Validating and Challenging No-go Theorems in Extra-Dimensional Theories of Cosmic Expansion

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Abstract: Explaining the accelerated expansion of the universe and inflation using extra-dimensional theories presents significant challenges. In this work, we employ the latest observational data on dark energy, the Newton constant, and the fine structure constant to test the no-go theorems of KK compactification and non-compactification models. Our results indicate that these no-go theorems are still valid for most models. However, these theorems are based on several assumptions that are not strictly valid and thus do not apply to more general models. We argue that when these assumptions are relaxed, the no-go theorems can no longer be derived.

Background

Previous studies have identified constraints imposed by the existence of dark energy and inflation for two overlapping classes of extra dimensional theories:

- **Metric-based constraints**
  There constraints are mathematically rigorous, derived from the metric to obtain the NEC equations

$$ds^2 = e^{2\Omega(t,y)} g^{FRW}_{\mu\nu}(t,x)dx^\mu dx^\nu + e^{-2\Omega(t,y)} h^{RF}_{\alpha\beta}(t,y)dy^\alpha dy^\beta \qquad (1)$$

The rate of change of h, the A-average and ζ_A must be defined.

$$\frac{1}{2}\frac{dh_{\alpha\beta}}{dt} = \frac{1}{k}\xi h_{\alpha\beta} + \sigma_{\alpha\beta}\,, \quad \langle Q\rangle_A = \frac{\int Q e^{A\Omega}\sqrt{g}d^ky}{\int e^{A\Omega}\sqrt{g}d^ky}\,, \quad \zeta_A = \frac{1}{H}\int e^{A\Omega}\xi\sqrt{h}d^ky \qquad (2)$$

NEC can be written in the following form, where (3) denotes NEC on large dimensions, and (4) denotes on extra dimensions.

$$\zeta_A^2 \leq \frac{6k(1+w)}{k+2} \qquad (3)$$

$$\frac{d\zeta_A}{dN} \geq \zeta_A^2 + \frac{3(w-1)}{2}\zeta_A - \frac{3k(1+3w)}{k+2} - \frac{1}{H^2}B.T - C(\partial\Omega)^2 \qquad (4)$$

- **Observation constraints**
  Observation constraints provide initial values for the NEC equations

$$\frac{\dot{G}}{G} \sim \frac{\dot{\alpha}}{\alpha} = H\zeta(N=0) \sim 10^{13}\mathrm{yr}^{-1}\,, \qquad (5)$$

dark energy density parameter $\Omega_\phi \sim 0.7$ and the EoS of dark energy.

Methods

We assume the evolution of dark energy follows the quintessence model and systematically examine points on the $w_0 w_a$ plane. Suitable points should satisfy at least the following conditions:

- Eq.(3) and (4) are obeyed.It means satisfying NEC both on large and extra dimensions.
- $w_\phi \leq 0$ as $a \sim 0$, so the universe is sure to be matter and radiation dominated at very early times
- $-1 \leq w_\phi \leq 1$, for all a.

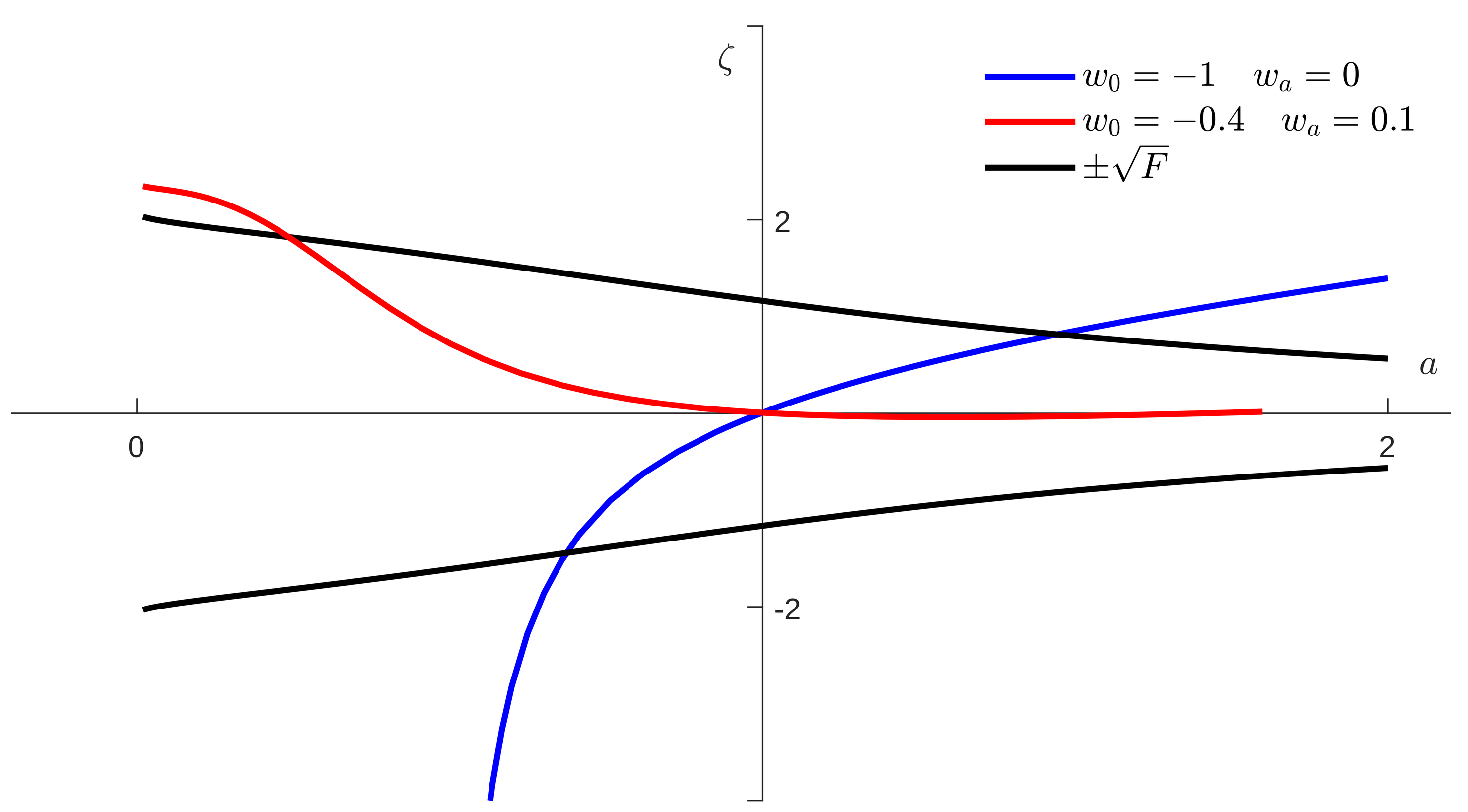

Figure 1:Considering $k=6$ and $\zeta_0=0.001$, we provide two examples: one satisfies the NEC equations on large and extra dimensions, while the other does not.

Comparing the regions that satisfy these conditions with the DESI 2024 dark energy observations.

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Results

- **KK-compactification**
  In KK compactification models, where the geometry of the extra dimensions is S^1 and the boundary term is zero, comparing the region satisfying the aforementioned conditions with the regions indicated by dark energy observations from DESI+BAO, we find that even with a lenient allowance of $\Omega_\phi \sim 0.6$, the two regions do not overlap.

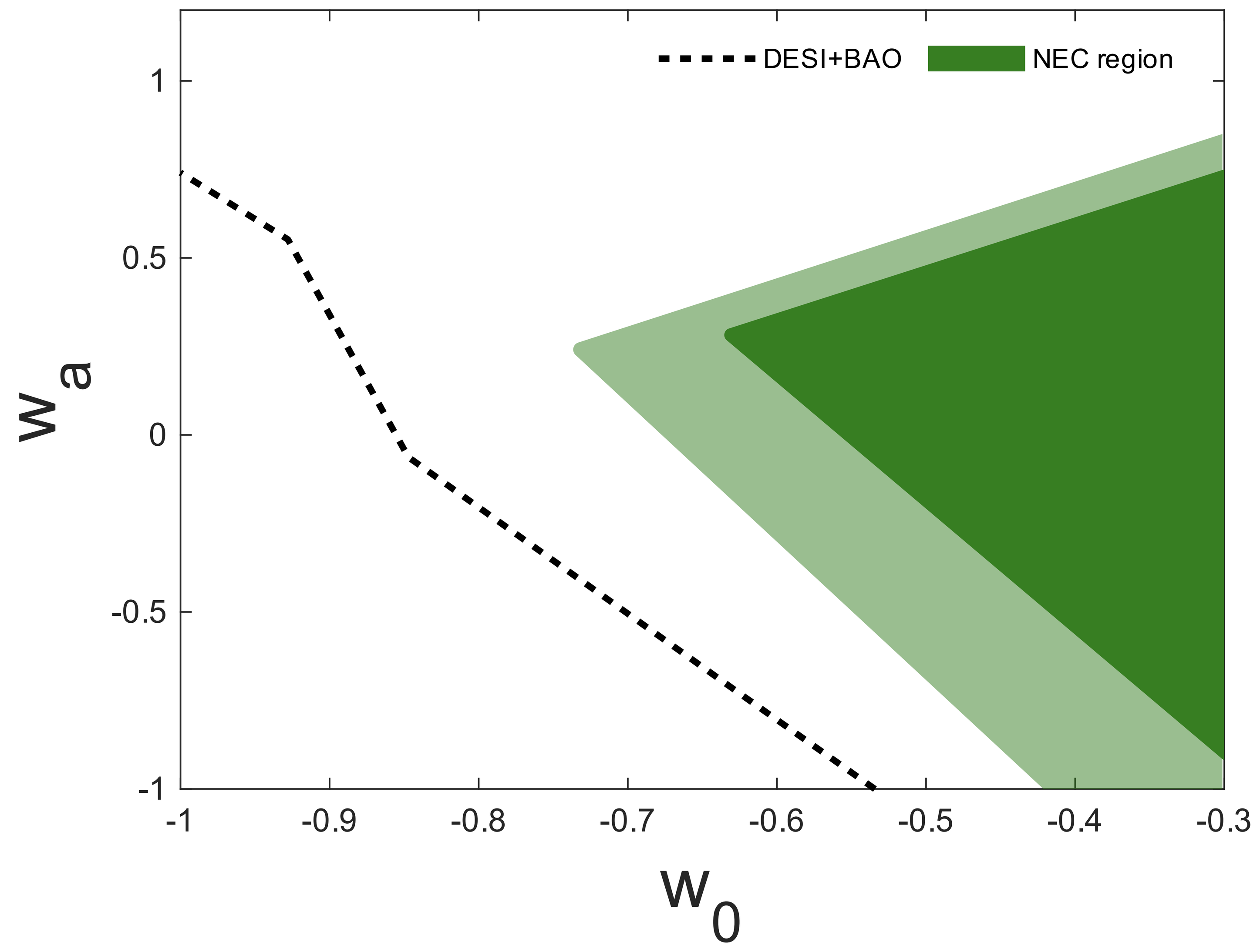

Figure 2: Considering $k=6$ and $\zeta_0 = 0.001$, regions are identified with $\Omega_\phi = 0.7$ and 0.6. These regions do not overlap with DESI observation, suggesting that under there theoretical assumptions, extra dimensions are not viable for explaining accelerated expansion of universe.

- **Non-compact** Boundary term cannot be considered zero, a universal no-go theorem cannot be established. For example, in the RS models, it is still possible to disregard B.T and $(\partial\Omega)^2$. However, in the football-shaped model, boundary terms render the no-go theorem no longer applicable.

Discussion

The above results are based on certain assumptions. Some assumptions are applicable to the vast majority of extra dimension models, but they may not apply to certain models, such as 1)-2). Some assumptions are considered imposed, for instance 4)-5).

- 1) Extra dimensions described by general relativity.
- 2) Higher dimensional manifold is RF or CRF.
- 3) Consider dynamic dark energy.
- 4) Warp factor does not change over time.
- 5) Select canonical average, A=2.
- 6) Assumptions associated with specific models.

Each assumption has models that do not conform to it, thus the derivation of a no-go theorem for extra dimensions still cannot universally apply to all models, including both compact and non-compact ones.

References

[1] A. Adame et al. *arXiv:2404.03002*.
[2] D.H.Wesley. *JCAP*. doi: 10.1088/1475-7516/2009/01/041.
[3] G.Montefalcone. *JHEP,arXiv:2005.01143*.
[4] R. Koster. *JCAP*. doi: 10.1088/1475-7516/2011/12/015.