



JOINT MARCH MEETING AND APRIL MEETING

Global Physics Summit

Non-Relativistic Interacting Fermions in $1 + 1$ Dimensions

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Introduction

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Introduction

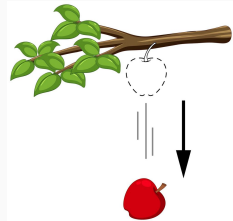
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- Using EFT to study three-particle systems consisting of two species in one dimension interacting via contact interactions.
- **Result:** We observe the formation of bound states and universal ratio between three-body and two-body binding energies.

Essences of an Effective Theory

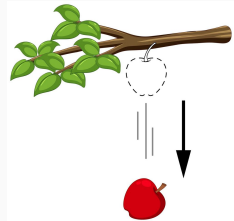
- Degrees of Freedom: Apple and Earth



$$F = G \frac{Mm}{(R+r)^2}$$

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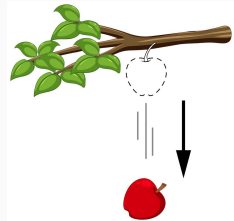
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- Clear separation of scales, h and R



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- Rotational symmetry

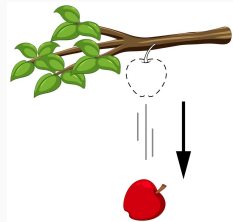


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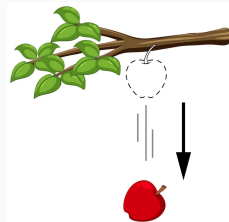
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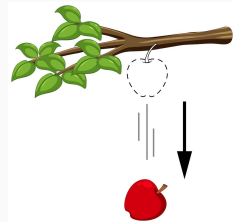
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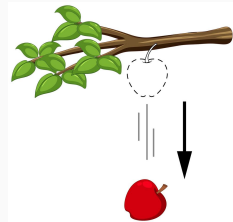
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- Theoretical error estimation



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- **Degrees of freedom:** ψ_1 and ψ_2 .
- Two-body contact potentials include

$$V_\lambda = -\lambda\delta(x)$$

$$V_c = -c\delta'(x)\partial_x$$

Two-Body Scattering Problem



Feynman diagrams for the full dimer propagator

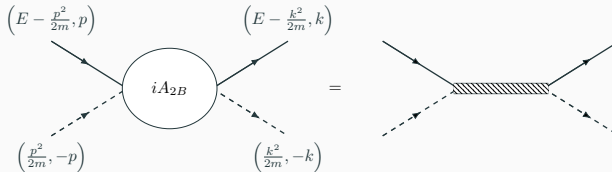


Figure 1: Relationship between two-body scattering amplitude and dressed propagators

Two-Body Scattering Amplitude

- Two-body scattering amplitude:

$$A_{2B}(p, k) = \frac{-i\lambda + c\lambda(B_2 + B_0pk) - icpk}{(1 + i\lambda B_0)(1 + icB_2)}$$

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- Renormalization behavior of the couplings in terms of momentum cutoff:

$$\lambda = \frac{2\gamma}{\Lambda}$$

$$c = -\frac{2\pi}{2\Lambda + \pi\mu}$$

Three-body scattering amplitude

The diagram illustrates the Faddeev equation for the three-body scattering amplitude. On the left, a large oval labeled iA_{3B} represents the full amplitude. It has four external lines: two incoming lines at the bottom labeled $(\frac{p^2}{2m}, -p)$ and $(\frac{k^2}{2m}, -k)$, and two outgoing lines at the top labeled $(\frac{p^2}{4m} - E_{2B}, p)$ and $(\frac{k^2}{4m} - E_{2B}, k)$. The top two lines are connected by a shaded rectangular region. This amplitude is equal to the sum of two terms. The first term is a diagram with two incoming lines at the bottom and two outgoing lines at the top, connected by a single diagonal line. The second term is a diagram where a shaded rectangular region labeled $D_{\phi/\sigma}$ is inserted into the bottom-left corner of the iA_{3B} oval. This region has an incoming line from the bottom and an outgoing line to the top, which then enters the oval. The top-right line of the oval is labeled (q_0, q) .

$$\left(\frac{p^2}{4m} - E_{2B}, p \right) \quad \left(\frac{k^2}{4m} - E_{2B}, k \right) \\ iA_{3B} \\ \left(\frac{p^2}{2m}, -p \right) \quad \left(\frac{k^2}{2m}, -k \right) = \text{diagram 1} + \text{diagram 2}$$

Figure 2: Faddeev equation for scattering amplitude under one interaction

Three-body scattering amplitude

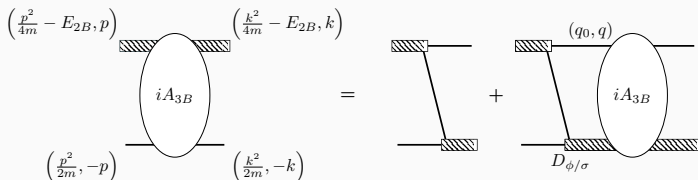


Figure 2: Faddeev equation for scattering amplitude under one interaction

- The above diagram can be expressed with an integral equation.

$$A_{3B}(p, k) = B(p, k) - \int_{-\Lambda}^{\Lambda} \frac{dq}{2\pi} D_{\sigma} \left(E - \frac{q^2}{2m}, -q \right) B(p, q) A_{3B}(q, k)$$

Delta Potential Interaction

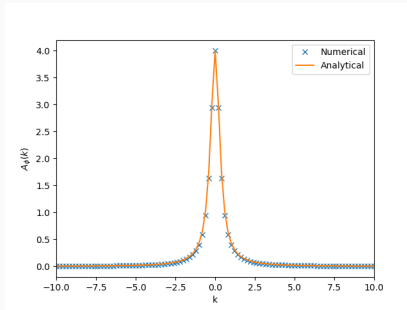


Figure 3: Scattering amplitude as a function of momentum

- The analytical solution is:

$$A_{\phi}(k) = \frac{m^2 \gamma^3}{4k^2 + m^2 \gamma^2}$$

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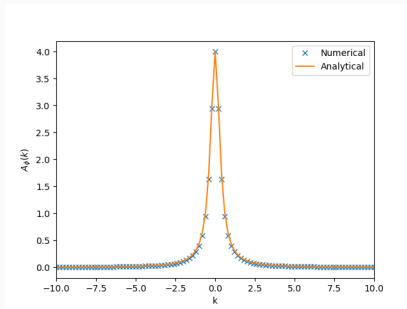


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- We observe the ratio between three-body and two-body binding energies:

$$\frac{E_{3B}}{E_{2B}} = \frac{7}{4}$$

Delta Prime Potential Interaction

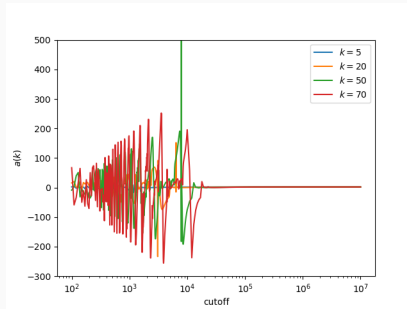


Figure 4: Scattering amplitude as a function of cutoff

- Interaction involving velocity of particles may involve three-body forces.

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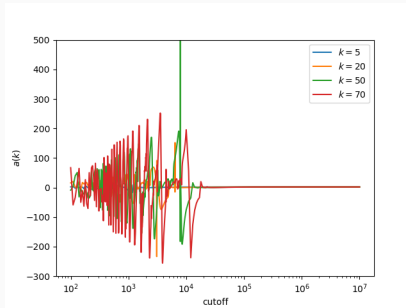


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- Interaction involving velocity of particles may involve three-body forces.
- We don't need a three-body force in this case as amplitude converges.

Delta Prime Potential Interaction

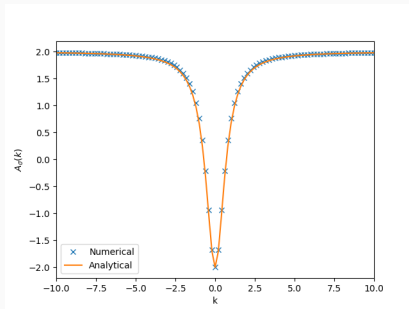


Figure 5: Scattering amplitude as a function of momentum under delta prime potential

- The analytical solution is:

$$A_\sigma(k) = -\frac{8\mu(2k^2 - \mu^2)}{m(2k^2 + \mu^2)}$$

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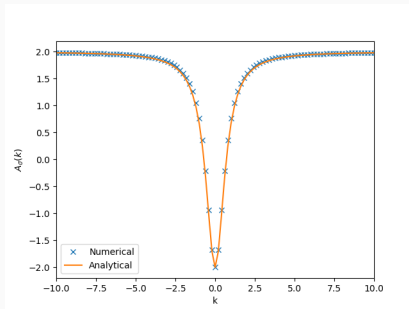


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- We observe:

$$\frac{E_{3B}}{E_{2B}} = \frac{11}{8}$$

Delta and Delta Prime Potential Interaction

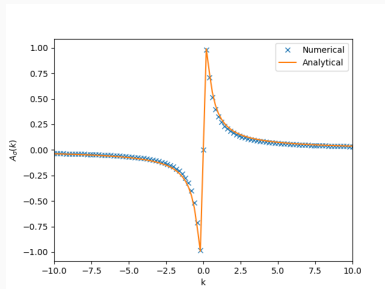
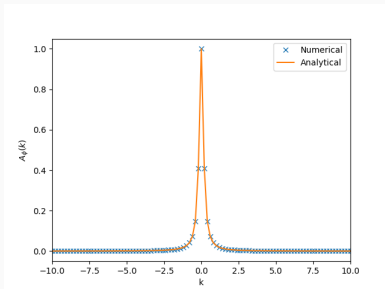


Figure 6: Scattering amplitude as a function of momentum for ϕ and σ field under δ and δ' potentials

- The analytical solutions are:

$$A_\phi(k) = \frac{2m^2\lambda^2}{m^2\lambda^2 + 9k^2}$$

$$A_\sigma(k) = \frac{12m^3k}{4\mu(m^2\lambda^2 + 9k^2)}$$

Delta and Delta Prime Potential Interaction

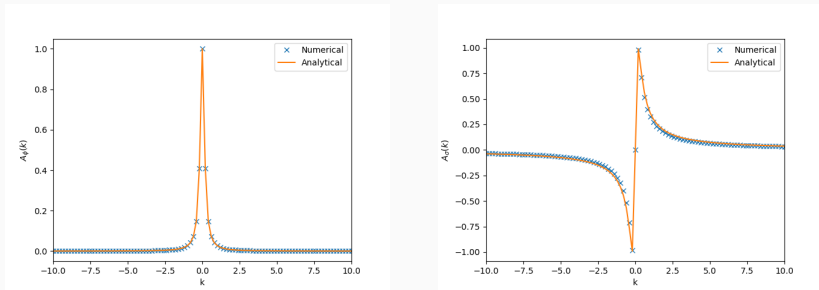


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- We observe:

$$\frac{E_{3B}}{E_{2B}} = \frac{4}{3}$$

Summary

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Summary

- We study a three-body scattering problem in one dimension with the Effective Field Theory approach.
- The system exhibits three-body bound state without the presence of a three-body force
- We observe universal ratios between three-body and two-body binding energies from the results.
- We consider studying four-body systems with the same approach.

Questions?

Two-body Dressed Propagator

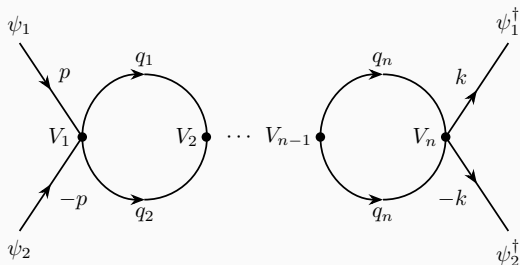


Figure 7: Diagram for two-body scattering problem

Two-Body Scattering Amplitude

- Renormalization behavior of the couplings:

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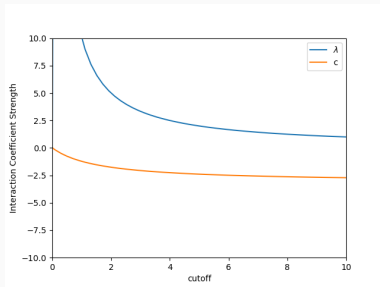


Figure 8: Interaction coefficient strength as function of cutoff

Thank You!