REVIEWS

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Math on Trial: How Mathematics Is Used and Abused in the Courtroom. By Leila Schneps and Coralie Colmez, Basic Books, New York, 2013, xi+256 pp., ISBN 978-0-465-03292-1, $26.99.

Reviewed by **Daniel Ullman**

The study of mathematics is often regarded as ideal undergraduate preparation for the study of law, since both disciplines require that practitioners reason carefully, pay precise attention to detail, and identify essential elements in complex situations. In both mathematics and the law, argument is used to deduce new conclusions from basic principles, or, put in a more pragmatic way, to reduce desired conclusions to accepted principles. In both mathematics and the law, reference to prior work is necessary in order not to have to argue every point from first principles. In mathematics, we have theorems from the literature; in law, we have precedent. The axiom systems that we favor in mathematics are not the only ones worthy of careful study, but we agree to accept them for most purposes as fundamental to our conception of reality. Similarly, the Constitution that we have adopted in America is not the only way to establish law, but it is one that we agree to accept as fundamental to our conception of a nation of laws.

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And yet the cultures of mathematics and law could not be more different. Where law is emotional and engaged, mathematics is dispassionate and detached. A legal argument implores the listener to agree; a mathematical argument *impels* the listener to agree. The modes of persuasion available to the litigator range far and wide, including appeals to emotion, bias, authority, and suggestion. The mathematician, in her turn, can appeal only to axioms (or to results already shown to follow from those axioms). Lawyers attempt to alter opinions, whereas mathematicians hold opinions in utter disdain.

So there is a curious clash of cultures when mathematicians find themselves in a court of law. Accustomed to dealing with truth and fallacy, the mathematician acting as expert witness may find her testimony used more for its emotional power than for its correctness. Often, juries may find themselves more impressed with a mathematician's credentials than with her explanations, which they might scarcely understand. When a mathematician overreaches and expounds on values, policy, or guilt, the jury may ascribe more weight to such remarks than is warranted. Knowing this, lawyers are inclined to feature mathematicians in a kind of abuse of authority.

The culture clash can lead to awkward interchanges in the courtroom. Will Kazez of the University of Georgia tells the story of his stint as an expert witness in a case involving a triangular parcel of land. In response to the lawyer's directive "And tell the court, Dr. Kazez, are you familiar with the theorem of Pythagoras?", Kazez planned the delicately sarcastic reply "Well, your honor, I don't mean to brag, but yes, I am familiar with the theorem of Pythagoras."

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I heard once about a mathematician brought to a courtroom as an expert witness on the theory of infinite series. The case involved a landlord/tenant dispute in which a subsidized rent was on a sliding scale depending on the income of the tenant. To complicate matters, the tenant received a government subsidy depending on the size of the rent. The landlord argued that the additional income that the tenant secured owing to the rent justified an increase in the rent. This increase in rent would of course justify an increase in the government subsidy, and a never-ending spiral of increases in rent and subsidy would ensue. Along came the mathematician hero to propose a solution to the conundrum: pass to the limit.

I received a phone message a few weeks ago from a Washington DC lawyer I did not know. The lawyer explained that he was looking for an expert who could testify that a certain fact pattern was "less likely than being struck by lightning". Not being any sort of expert on lightning, I didn't return the call. But two thoughts did come to my mind. The first was that the unlikely fact pattern that the lawyer had in mind was probably the one that had actually occurred, in which case its probability (in retrospect) is equal to unity. The second was that fancy law firms probably pay expert witnesses generously.

How certain must a juror be to vote for a criminal conviction? The phrase "proof beyond a reasonable doubt", which expresses the familiar standard, grates on the mathematical ear. How much doubt is reasonable? Judges refuse to say. Besides, proof in the mathematical sense is meant to be beyond *all* doubt. Proof leads to absolute certainly (allegedly, anyway). Like uniqueness and pregnancy, proof does not accept modifying phrases gracefully. While it must be acknowledged that absolute certainty is never possible in the real (non-mathematical) world, we don't solve any problem by using a phrase like "proof beyond a reasonable doubt" while refusing to define it. Here is what I would propose: Jurors will be instructed to regard something as proved beyond a reasonable doubt if their personal assessment of the probability of the truth of the assertion exceeds some predetermined value, say 98%. To measure whether this standard is achieved, the jurors are asked if they would be willing to risk $49 if the assertion were to be revealed as false against a $1 gain if the assertion were it revealed to be true. This remains a subjective judgement, as it must be, but at least the degree of certainty expected is quantified. Whether 98% is the right number is a matter of policy, not of mathematics, but leaving this number unspecified relegates the standard of certainty to a vague "pretty sure".

The standard for winning in court in many civil matters is "by a preponderance of the evidence", which is to say "more likely than not". In contrast to "reasonable doubt", this standard is clearly defined, and the test is a willingness to accept a bet at even odds ... or, rather, to be more precise, for some $\epsilon > 0$, a bet in which one risks $1 + \epsilon$ dollars against a gain of one dollar. Still, we wonder whether some circumstances might best be subject to, say, a 60% standard. We could call it "beyond a reasonable filibuster."

The only place we expect to find absolute certainty is within mathematics. Yet it must be said that proof, even in the mathematical sense, often does seem to leave some room for uncertainty. The announcements of the proofs of the Four Color Theorem and Fermat's Last Theorem did not immediately extinguish all doubt. Thomas Hales's proof of the Kepler conjecture demonstrates that it is true, but is it *undoubtedly* true? And if there remains doubt, is that doubt "reasonable"? The announcement in 1980 of the classification of finite simple groups most certainly did not settle that question definitively. In 2004, at his AMS Invited Address at the Joint Mathematics Meetings in Phoenix, Michael Aschbacher described his 1300-page manuscripts plugging a hole in what had been regarded as the complete proof. At the conclusion of the talk, an audience member asked Aschbacher whether he now felt certain that the classification was in fact correct. As I recall, he responded, "I'd bet my car on it, but I wouldn't bet my house on it." So much for certainty. Perhaps "proof beyond a reasonable doubt" exists within mathematics after all.

Mathematicians brought into the judicial system would be wise to heed the same warning that is given to mathematicians who work in the policy arena: Limit your pronouncements to the mathematical facts and leave the interpretation of those facts to others. It is appropriate, for example, to describe the rate of planetary temperature increase and to explain how this rate is determined from data. But it is dangerous to assert then that, based on mathematics, we should implement a cap-and-trade system for containing greenhouse gases. In fact, it is best to avoid the word "should" in its entirety. Mathematics does not tell us that we "should" do anything. What we should do can be informed by mathematics, but what we should do also involves values on which mathematics is utterly ill-equipped to comment. When mathematicians cross the line and become advocates for or against certain policies (or defendants), they threaten their position as trusted and dispassionate arbiters of indisputable facts.

The clash of cultures between mathematics and the law emerges clearly in *Math on Trial*. The authors, Leila Schneps and her daughter Coralie Colmez, have divided the book into ten chapters, each of which focuses on a classical mathematical error and a particular legal case. For example, the first chapter is entitled "Math Error Number 1: Multiplying Non-independent Probabilities" and subtitled "The Case of Sally Clark: Motherhood Under Attack". Here's a quick synopsis: Sally Clark gave birth to two children in the 1990s who died as infants. Double crib deaths happen only very rarely, and Sally was (therefore?) accused of murdering the children. The estimations of the probability of double crib deaths occurring simply "at random" played a pivotal role in the trial. The jury was persuaded that this probability was minuscule and, on that basis, ... well, there is more to the story, but I won't give it all away here.

In another chapter focusing on gender bias in graduate admissions at Berkeley in the 1970s, mathematics is used to exculpate the institution. It is shown how Simpson's paradox can lead to a seemingly impossible circumstance in which each department admits a percentage of female applicants into its graduate program that is greater than the percentage of male applicants who are admitted, and yet taken together the percentage of women admitted to graduate programs in general is lower for women than for men. In effect, this is what was happening at Berkeley. The lower admissions rate for women across all Berkeley departments collectively resulted principally from the fact that women were applying in disproportionate numbers to more competitive graduate programs.

The book is beautifully written. The story telling is powerful. The pages turn swiftly and easily. While the stories involve courtroom trials, they are not delivered in the form of tales of suspense; instead, the outcomes of these trials are strongly foreshadowed in the narrative. As soon as Sally Clark is introduced, the reader discerns that she is innocent of wrongdoing but about to be wrapped up in a travesty of a trial and convicted unfairly. The key expert witness, not a mathematician but a pediatrician, is quickly seen to be a charming but self-promoting charlatan with nearly hypnotic power over the jury. Breaking the suspense in this way allows the focus to return to the theme, which is mathematical error.

The book is also carefully researched. The authors studied the ten cases with great care, pouring over books, scholarly articles, courtroom transcripts, and newspaper archives. In some cases, they interviewed individuals involved in the trials. They point to a substantial literature on the topic of mathematics and statistics at trial. In particular, they mention the work of Laurence Tribe, the well-known Harvard Law professor, who studied mathematics as an undergraduate but who generally argues against permitting mathematical testimony at trial.

At the top of the front cover of the book appears the sentence: "When math becomes a matter of life and death, you'd better check your sums." In fact, there is no summing at all in the book, and this glib sentence gives away that the book targets a non-mathematical audience. In fact, the sentence also perpetuates a myth among the ignorant that mathematicians are people who "do sums". A more apt sentence would replace "sums" with "reasoning". In any case, the sentence is corny and belies the seriousness and the engaging exposition one finds when one opens the book.

Math on Trial is about mathematical errors. Unfortunately, the book makes a number of mathematical errors of its own. On page 1, we are told, "If you multiply the probabilities of events that are not independent of each other, you will get a significantly smaller probability than is accurate." No, not always smaller. On page 62, we read, "Suppose you are given a coin and told that it is of one of two types: either fair and balanced or weighted to come up heads 70% of the time. You are allowed one toss, and it falls on heads. Let us ... investigate the probability that the coin is biased after this result." They calculate the answer, despite the fact that the question is nonsensical without an a priori distribution on the two coins. In chapter 5, we are told that if the probability of a single DNA sample matching a target sample is $1/n$, then the probability of finding a match among m random samples is m/n. This is false but approximately correct when $m \ll n$, though it must be very wrong when $m > n$, of course. And on page 190, we read that the probability of throwing 3 sixes in 6 rolls of a die is 20/216. Not quite. The probability is $(20/216)(5/6)^3$. It may be that "throwing *at least* 3 sixes in 6 rolls" was intended, but 20/216 isn't the correct value for that either.

The irony is that these errors won't matter much to the non-mathematical readers, who, like most jurors faced with mathematical testimony, won't question any of the assertions or identify any errors. They will merely take in the broad conclusions professed by the authors, whom they will identify as people of substantial intellect and with whom they will therefore be inclined to agree. In some cases, this can lead to the conviction of innocent defendants. Indeed, the main point that the judicial enterprise must be vigilant to the misuse of mathematics is not diminished by any mathematical errors in *Math on Trial*.

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In the preface, Schneps and Colmez refer to "mathematics' disastrous record of causing judicial error". I take exception to this characterization. It may well be true that there is a long history of legal argument inspired by misapplied, misinterpreted, misused mathematics. But mathematics itself is not to blame. Whatever disasters are on the record do not belong to mathematics. The culprit is not the use but the abuse of mathematics. It is critically important to permit sound mathematics and science to inform legal proceedings. But the translation from facts in the mathematical world to conclusions in the real world is fraught, and it is best left in the hands of those who are positioned to hold values, biases, and opinions. Mathematics remains untarnished. Pythagaras has nothing to be ashamed of.

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