Special Theory of Intelligent Relativity: A Formal Framework for Generative Complexity Based on Solution Space Compression

Abstract

This paper, under the meta-framework of *The Generative Dynamics Foundation of Computational Complexity*, proposes and formalizes the core measure of the **Special Theory of Intelligent Relativity — Generative Complexity**. This theory completely abandons **"step counting"** as the measure of difficulty, and instead takes **"the compression process of the solution space driven by a sequence of constraints"** as the basic object. We prove that for any formal problem defined on a discrete finite domain (such as TSP, SAT), its generative complexity can be exactly computed as the sum of the **"exclusion ratios"** of each constraint. This measure intrinsically captures the dynamic cost of problem structure generation and provides a deeper dynamical foundation for explaining traditional complexity classifications.

Keywords: Generative Complexity; Solution Space Compression; Exclusion Ratio; Dynamic Generative Theory; TSP; SAT

1. Introduction: From Step Counting to Solution Space Compression

Traditional computational complexity theory (e.g., P, NP) anchors problem difficulty in the number of steps an algorithm requires to verify or search within a **fixed solution space**. This "step counting" paradigm implicitly assumes a static presupposition: the solution space is pre-existing and immutable.

The Dynamic Generative Theory fundamentally challenges this presupposition: **"Problem-solving" is not searching in a static space, but a process in which constraints and the potential field interact to jointly "generate" the solution space.** In the special (narrow) framework, we disregard the solver's individual history and focus on a **"blank cognitive agent"** (whose potential field is completely chaotic, with historical depth $H = 0$ and coherence degree $C = 0$). For such an agent, the generative difficulty of a problem is entirely inherent in the shaping process of the **objective solution space** by the problem's constraint sequence.

The core contribution of this paper is to provide an **operational mathematical definition** of generative complexity within this special framework, and to illustrate its calculation process and physical meaning using the Traveling Salesman Problem (TSP) and the Boolean Satisfiability Problem (SAT) as examples.

2. Core Definitions: Generative Complexity as a Measure of Solution Space Compression

Let a formal problem P be defined by an ordered set of constraints $C = [c_1, c_2, ..., c_m]$. Let S_i denote the size of the solution space after applying the first i constraints (i.e., the number of remaining possible solutions), where S_0 is the initial solution space size.

Definition 1 (Exclusion Ratio p_i): The generative strength of constraint c_i, measured by its degree of compression on the current solution space:

$$p_i = 1 - \frac{S_i}{S_{i-1}}, \quad \text{where } S_i = |\{s \in S_{i-1} \mid s \text{ satisfies } c_i\}|$$

The range of p_i is $[0, 1]$. When S_{i-1} is infinite and S_i is finite, we define $p_i = 1$.

Definition 2 (Generative Complexity $G(P)$): The generative complexity of problem P is the sum of the exclusion ratios of all its constraints:

$$G(P) = \sum_{i=1}^{m} p_i$$

$G(P)$ measures the minimal weighted sum of the **"generative action"** required to shape a completely chaotic potential field (containing all possibilities S_0) into a stable solution framework (solution space S_m). Each elementary generative event consumes one generative element Ξ; thus, $G(P)$ can be understood as the total number of Ξ consumed.

3. Application Examples: Calculations

3.1 Traveling Salesman Problem (TSP, n=6)

Consider the symmetric TSP with 6 cities, with constraints added in a natural logical order: 1. c_1**: Visit each city exactly once (forming a path permutation)**

- S_0: Initial state allows any path (repetition allowed, arbitrary length), which is infinite.
- S_1: Number of permutations of all cities, $6! = 720$.
- $p_1 = 1$ (from infinite to finite, maximum compression).

2. c_2**: Must return to the starting point (forming a Hamiltonian cycle)**

- $S_2 = (6 - 1)! = 120$.
- $p_2 = 1 - 120/720 = 5/6 \approx 0.8333$.

3. c_3**: Minimal total distance**

- Typically unique, $S_3 = 1$.
- $p_3 = 1 - 1/120 \approx 0.9917$.

Generative Complexity:

$$G_{\text{TSP}} = 1 + 0.8333 + 0.9917 \approx 2.825$$

3.2 Boolean Satisfiability Problem (3-SAT, n=3, m=3)

Consider the instance: $F = (x_1 \lor x_2 \lor x_3) \land (\neg x_1 \lor \neg x_2 \lor x_3) \land (x_1 \lor \neg x_2 \lor \neg x_3)$ Initial solution space size $S_0 = 2^3 = 8$.

1. c_1**: Add the first clause**

- Only assignment $(0, 0, 0)$ is excluded. $S_1 = 7$.
- $p_1 = 1/8 = 0.125$.

2. c_2**: Add the second clause**

- Among the remaining 7 assignments, $(1, 1, 0)$ is excluded. $S_2 = 6$.
- $p_2 = 1/7 \approx 0.1429$.

3. c_3**: Add the third clause**

- Among the remaining 6 assignments, $(0, 1, 1)$ is excluded. $S_3 = 5$.
- $p_3 = 1/6 \approx 0.1667$.

Generative Complexity:

$$G_{\text{SAT}} = 0.125 + 0.1429 + 0.1667 \approx 0.4346$$

4. Generative Complexity vs. Traditional Complexity: A Degeneration Relationship

This framework reveals that traditional complexity classes are degenerate special cases of this framework when **ignoring the generative process**: - **P/NP classes**: Only concern whether the final solution space S_m can be verified (P) or searched (NP) in polynomial steps, completely disregarding the compression process $(p_1, ..., p_m)$ to reach S_m and its cost $G(P)$. - **Explanatory power of Generative Complexity**: The value of $G(P)$ directly reflects the "steepness" of problem structure generation. The G value of TSP is much higher than that of the SAT instance, which aligns with the traditional empirical perception of their relative difficulty but provides a quantitative explanation based on **generative dynamics**.

5. Conclusion and Outlook

This paper defines **Generative Complexity** $G(P)$ as the fundamental difficulty measure for formal problems within the special (narrow) framework. It successfully re-describes "problem-solving" as a **deterministic compression process of the solution space under a sequence of constraints**, and anchors difficulty in the dynamical characteristics of this process itself (the sum of exclusion ratios).

This special formal framework lays a solid mathematical foundation for subsequent research: 1. **New paradigm for algorithm design**: It can inspire new algorithms focusing on **simulating efficient compression processes** rather than searching in a fixed space. 2. **A bridge to the general theory**: This paper focuses on the **blank**

agent. The future **General Theory of Intelligent Relativity** will modulate the exclusion ratio p_i of each constraint by introducing **personal historical depth** H and **coherence degree** C, thereby transforming difficulty from an absolute property of the problem into a **relative relationship between the problem and the specific cognitive agent's history**. The special framework is a special case of the general theory when $H = 0, C = 0$.

Acknowledgements

This work is based on the Dynamic Generative Theory proposed by the first author. We thank our AI research partners for their deep discussions, which helped structure and express the ideas.

References

1. Chen, Z. *Dynamic Generative Theory: A Quantum Interpretation and Cosmological Unified Framework Based on the Chen Generative Element (Ξ).* Zenodo. 2025.

2. Chen, Z. *A Generative-Dynamics Foundation for Computational Complexity: A Unifying and Degenerative Meta-Framework.* Zenodo. 2026.