ECE 1513 Saturday Session Assignment 1

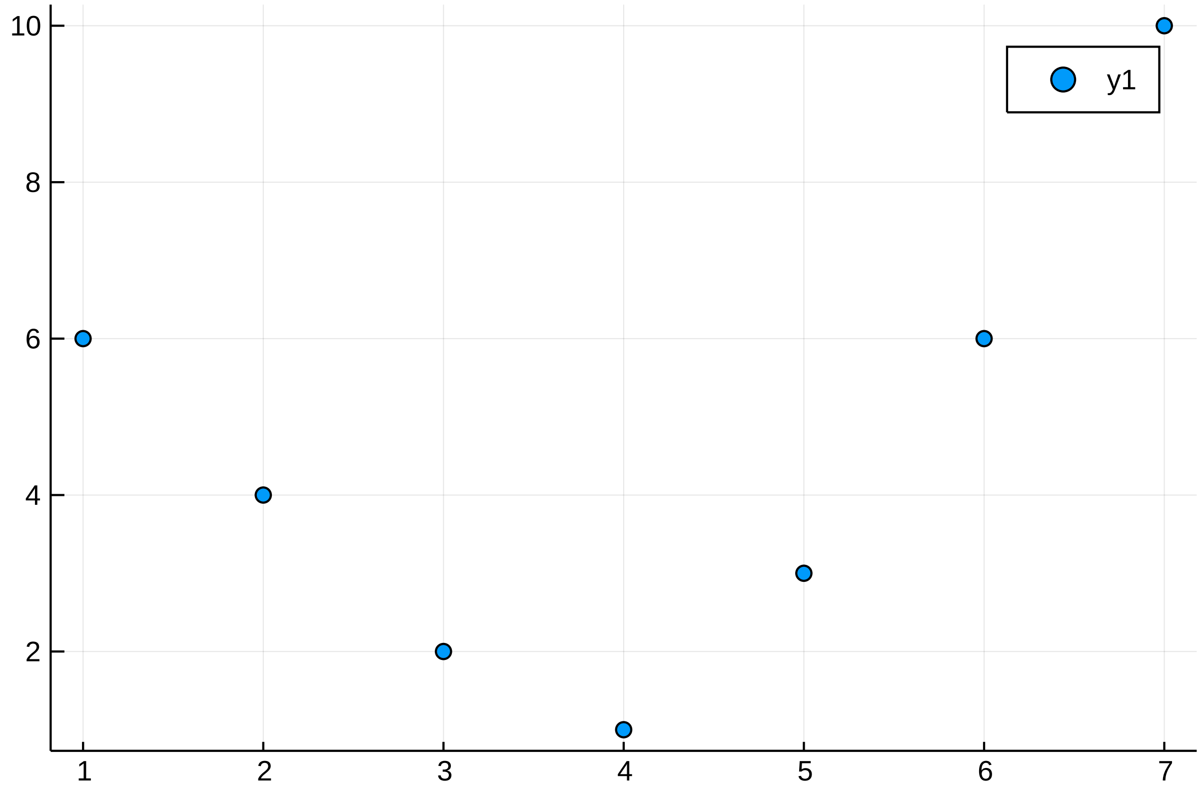
Name: Xiaochen Sun

Student ID: 1005747113

Problem 1(1)

```
using Plots

X = [1,2,3,4,5,6,7]
Y = [6,4,2,1,3,6,10]
scatter(X,Y)
```



Problem 1(2)

$$\mathbb{L} = \frac{1}{2N}\sum_{i=1}^{N}(y^{(i)} - t^{i})^2 \tag{1}$$

$$y = g_{(w,b)}(x) = wx + b \tag{2}$$

$$\mathbb{L} = \frac{1}{2N}\sum_{i=1}^{N}(wx^{i} + b - t^{i})^2 \tag{3}$$

$$\mathbb{L} = \frac{1}{2N}\sum_{i=1}^{N} w^2x^{(i)2} + b^2 + 2xbw - 2txw - 2tb + t^2 \tag{4}$$

$$\text{Setting A,B,C,D,E,F according to problem} \tag{5}$$

$$A = x^2, B = 1, C = 2x, D = -2tx, E = -2t, F = t^2 \tag{6}$$

$$\mathbb{L} = \frac{1}{2N}\sum_{i=1}^{N}(Aw^2 + Bb^2 + CwB + Dw + Eb + F) \tag{7}$$

Problem 1(3)

$$\mathbb{L} = \frac{1}{2N}\sum_{i=1}^{N}(Aw^2 + Bb^2 + CwB + Dw + Eb + F) \tag{8}$$

$$\frac{\partial \mathbb{L}}{\partial w} = \frac{1}{2N}\sum_{i=1}^{N} 2Aw + Cb + D \tag{9}$$

$$0 = \frac{1}{2N}\sum_{i=1}^{N} 2Aw + Cb + D \tag{10}$$

$$\frac{\partial \mathbb{L}}{\partial b} = \frac{1}{2N}\sum_{i=1}^{N} 2Bb + Cw + E \tag{11}$$

$$0 = \frac{1}{2N}\sum_{i=1}^{N} 2Bb + Cw + E \tag{12}$$

Problem 1(4)

$$A = \sum_{i=1}^{N} x^2 \tag{13}$$

$$\text{Substituting values x} \tag{14}$$

$$A = 140 \tag{15}$$

$$\text{computing B,C,D,E,F} \tag{16}$$

$$B = 7, C = 56, D = -290, E = -64, F = 202 \tag{17}$$

$$0 = \frac{1}{2N}\sum_{i=1}^{N} 2Bb + Cw + E \tag{18}$$

$$b = \frac{64 - 56w}{14} \tag{19}$$

$$0 = \frac{1}{2N}\sum_{i=1}^{N} 2Aw + Cb + D \tag{20}$$

$$0 = 2w(140) + b(56) - (290) \tag{21}$$

$$\text{solving for w:} \tag{22}$$

$$\frac{56(64 - 56w)}{14} = 290 - 280w \tag{23}$$

$$256 - 224w = 290 - 280w \tag{24}$$

$$w = 0.607142857 \tag{25}$$

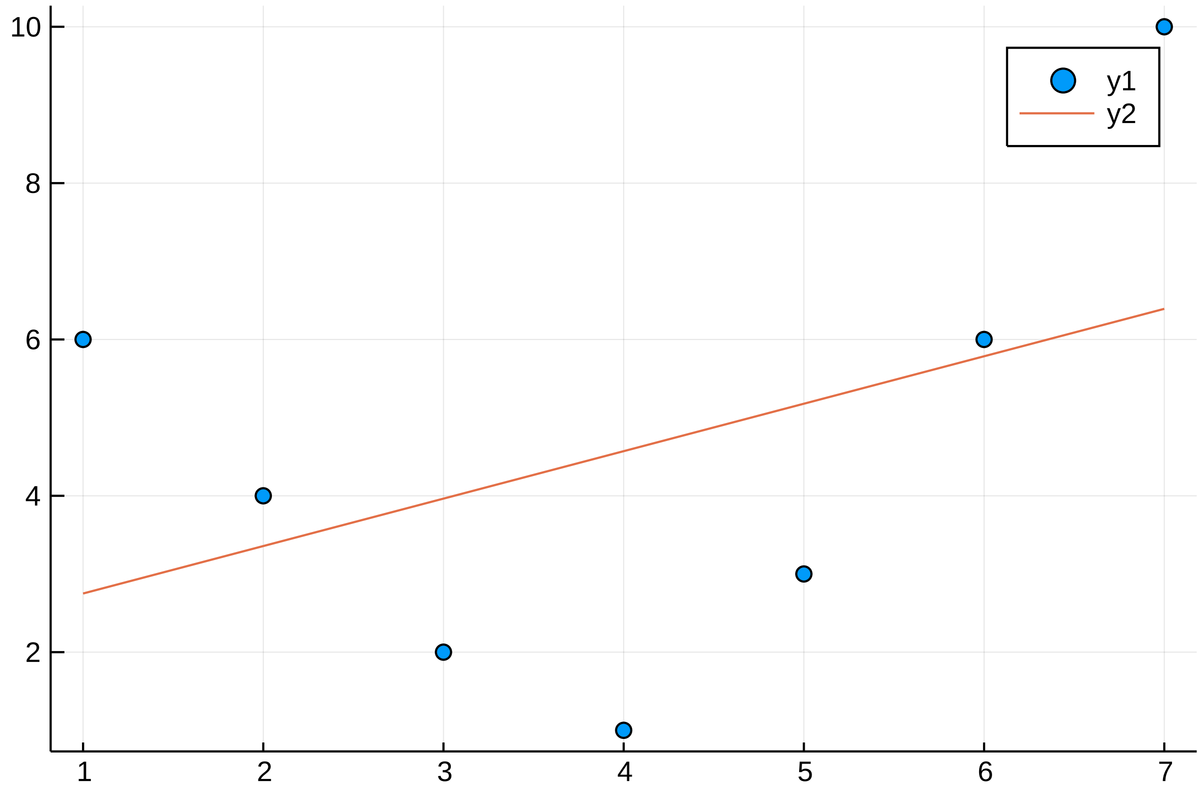
$$\text{solving for b:} \tag{26}$$

$$b = 2.142857 \tag{27}$$

Problem 1(5)

```
using Plots

X = [1,2,3,4,5,6,7]
Y = [6,4,2,1,3,6,10]
f(x) = x*0.607 + 2.143
scatter(X,Y)
plot!(X,f.(X))
```



Problem 2(1)

$$X \in \mathbb{R}^{dXN}, g_w(X) = wX \tag{28}$$

$$A = \sum_{i=1}^{N} X_i X_i^T \tag{29}$$

$$= \sum_{i=1}^{N} \begin{bmatrix} x_1 \\ : \\ x_d \end{bmatrix} \begin{bmatrix} x_1 & : & x_d \end{bmatrix} \tag{30}$$

$$= \sum_{i=1}^{N} \begin{bmatrix} x_1^2 & x_1x_2 & \dots & x_1x_d \\ x_2x_1 & : & : & : \\ : & : & : & : \\ x_dx_1 & \dots & \dots & x_d^2 \end{bmatrix} \tag{31}$$

$$\text{element of A:} \tag{32}$$

$$A = \sum_{i=1}^{N} \sum_{j=1}^{d} x_{ij}x_{ji} \tag{33}$$

$$A = XX^T \tag{34}$$

Problem 2(2)

$$\mathbb{L} = \frac{1}{2N}\sum_{i\in 1..N}(Y_i - \vec{w}X_i)^2 + \frac{\lambda}{2}||\vec{w}||_2^2 \quad (35)$$

$$\nabla\mathbb{L} = \frac{1}{2N}\sum_{i=1}^{N} -2X_i(Y_i - \vec{w}X_i) + \lambda\vec{w} \quad (36)$$

$$= \frac{1}{N}[X(\vec{w}X - Y) + \lambda\vec{w}] \quad (37)$$

$$\text{shape:[dXN]([1Xd][dXN]-[1XN])} \quad (38)$$

$$= \frac{1}{N}[\vec{w}XX^T - YX^T] + \lambda\vec{w} \quad (39)$$

$$\text{shape:([1Xd][dXN][NXd]-[1XN][NXd])} \quad (40)$$

$$\text{Assigning A and B as stated by the problem:} \quad (41)$$

$$A = XX^T, B = X^TY \quad (42)$$

$$\nabla\mathbb{L} = \frac{1}{N}[A\vec{w} - B] + \lambda\vec{w} \quad (43)$$

Problem 2(3)

$$w^* = argmin\mathbb{L}(\vec{w}, D) \rightarrow 0 = \frac{1}{N}[A\vec{w} - B] + \lambda\vec{w} \quad (44)$$

$$0 = \frac{1}{N}A\vec{w} - \frac{1}{N}B + \lambda w \quad (45)$$

$$\frac{1}{N}B = \vec{w}(\frac{1}{N}A + \lambda) \quad (46)$$

$$\lambda \text{ must multiply anidentity matrix to operate with matrix} \quad (47)$$

$$w^* = \frac{\frac{1}{N}B}{\frac{1}{N}A + \lambda\mathbb{I}} * \frac{N}{N} \quad (48)$$

$$w^* = \frac{B}{A + \lambda N\mathbb{I}} \quad (49)$$

Problem 2(4)

$$A = XX^T \quad (50)$$

$$\text{Eigenvalue is defined as:} \quad (51)$$

$$Ax = \lambda x \quad (52)$$

$$\text{Multiply both sides by } x^T\text{:} \quad (53)$$

$$x^TAx = \lambda x^Tx \quad (54)$$

$$x^TAx = \lambda||x||^2 \quad (55)$$

$$\text{Therefore:} \quad (56)$$

$$x^TAx > 0 \quad (57)$$

$$\text{Thus, A is positive definite, and will have positive eigenvalues} \quad (58)$$

Problem 2(5)

Knowing A is positive definite, assume $\lambda = 0$

$$A + 0N\mathbb{I}_d = 0$$

for Av = 0 exists eigenvector v = 0, which contradicts with A being positive definite

Thus, for A to be invertible, λ must not be 0

Problem 5(6)

$$\nabla\mathbb{L} = \frac{1}{N}[\vec{w}XX^T - YX^T) + \lambda\vec{w}] \tag{59}$$

$$0 = \frac{1}{N}[\vec{w}XX^T - YX^T] + \lambda\vec{w} \tag{60}$$

$$0 = \frac{1}{N}\vec{w}XX^T - \frac{1}{N}YX^T + \lambda\vec{w} \tag{61}$$

$$0 = w(\frac{1}{N}XX^T + \lambda\mathbb{I}) - \frac{1}{N}YX^T \tag{62}$$

$$w(\frac{1}{N}XX^T + \lambda\mathbb{I}) = \frac{1}{N}YX^T \tag{63}$$

$$w = (\frac{1}{N}XX^T + \lambda\mathbb{I})^{-1}\frac{1}{N}YX^T \tag{64}$$

$$w = \frac{1}{N}(XX^T + \lambda N\mathbb{I})^{-1}(YX^T) \tag{65}$$