ECE 1513 Introduction to Machine Learning

Assignment 6

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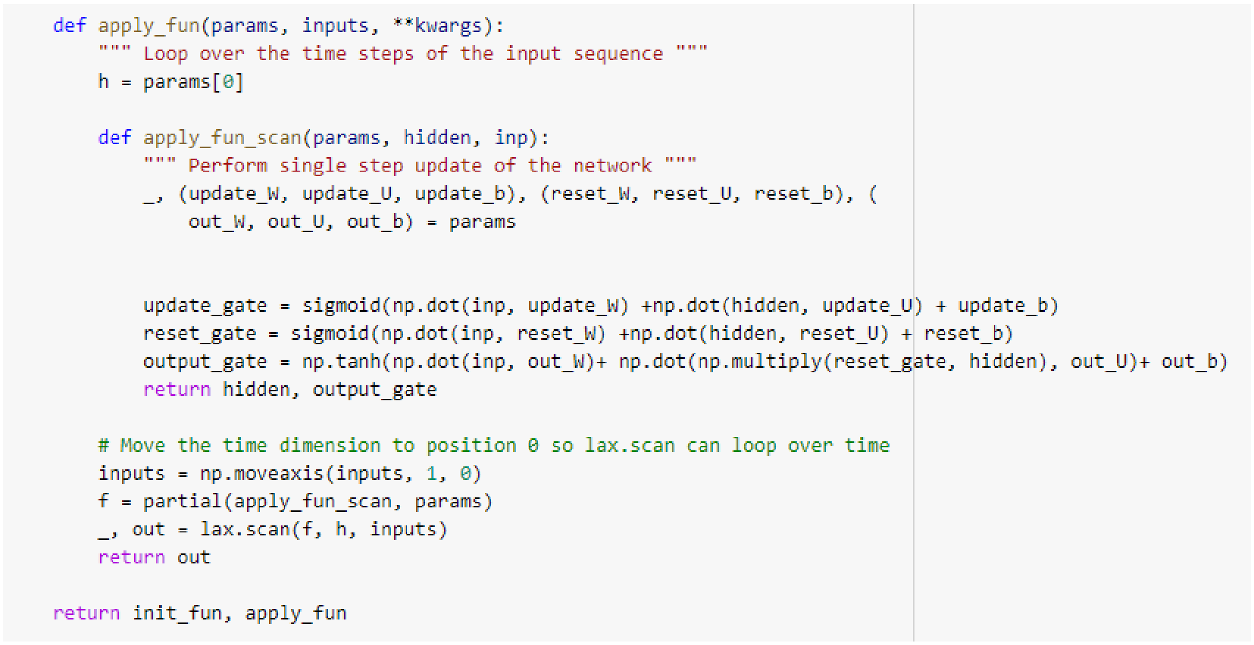
1. (1 point) Fill the line implementing the forward pass for the update gate in apply fun scan.

2. (1 point) Fill the line implementing the forward pass for the reset gate in apply fun scan.

3. (1 point) Fill the line implementing the forward pass for the output gate in apply fun scan.

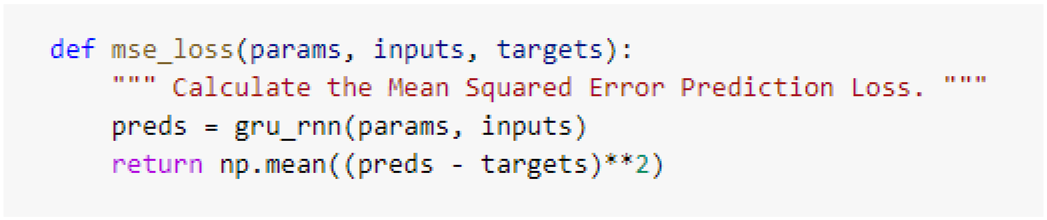
Solutions of above three questions:



```
def apply_fun(params, inputs, **kwargs):
    """ Loop over the time steps of the input sequence """
    h = params[0]

    def apply_fun_scan(params, hidden, inp):
        """ Perform single step update of the network """
        _, (update_W, update_U, update_b), (reset_W, reset_U, reset_b), (
            out_W, out_U, out_b) = params


        update_gate = sigmoid(np.dot(inp, update_W) +np.dot(hidden, update_U) + update_b)
        reset_gate = sigmoid(np.dot(inp, reset_W) +np.dot(hidden, reset_U) + reset_b)
        output_gate = np.tanh(np.dot(inp, out_W)+ np.dot(np.multiply(reset_gate, hidden), out_U)+ out_b)
        return hidden, output_gate

    # Move the time dimension to position 0 so lax.scan can loop over time
    inputs = np.moveaxis(inputs, 1, 0)
    f = partial(apply_fun_scan, params)
    _, out = lax.scan(f, h, inputs)
    return out

return init_fun, apply_fun
```

4. (1 point) Fill the missing line in the function mse loss. The function returns the mean squared error loss between the model's predictions (i.e., preds) and the target sequence (i.e., targets).

Solution:



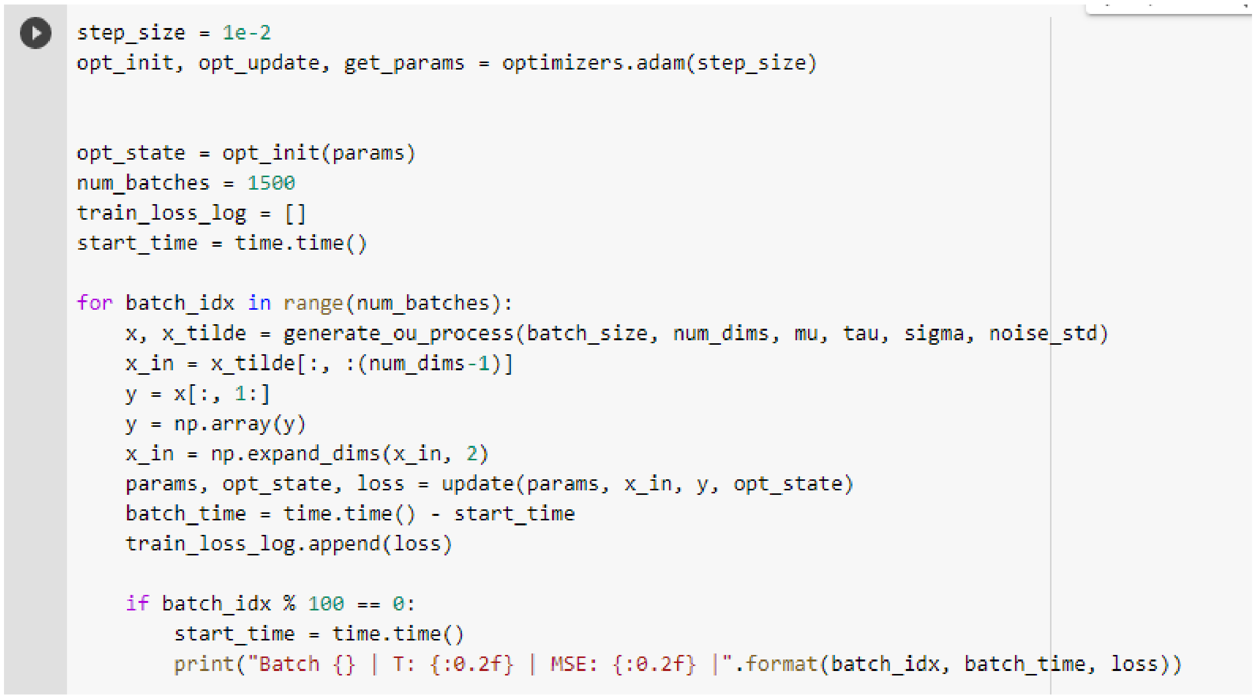
```
def mse_loss(params, inputs, targets):
    """ Calculate the Mean Squared Error Prediction Loss. """
    preds = gru_rnn(params, inputs)
    return np.mean((preds - targets)**2)
```

5. (2 points) Fill the missing lines at the top of the cell titled "Training the RNN". These lines should use the optimizers pre-built into JAX to instantiate an Adam optimizer. As seen in previous homeworks, you should obtain three things from the pre-built JAX optimizer: a method opt init that takes in a set of initial parameter values returned by init fun and returns the initial optimizer state opt state, a method opt update which takes in gradients and parameters and updates the optimizer states by applying one step of optimization, and a method get params which takes in an optimizer state and returns current parameter values.

6. (1 point) Fill the lines that define x in (the input) and y (the output) of our recurrent neural network. The RNN should take in all but the last step of the noisy time series, and predict all but the first step of the ground truth time series (i.e., the time series before it was noised).

7. (1 point) Fill the line that calls update to take one step of gradient descent on the batch of training data sampled.

Solutions of above three questions:

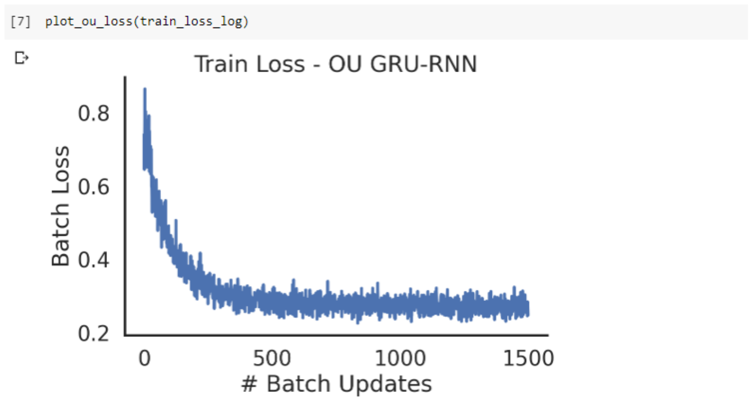


```
step_size = 1e-2
opt_init, opt_update, get_params = optimizers.adam(step_size)


opt_state = opt_init(params)
num_batches = 1500
train_loss_log = []
start_time = time.time()

for batch_idx in range(num_batches):
    x, x_tilde = generate_ou_process(batch_size, num_dims, mu, tau, sigma, noise_std)
    x_in = x_tilde[:, :(num_dims-1)]
    y = x[:, 1:]
    y = np.array(y)
    x_in = np.expand_dims(x_in, 2)
    params, opt_state, loss = update(params, x_in, y, opt_state)
    batch_time = time.time() - start_time
    train_loss_log.append(loss)

    if batch_idx % 100 == 0:
        start_time = time.time()
        print("Batch {} | T: {:0.2f} | MSE: {:0.2f} |".format(batch_idx, batch_time, loss))
```

8. (2 points) As done in prior assignments, perform a hyperparameter search to find a good value for your learning rate. Describe briefly how you conducted the search, the value you chose, and why you chose that value. You may find it useful to call plot ou loss(train loss log)

Solution:

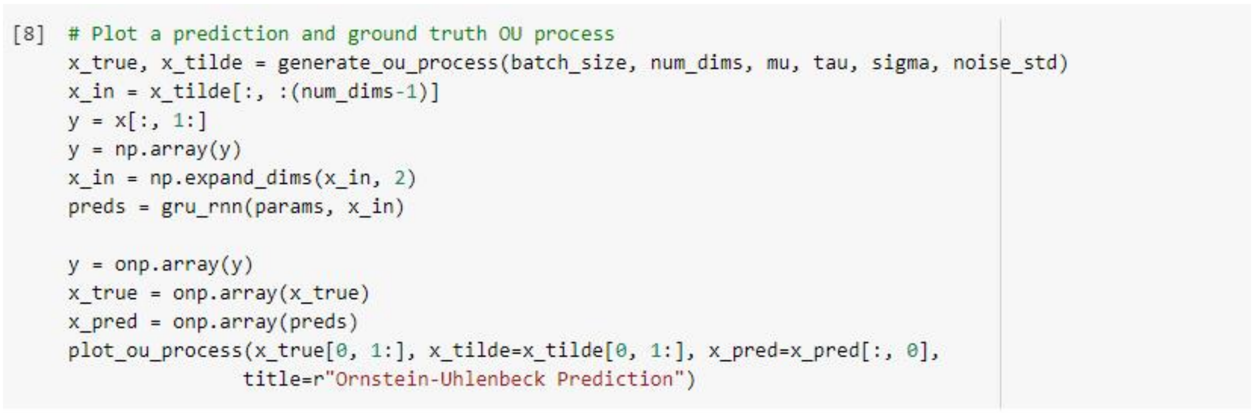
I have tried different step size like 1e-2, 1e-3, 1e-4, 1e-5, 1e-6, and after running the plot_ou_loss function, it turned out that 1e-4 is the best step size for training. Because by looking at the MSE value from the output and the train loss plot, this value gives us convergence performance. Others made the MSE value fluctuating and oscillation, meaning that select bad step size we don't actually trained the RNN model.



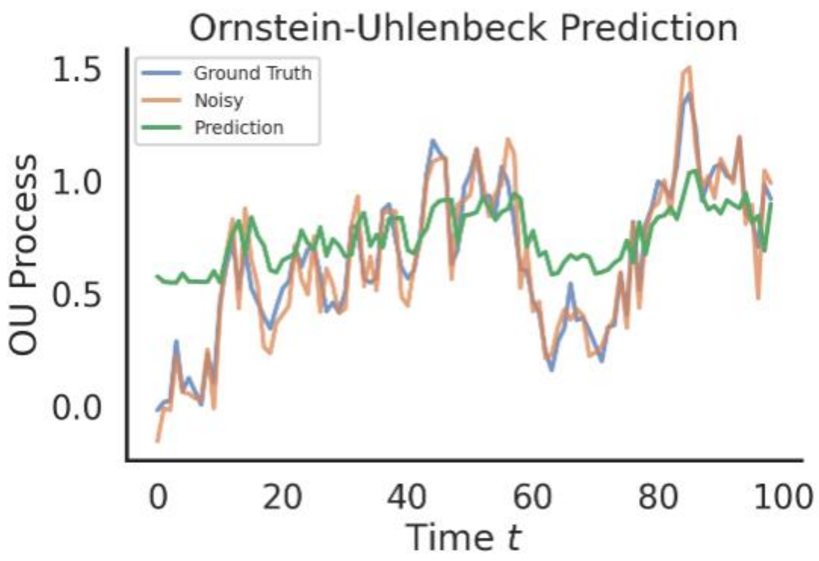
```
[7] plot_ou_loss(train_loss_log)
```

9. (1 point) Using the last cell of the notebook, comment qualitatively on the difference between the predicted time series, the ground truth, and the noisy time series. You will have to reuse the definition of x in (the input) and y from the question above.

Solution:

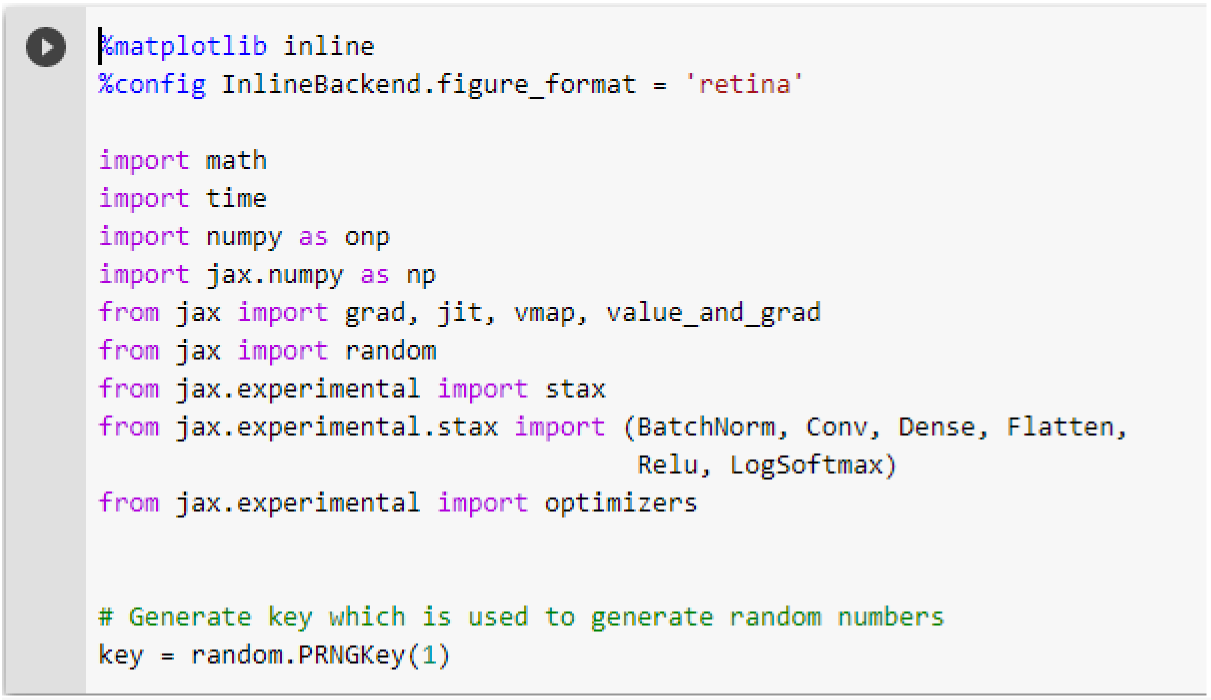


```
[8]  # Plot a prediction and ground truth OU process
     x_true, x_tilde = generate_ou_process(batch_size, num_dims, mu, tau, sigma, noise_std)
     x_in = x_tilde[:, :(num_dims-1)]
     y = x[:, 1:]
     y = np.array(y)
     x_in = np.expand_dims(x_in, 2)
     preds = gru_rnn(params, x_in)

     y = onp.array(y)
     x_true = onp.array(x_true)
     x_pred = onp.array(preds)
     plot_ou_process(x_true[0, 1:], x_tilde=x_tilde[0, 1:], x_pred=x_pred[:, 0],
                     title=r"Ornstein-Uhlenbeck Prediction")
```

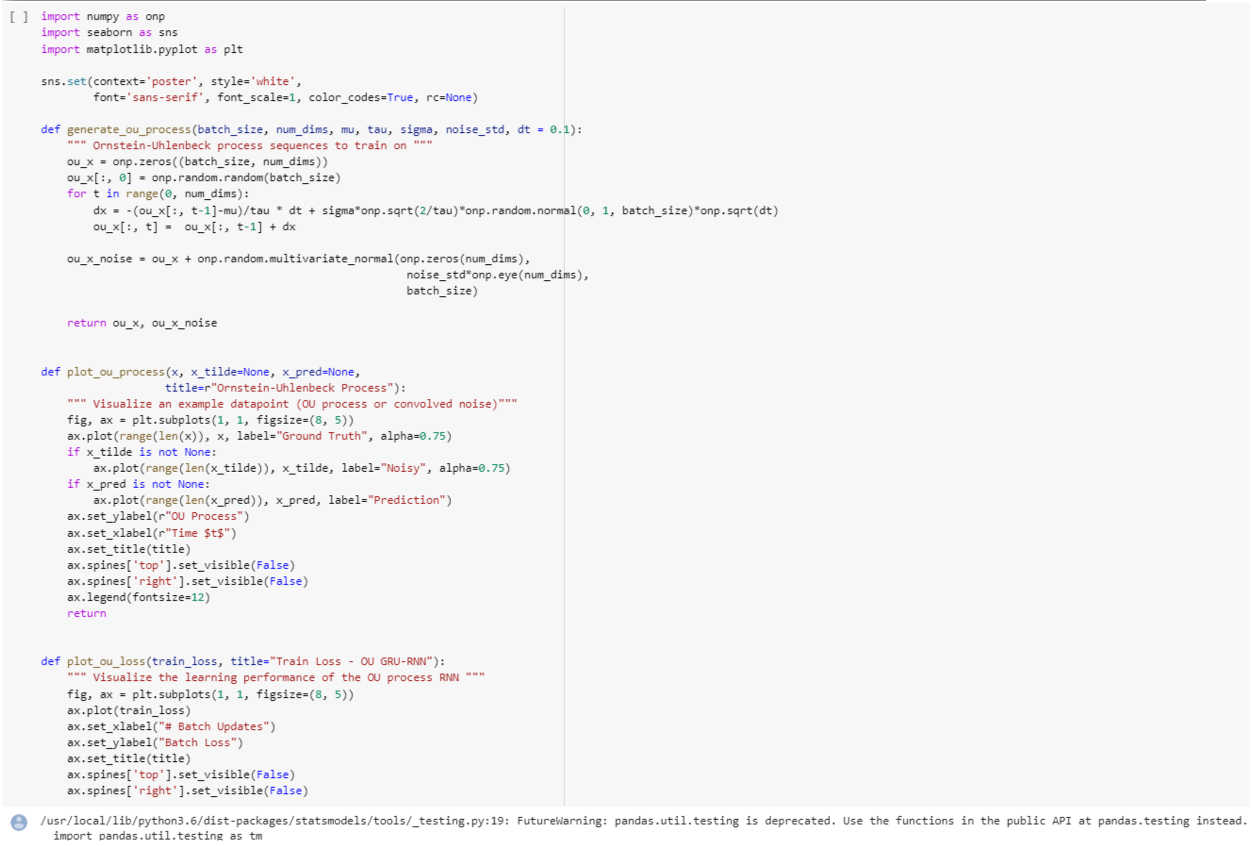


From the plot above, the difference between the predicted time series, the ground truth, and the noisy time series is that predicted time series is smoother than the other two. The predicted line seems to lie in with range mean μ=0.5 to μ=1 which can be considered as desired behavior.

Full code:



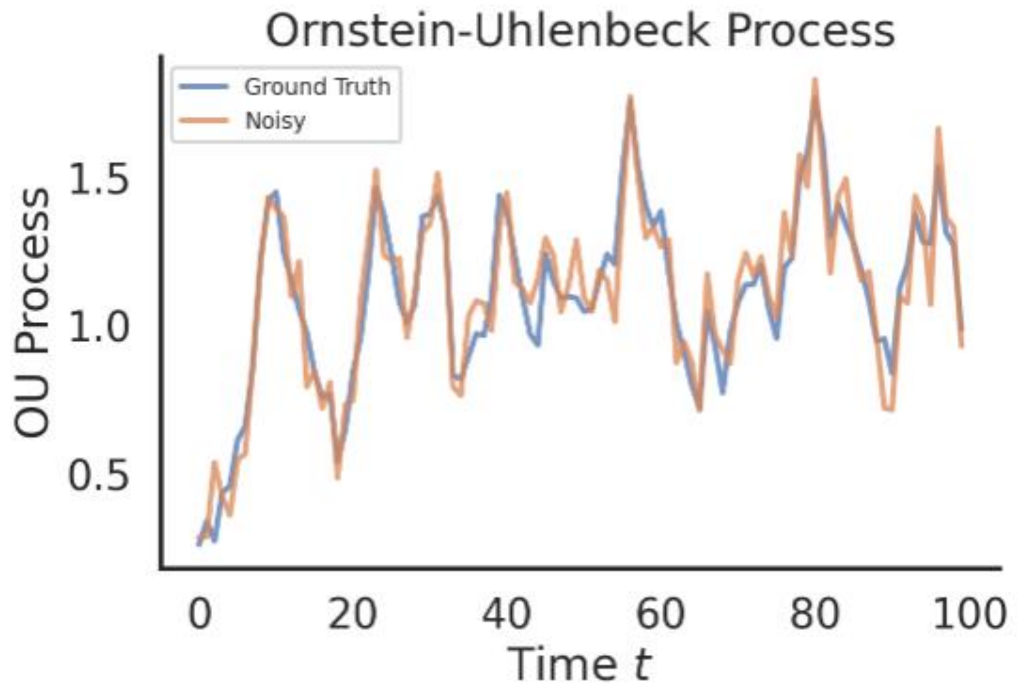
```python
%matplotlib inline
%config InlineBackend.figure_format = 'retina'

import math
import time
import numpy as onp
import jax.numpy as np
from jax import grad, jit, vmap, value_and_grad
from jax import random
from jax.experimental import stax
from jax.experimental.stax import (BatchNorm, Conv, Dense, Flatten,
                                   Relu, LogSoftmax)
from jax.experimental import optimizers


# Generate key which is used to generate random numbers
key = random.PRNGKey(1)
```



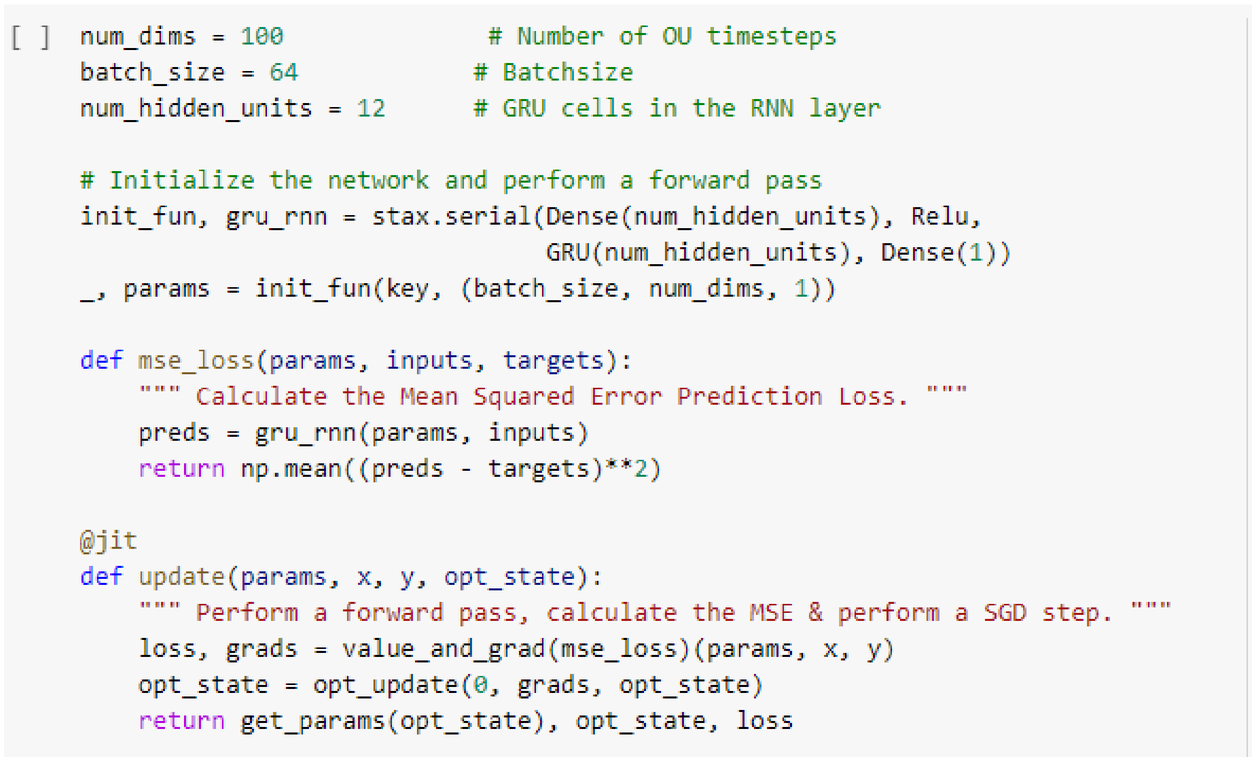
```python
[ ] import numpy as onp
    import seaborn as sns
    import matplotlib.pyplot as plt

    sns.set(context='poster', style='white',
            font='sans-serif', font_scale=1, color_codes=True, rc=None)

    def generate_ou_process(batch_size, num_dims, mu, tau, sigma, noise_std, dt = 0.1):
        """ Ornstein-Uhlenbeck process sequences to train on """
        ou_x = onp.zeros((batch_size, num_dims))
        ou_x[:, 0] = onp.random.random(batch_size)
        for t in range(0, num_dims):
            dx = -(ou_x[:, t-1]-mu)/tau * dt + sigma*onp.sqrt(2/tau)*onp.random.normal(0, 1, batch_size)*onp.sqrt(dt)
            ou_x[:, t] =  ou_x[:, t-1] + dx

        ou_x_noise = ou_x + onp.random.multivariate_normal(onp.zeros(num_dims),
                                                            noise_std*onp.eye(num_dims),
                                                            batch_size)

        return ou_x, ou_x_noise


    def plot_ou_process(x, x_tilde=None, x_pred=None,
                        title=r"Ornstein-Uhlenbeck Process"):
        """ Visualize an example datapoint (OU process or convolved noise)"""
        fig, ax = plt.subplots(1, 1, figsize=(8, 5))
        ax.plot(range(len(x)), x, label="Ground Truth", alpha=0.75)
        if x_tilde is not None:
            ax.plot(range(len(x_tilde)), x_tilde, label="Noisy", alpha=0.75)
        if x_pred is not None:
            ax.plot(range(len(x_pred)), x_pred, label="Prediction")
        ax.set_ylabel(r"OU Process")
        ax.set_xlabel(r"Time $t$")
        ax.set_title(title)
        ax.spines['top'].set_visible(False)
        ax.spines['right'].set_visible(False)
        ax.legend(fontsize=12)
        return


    def plot_ou_loss(train_loss, title="Train Loss - OU GRU-RNN"):
        """ Visualize the learning performance of the OU process RNN """
        fig, ax = plt.subplots(1, 1, figsize=(8, 5))
        ax.plot(train_loss)
        ax.set_xlabel("# Batch Updates")
        ax.set_ylabel("Batch Loss")
        ax.set_title(title)
        ax.spines['top'].set_visible(False)
        ax.spines['right'].set_visible(False)
```

```
/usr/local/lib/python3.6/dist-packages/statsmodels/tools/_testing.py:19: FutureWarning: pandas.util.testing is deprecated. Use the functions in the public API at pandas.testing instead.
  import pandas.util.testing as tm
```

```
[ ] # Generate & plot a time series generated by the OU process
    x_0, mu, tau, sigma, dt = 0, 1, 2, 0.5, 0.1
    noise_std = 0.01
    num_dims, batch_size = 100, 64  # Number of timesteps in process

    x, x_tilde = generate_ou_process(batch_size, num_dims, mu, tau,
                                     sigma, noise_std, dt)
    plot_ou_process(x[0, :], x_tilde[0, :])
```



```python
from jax.nn import sigmoid
from jax.nn.initializers import glorot_normal, normal

from functools import partial
from jax import lax

def GRU(out_dim, W_init=glorot_normal(), b_init=normal()):
    def init_fun(rng, input_shape):
        """ Initialize the GRU layer for stax """
        hidden = b_init(rng, (input_shape[0], out_dim))

        k1, k2, k3 = random.split(rng, num=3)
        update_W, update_U, update_b = (
            W_init(k1, (input_shape[2], out_dim)),
            W_init(k2, (out_dim, out_dim)),
            b_init(k3, (out_dim,)),)

        k1, k2, k3 = random.split(rng, num=3)
        reset_W, reset_U, reset_b = (
            W_init(k1, (input_shape[2], out_dim)),
            W_init(k2, (out_dim, out_dim)),
            b_init(k3, (out_dim,)),)

        k1, k2, k3 = random.split(rng, num=3)
        out_W, out_U, out_b = (
            W_init(k1, (input_shape[2], out_dim)),
            W_init(k2, (out_dim, out_dim)),
            b_init(k3, (out_dim,)),)
        # Input dim 0 represents the batch dimension
        # Input dim 1 represents the time dimension (before scan moveaxis)
        output_shape = (input_shape[0], input_shape[1], out_dim)
        return (output_shape,
            (hidden,
             (update_W, update_U, update_b),
             (reset_W, reset_U, reset_b),
             (out_W, out_U, out_b),),)

    def apply_fun(params, inputs, **kwargs):
        """ Loop over the time steps of the input sequence """
        h = params[0]

        def apply_fun_scan(params, hidden, inp):
            """ Perform single step update of the network """
            _, (update_W, update_U, update_b), (reset_W, reset_U, reset_b), (
                out_W, out_U, out_b) = params


            update_gate = sigmoid(np.dot(inp, update_W) +np.dot(hidden, update_U) + update_b)
            reset_gate = sigmoid(np.dot(inp, reset_W) +np.dot(hidden, reset_U) + reset_b)
            output_gate = np.tanh(np.dot(inp, out_W)+ np.dot(np.multiply(reset_gate, hidden), out_U)+ out_b)
            return hidden, output_gate

        # Move the time dimension to position 0 so lax.scan can loop over time
        inputs = np.moveaxis(inputs, 1, 0)
        f = partial(apply_fun_scan, params)
        _, out = lax.scan(f, h, inputs)
        return out

    return init_fun, apply_fun
```



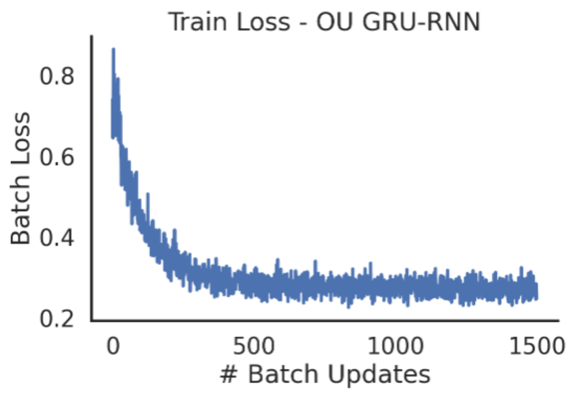
```
[ ] num_dims = 100              # Number of OU timesteps
    batch_size = 64            # Batchsize
    num_hidden_units = 12      # GRU cells in the RNN layer

    # Initialize the network and perform a forward pass
    init_fun, gru_rnn = stax.serial(Dense(num_hidden_units), Relu,
                                    GRU(num_hidden_units), Dense(1))
    _, params = init_fun(key, (batch_size, num_dims, 1))

    def mse_loss(params, inputs, targets):
        """ Calculate the Mean Squared Error Prediction Loss. """
        preds = gru_rnn(params, inputs)
        return np.mean((preds - targets)**2)

    @jit
    def update(params, x, y, opt_state):
        """ Perform a forward pass, calculate the MSE & perform a SGD step. """
        loss, grads = value_and_grad(mse_loss)(params, x, y)
        opt_state = opt_update(0, grads, opt_state)
        return get_params(opt_state), opt_state, loss
```

Training the RNN

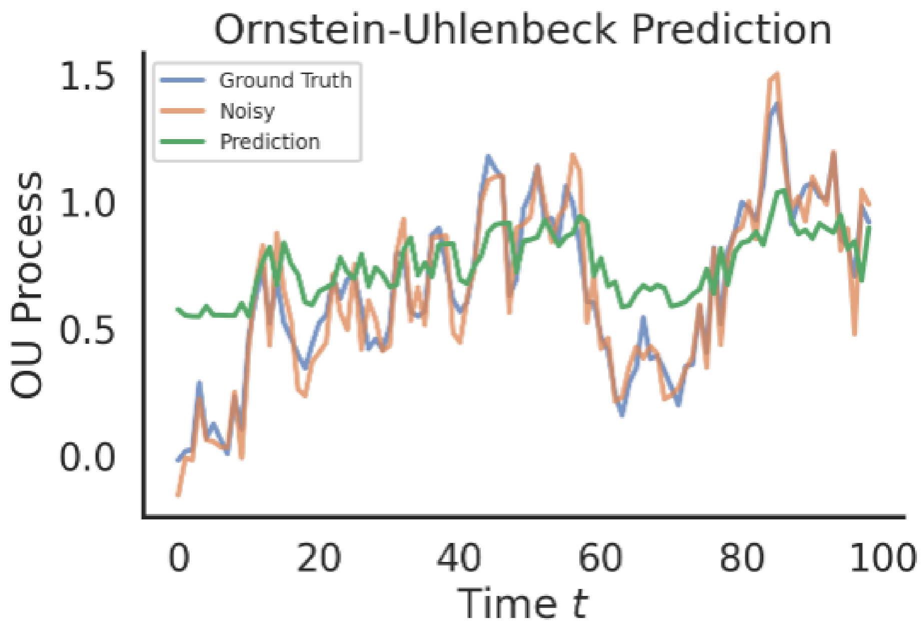
```
[ ]  step_size = 1e-4
     opt_init, opt_update, get_params = optimizers.adam(step_size)


     opt_state = opt_init(params)
     num_batches = 1500
     train_loss_log = []
     start_time = time.time()

     for batch_idx in range(num_batches):
         x, x_tilde = generate_ou_process(batch_size, num_dims, mu, tau, sigma, noise_std)
         x_in = x_tilde[:, :(num_dims-1)]
         y = x[:, 1:]
         y = np.array(y)
         x_in = np.expand_dims(x_in, 2)
         params, opt_state, loss = update(params, x_in, y, opt_state)
         batch_time = time.time() - start_time
         train_loss_log.append(loss)

         if batch_idx % 100 == 0:
             start_time = time.time()
             print("Batch {} | T: {:0.2f} | MSE: {:0.2f} |".format(batch_idx, batch_time, loss))
```

```
Batch 0 | T: 1.97 | MSE: 0.74 |
Batch 100 | T: 3.05 | MSE: 0.44 |
Batch 200 | T: 3.03 | MSE: 0.33 |
Batch 300 | T: 3.04 | MSE: 0.31 |
Batch 400 | T: 3.04 | MSE: 0.31 |
Batch 500 | T: 3.01 | MSE: 0.28 |
Batch 600 | T: 3.09 | MSE: 0.29 |
Batch 700 | T: 3.06 | MSE: 0.28 |
Batch 800 | T: 2.99 | MSE: 0.27 |
Batch 900 | T: 3.00 | MSE: 0.31 |
Batch 1000 | T: 3.04 | MSE: 0.26 |
Batch 1100 | T: 3.00 | MSE: 0.28 |
Batch 1200 | T: 2.96 | MSE: 0.27 |
Batch 1300 | T: 3.00 | MSE: 0.31 |
Batch 1400 | T: 3.00 | MSE: 0.27 |
```

```
[ ]  plot_ou_loss(train_loss_log)
```



```
[ ] # Plot a prediction and ground truth OU process
    x_true, x_tilde = generate_ou_process(batch_size, num_dims, mu, tau, sigma, noise_std)
    x_in = x_tilde[:, :(num_dims-1)]
    y = x[:, 1:]
    y = np.array(y)
    x_in = np.expand_dims(x_in, 2)
    preds = gru_rnn(params, x_in)

    y = onp.array(y)
    x_true = onp.array(x_true)
    x_pred = onp.array(preds)
    plot_ou_process(x_true[0, 1:], x_tilde=x_tilde[0, 1:], x_pred=x_pred[:, 0],
                    title=r"Ornstein-Uhlenbeck Prediction")
```


Ornstein-Uhlenbeck Prediction
Ground Truth
Noisy
Prediction
OU Process
Time t