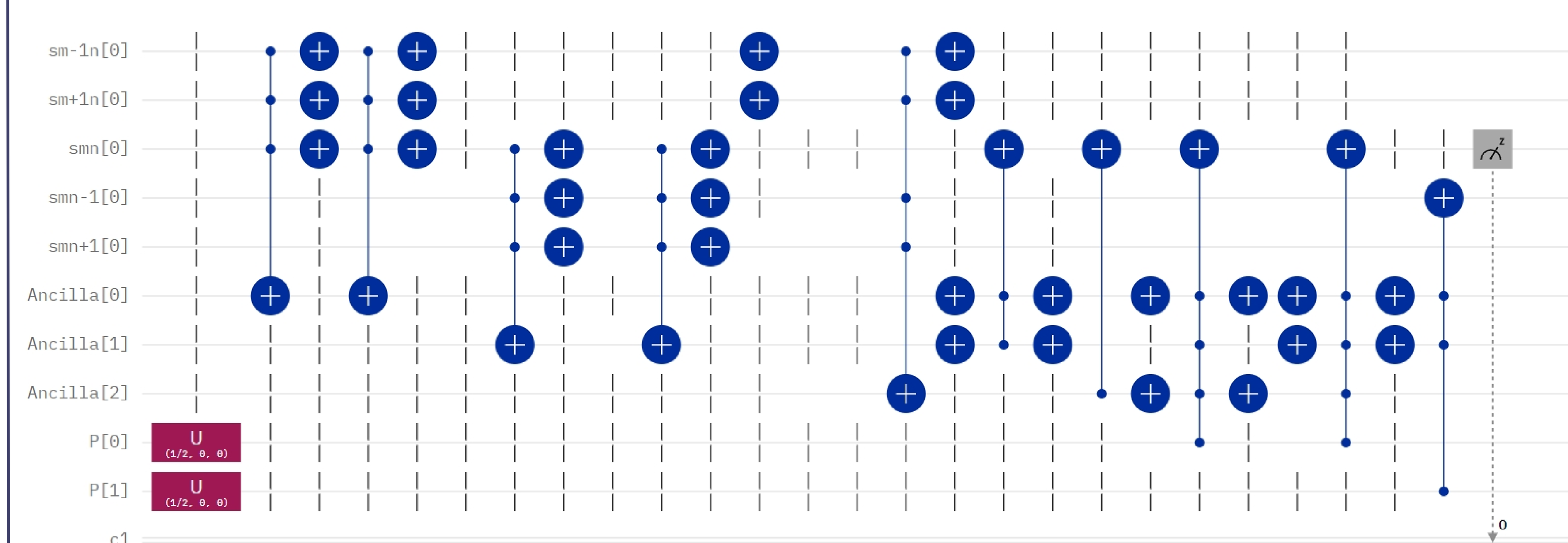

Figure III. Quantum circuit for single two-dimensional Ising model unit

- The streaming of the circuit is the same.

Simulation Results

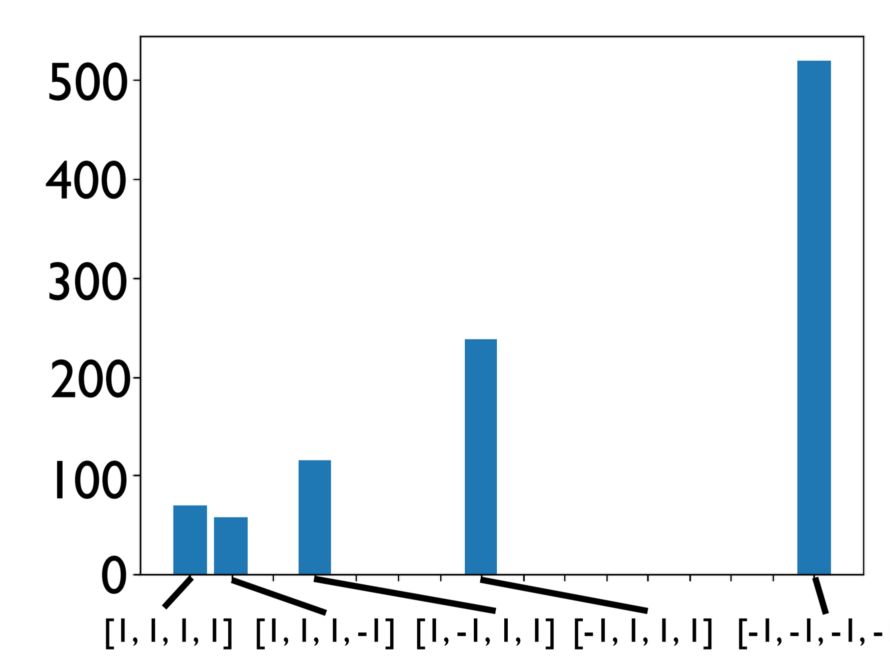

Figure IV. Simulation results for one-dimensional Ising model without external magnetic field with Monte Carlo Method

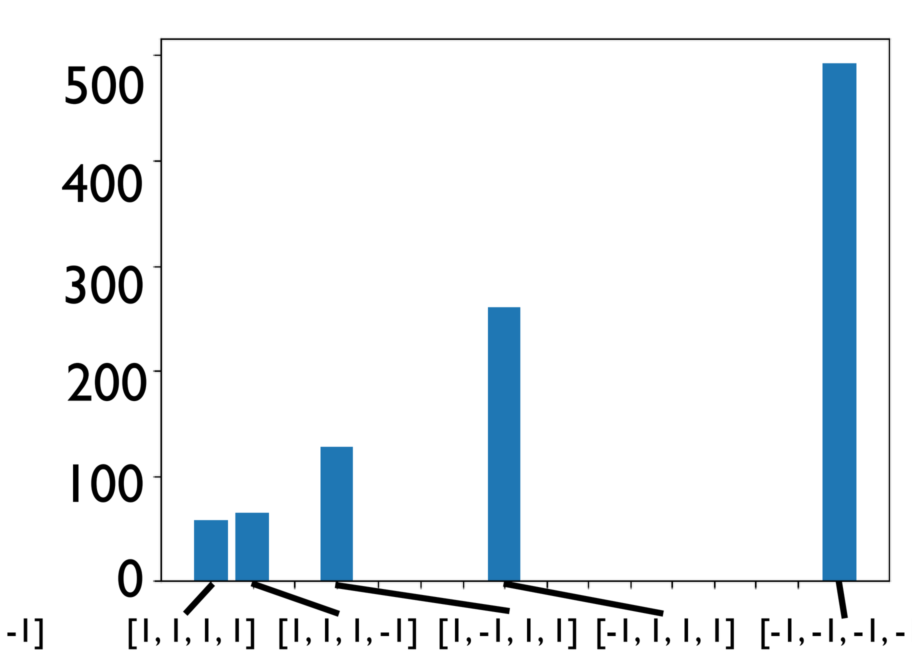

Figure V. Simulation results for one-dimensional Ising model without external magnetic field with quantum algorithms

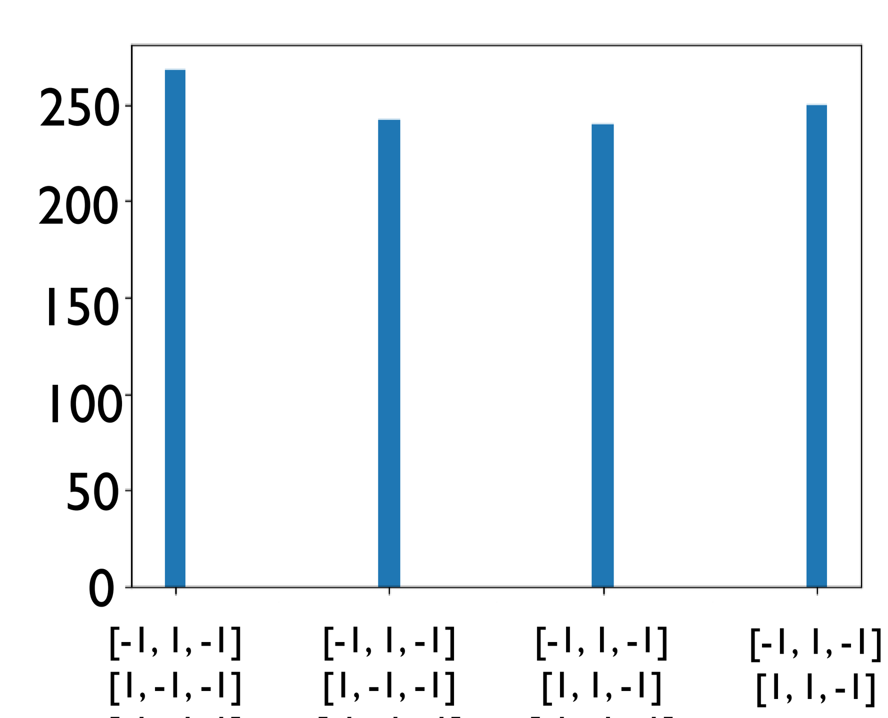

Figure VI. Simulation results for two-dimensional Ising model without external magnetic field with Monte Carlo Method

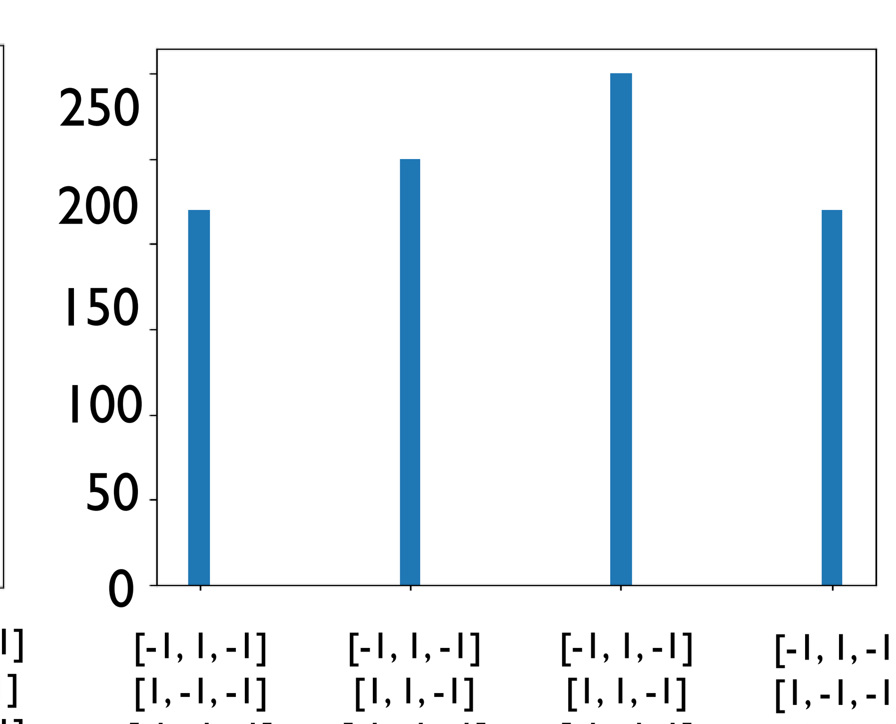

Figure VII. Simulation results for two-dimensional Ising model without external magnetic field with quantum algorithms

Conclusions

We constructed quantum circuits for the Ising model with Qiskit and obtained results similar to the classical Monte Carlo method. The quantum algorithm, although not as fast as the classic method, can still be a good demonstration of quantum circuits in an introductory class. It is also possible to develop a faster quantum algorithm in the future. The new method of breaking the high-dimensional Ising model into multiple one-dimensional Ising models may also provide a new way of examining other important statistical physics properties, such as magnetization and susceptibility.

References

[1] J. H. Cole, L.C.L. Hollenberg, S. Prawer, Computer Physics Communications. 161, (2004).