

# Announcements

- **Class Website:**

<http://www.teach.cs.toronto.edu/~csc420h/fall/>

- Website link is on Piazza

- **Tutorial:**

First Tutorial on Thursday, Sep 14, at 4pm (Matlab for Images) - Jake

- **Waitlist....:**

# Syllabus Overview

## Image Processing

- Linear Filters
- Edge Detection
- Image Pyramids

## Features & Matching

- KeyPoint Detection
- Scale Invariance
- Local Descriptors

## Geometry

- Camera Models
- Stereo Vision

## Recognition and Detection

- Object Recognition
- Neural Nets
- Deep Learning

- Shape Models
- HoG detectors
- Deformable Parts

- Segmentation

# Images



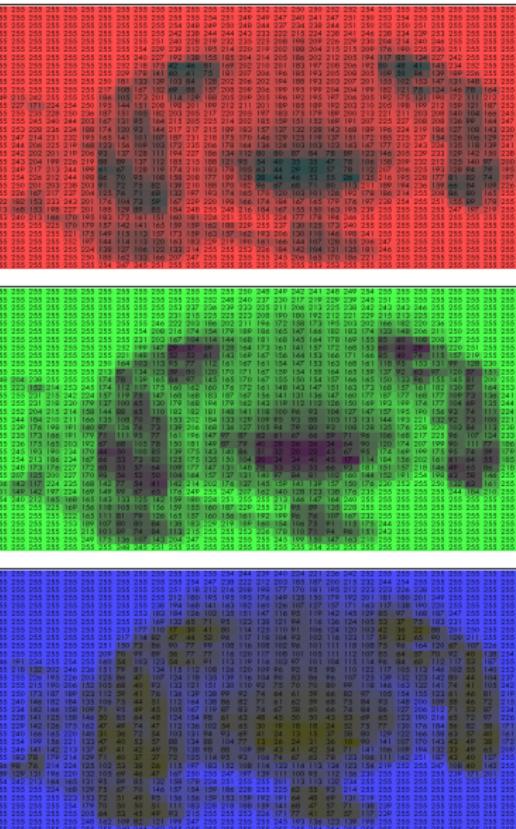




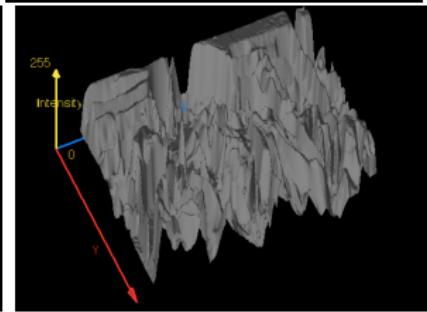
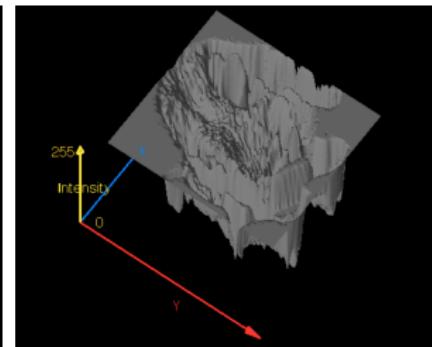
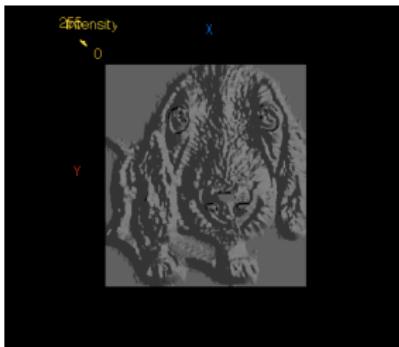


# Digital Image

- Image is a matrix with integer values
- We will typically denote it with  $I$
- $I(i,j)$  is called **intensity**
- Matrix  $I$  can be  $m \times n$  (grayscale)
- or  $m \times n \times 3$  (colour)



# Intensity



- We can think of a (grayscale) image as a function  $f : \mathbb{R}^2 \rightarrow \mathbb{R}$  giving the intensity at position  $(i, j)$
- Intensity 0 is black and 255 is white

# Image Transformations

- As with any function, we can apply operators to an image, e.g.:



$$I(i, j)$$

$$J(i, j) = I(i, j) + 50$$

- We'll talk about special kinds of operators, **correlation** and **convolution** (linear filtering)

[Adapted from: N. Snavely]

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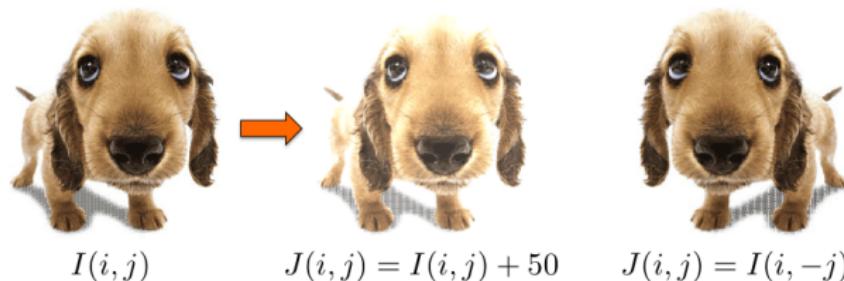
$$J(i, j) = I(i, -j)$$

- We'll talk about special kinds of operators, **correlation** and **convolution** (linear filtering)

[Adapted from: N. Snavely]

# Image Transformations

- As with any function, we can apply operators to an image, e.g.:



- We'll talk about special kinds of operators, **correlation** and **convolution** (linear filtering)

[Adapted from: N. Snavely]

# Linear Filters

Reading: Szeliski book, Chapter 3.2

# Motivation: Finding Waldo

- How can we find Waldo?



[Source: R. Urtasun]

# Answer

- Slide and compare!
- In formal language: **filtering**

# Motivation: Noise reduction

- Given a camera and a still scene, how can you reduce noise?



[Source: S. Seitz]

# Image Filtering

- Modify the pixels in an image based on some function of a local neighborhood of each pixel
- In other words... Filtering

10	5	3
4	5	1
1	1	7

Local image data

Some function



	7	

Modified image data

[Source: L. Zhang]

# Applications of Filtering

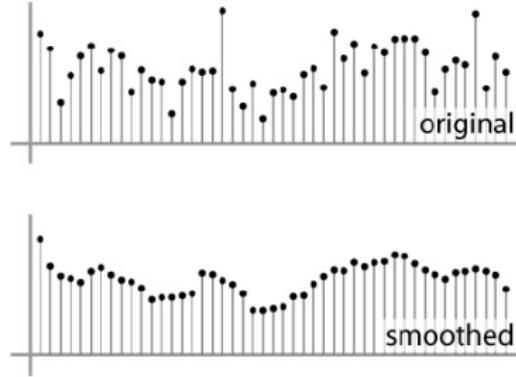
- Enhance an image, e.g., **denoise**.
- Detect patterns, e.g., **template matching**.
- Extract information, e.g., **texture, edges**.

# Applications of Filtering

- Enhance an image, e.g., **denoise**. Let's talk about this first
- Detect patterns, e.g., **template matching**.
- Extract information, e.g., **texture, edges**.

# Noise reduction

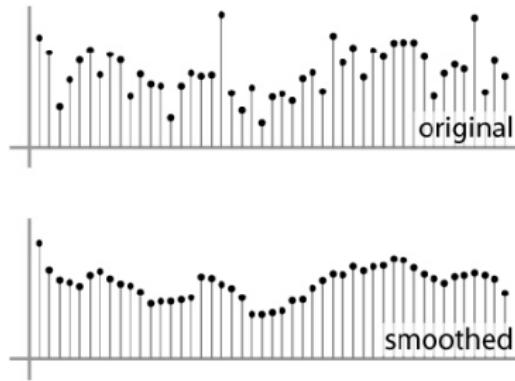
- Simplest thing: replace each pixel by the average of its neighbors.
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.



[Source: S. Marschner]

# Noise reduction

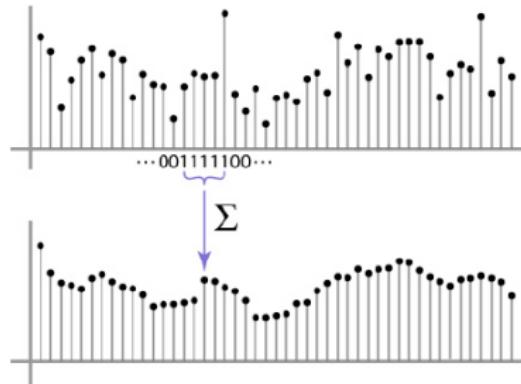
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# Noise reduction

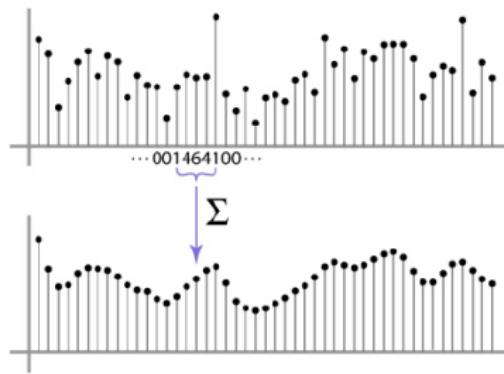
- Simplest thing: replace each pixel by the average of its neighbors
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- **Moving average** in 1D:  $[1, 1, 1, 1, 1]/5$



[Source: S. Marschner]

# Noise reduction

- Simplest thing: replace each pixel by the average of its neighbors
- This assumes that neighboring pixels are similar, and the noise to be independent from pixel to pixel.
- Non-uniform weights  $[1, 4, 6, 4, 1] / 16$



[Source: S. Marschner]

# Moving Average in 2D

$I(i, j)$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$G(i, j)$

0									

[Source: S. Seitz]

# Moving Average in 2D

$$I(i, j)$$

0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

0	10										

[Source: S. Seitz]

# Moving Average in 2D

$$I(i, j)$$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

0	10	20								

[Source: S. Seitz]

# Moving Average in 2D

$$I(i, j)$$

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

0	10	20	30						

[Source: S. Seitz]

# Moving Average in 2D

 $I(i, j)$ 

0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	90	0	90	90	90	0	0
0	0	0	90	90	90	90	90	0	0
0	0	0	0	0	0	0	0	0	0
0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0

 $G(i, j)$ 

0	10	20	30	30
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0
0	0	0	0	0

[Source: S. Seitz]

# Moving Average in 2D

$$I(i, j)$$

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

$$G(i, j)$$

	0	10	20	30	30	30	20	10		
	0	20	40	60	60	60	40	20		
	0	30	60	90	90	90	60	30		
	0	30	50	80	80	90	60	30		
	0	30	50	80	80	90	60	30		
	0	20	30	50	50	60	40	20		
	10	20	30	30	30	30	20	10		
	10	10	10	0	0	0	0	0		

[Source: S. Seitz]

## Linear Filtering: Correlation

- Involves weighted combinations of pixels in small neighborhoods:

$$G(i, j) = \frac{1}{(2k+1)^2} \sum_{u=-k}^k \sum_{v=-k}^k I(i+u, j+v)$$

- The output pixels value is determined as a weighted sum of input pixel values

$$G(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k F(u, v) \cdot I(i+u, j+v)$$

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- This operator is the **correlation** operator

$$G = F \otimes I$$

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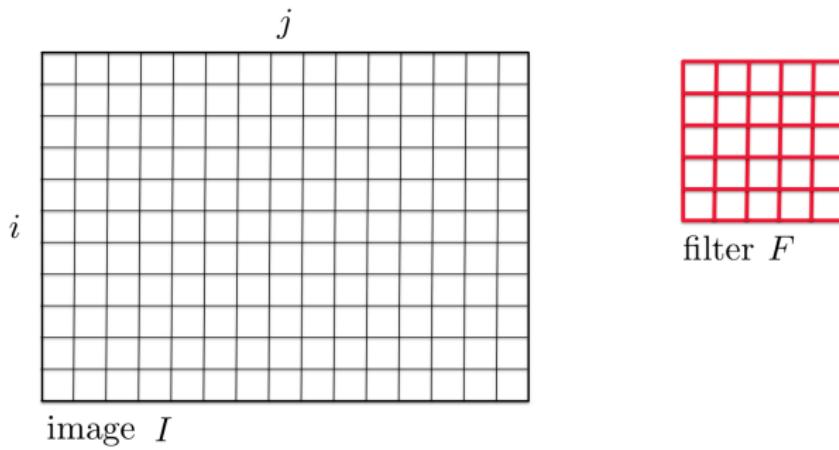
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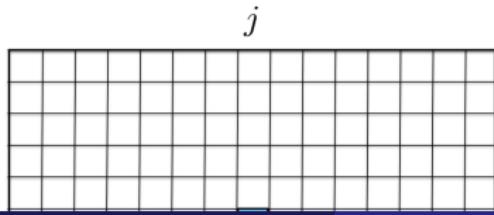
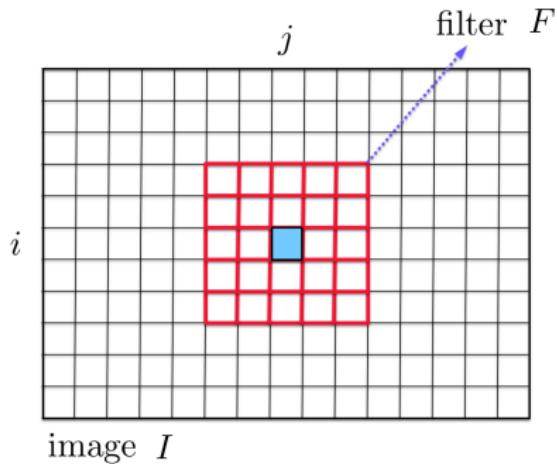
# Linear Filtering: Correlation

- It's really easy!



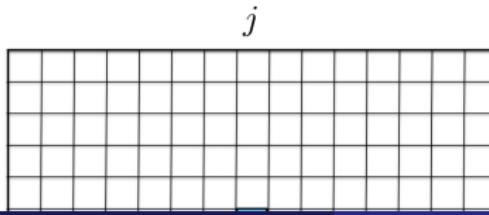
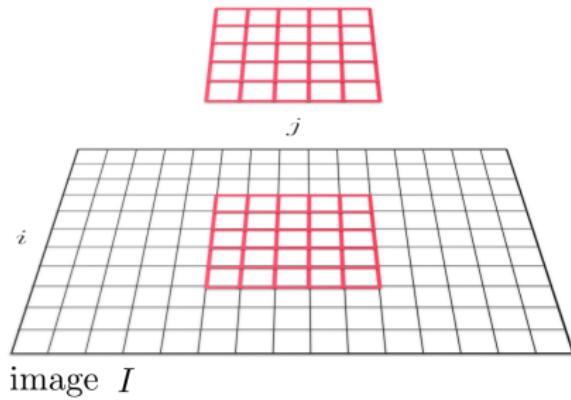
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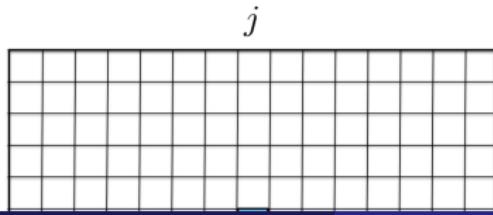
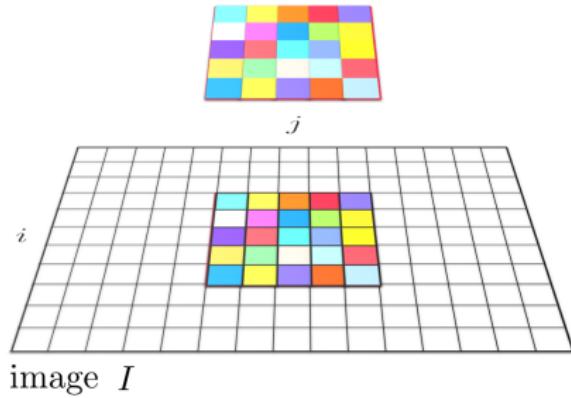
# Linear Filtering: Correlation

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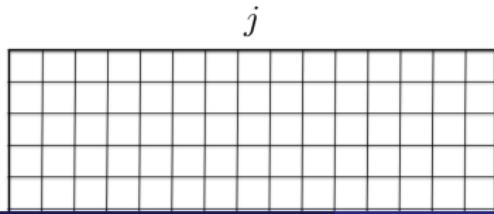
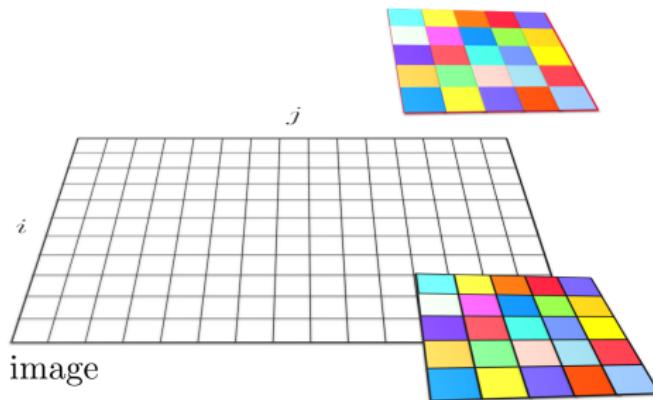
# Linear Filtering: Correlation

- It's really easy!



# Linear Filtering: Correlation

- What happens along the borders of the image?



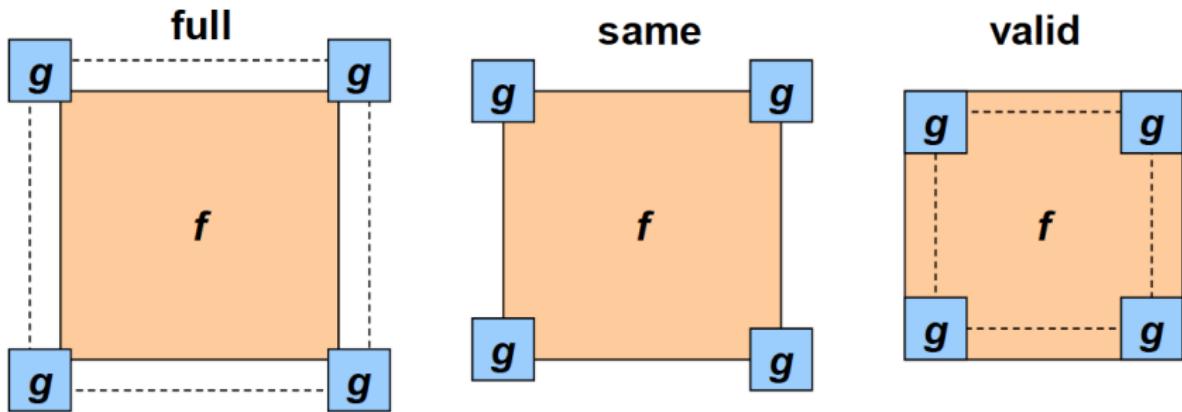
# Boundary Effects

- What happens at the border of the image? What's the size of the output matrix?
- MATLAB: `FILTER2(G, F, SHAPE)`
- `shape = "full"`: output size is bigger than the image
- `shape = "same"`: output size is same as  $f$
- `shape = "valid"`: output size is smaller than the image

[Source: S. Lazebnik]

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[Source: S. Lazebnik]

## Filtering with Correlation: Example

- What's the result?



0	0	0
0	1	0
0	0	0

?

[Source: D. Lowe]

## Filtering with Correlation: Example

- What's the result?



**Original**

0	0	0
0	1	0
0	0	0



**Filtered  
(no change)**

[Source: D. Lowe]

## Filtering with Correlation: Example

- What's the result?



0	0	0
0	0	1
0	0	0

?

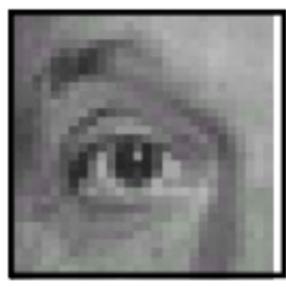
[Source: D. Lowe]

## Filtering with Correlation: Example

- What's the result?



0	0	0
0	0	1
0	0	0



[Source: D. Lowe]

# Filtering with Correlation: Example

- What's the result?


$$\text{Original} \quad * \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{array} - \frac{1}{9} \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right) =$$

[Source: D. Lowe]

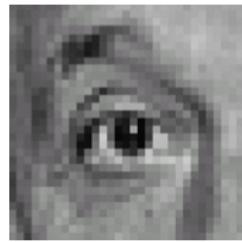
# Filtering with Correlation: Example

- What's the result?



Original

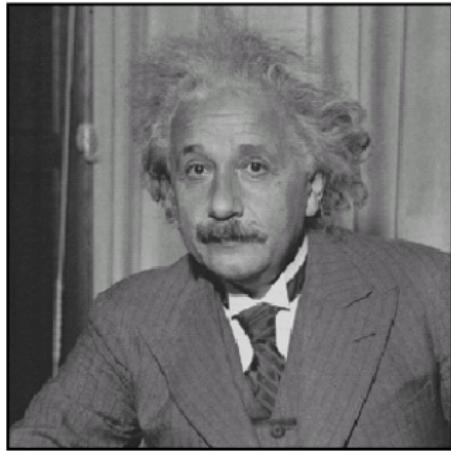
$$\text{Original} * \left( \begin{array}{ccc} 0 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 0 \end{array} - \frac{1}{9} \begin{array}{ccc} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{array} \right) =$$



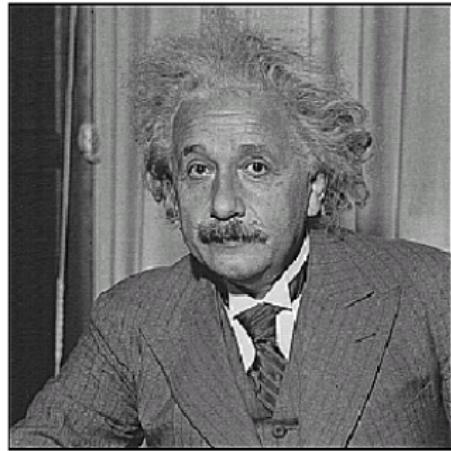
Sharpening filter  
(accentuates edges)

[Source: D. Lowe]

# Sharpening



before



after

This is a prelude to edge detection (next time)! [Source: D. Lowe]

# Sharpening



[Source: N. Snavely]

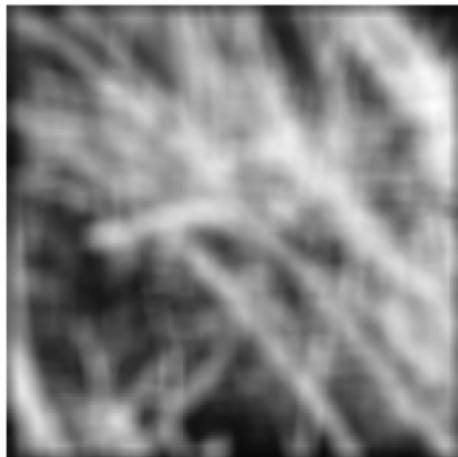
# Smoothing by averaging



depicts box filter:  
white = high value, black = low value



original



filtered

- What if the filter size was  $5 \times 5$  instead of  $3 \times 3$ ?

[Source: K. Graumann]

# Gaussian filter

- What if we want nearest neighboring pixels to have the most influence on the output?
- Removes high-frequency components from the image (low-pass filter).

0	0	0	0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	90	0	90	90	90	0	0	0
0	0	0	90	90	90	90	90	0	0	0
0	0	0	0	0	0	0	0	0	0	0
0	0	0	90	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0	0	0	0

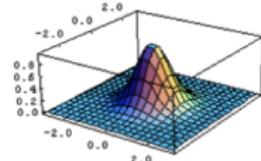
$$I(i, j)$$

$$\frac{1}{16} \begin{matrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{matrix}$$

$$F(i, j)$$

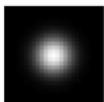
This kernel is an approximation of a 2d Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



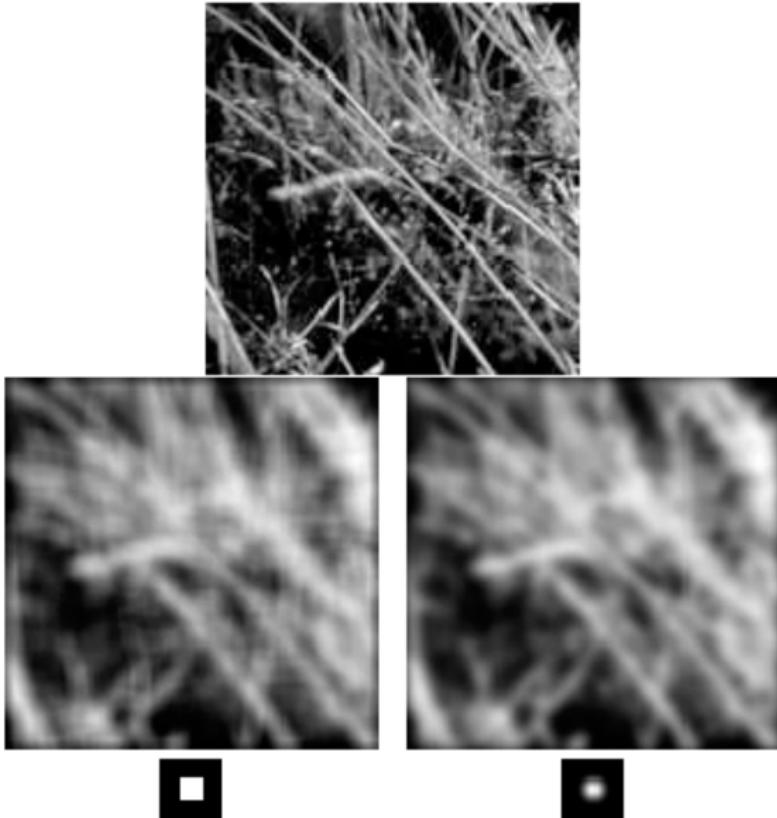
[Source: S. Seitz]

# Smoothing with a Gaussian



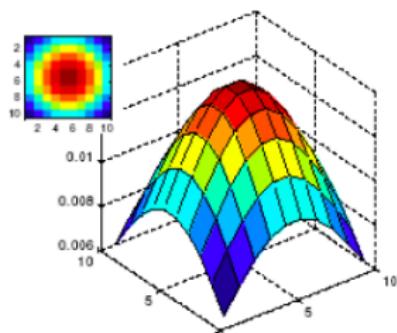
[Source: K. Grauman]

# Mean vs Gaussian

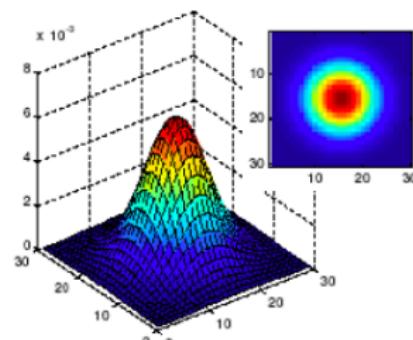


# Gaussian filter: Parameters

- **Size of filter or mask:** Gaussian function has infinite support, but discrete filters use finite kernels.



$\sigma = 5$  with  
10 x 10  
kernel

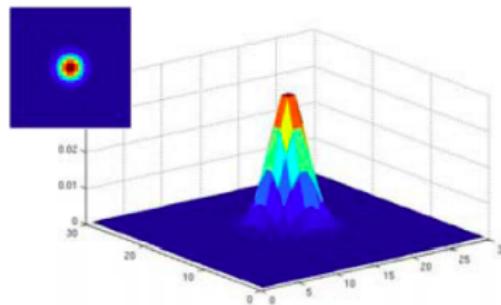


$\sigma = 5$  with  
30 x 30  
kernel

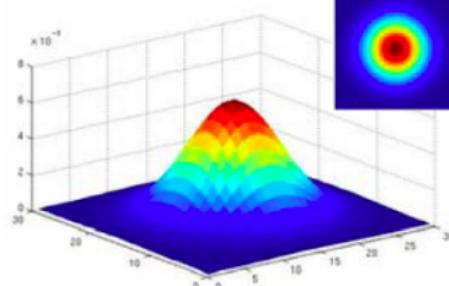
[Source: K. Grauman]

# Gaussian filter: Parameters

- **Variance of the Gaussian:** determines extent of smoothing.



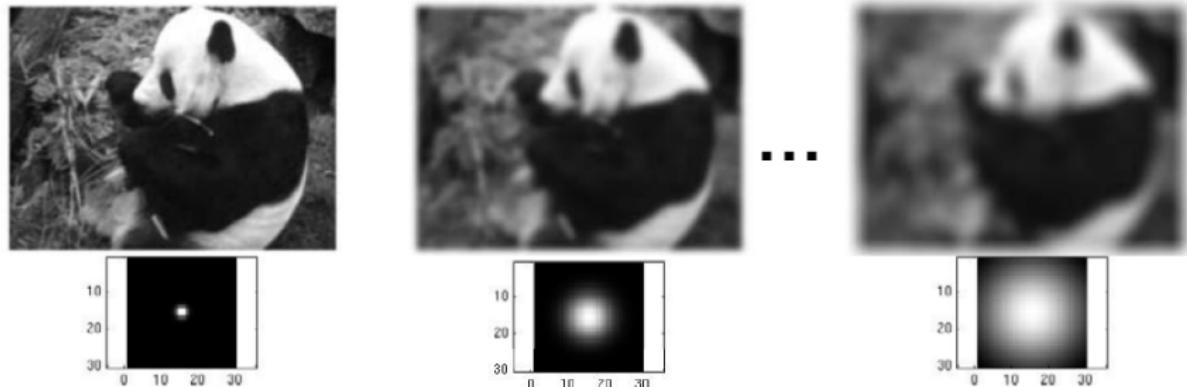
$\sigma = 2$  with  
30 x 30  
kernel



$\sigma = 5$  with  
30 x 30  
kernel

[Source: K. Grauman]

# Gaussian filter: Parameters



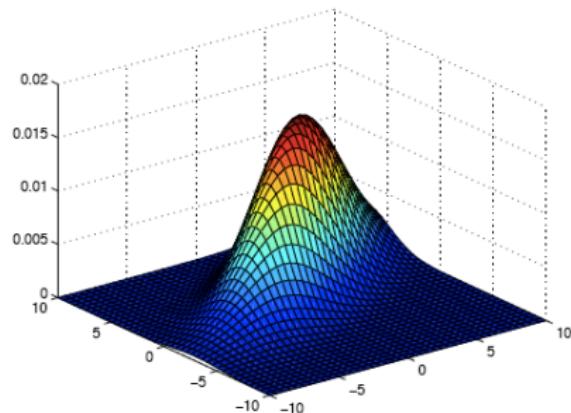
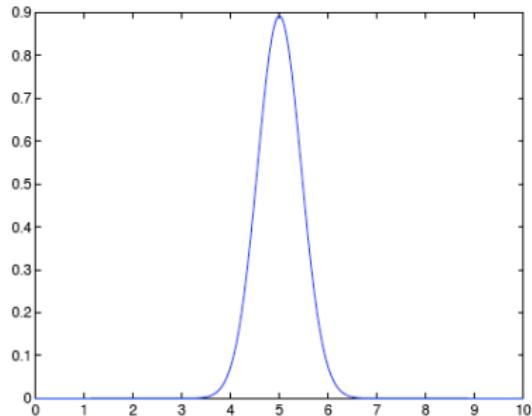
```
for sigma=1:3:10
    h = fspecial('gaussian', fsize, sigma);
    out = imfilter(im, h);
    imshow(out);
    pause;
end
```

[Source: K. Grauman]

# Is this the most general Gaussian?

- No, the most general form is anisotropic (i.e not symmetric)  $\mathbf{x} \in \Re^d$

$$\mathcal{N}(\mathbf{x}; \mu, \Sigma) = \frac{1}{(2\pi)^{d/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}(\mathbf{x} - \mu)^T \Sigma^{-1} (\mathbf{x} - \mu)\right)$$



- But the simplified version is typically used for filtering.

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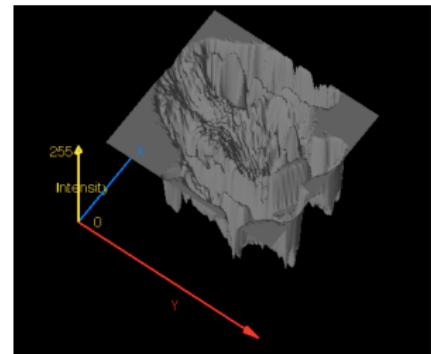
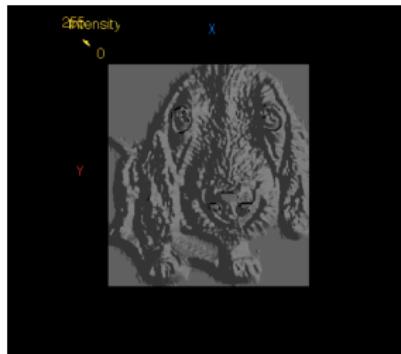
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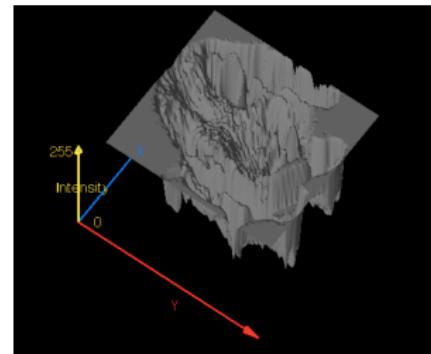
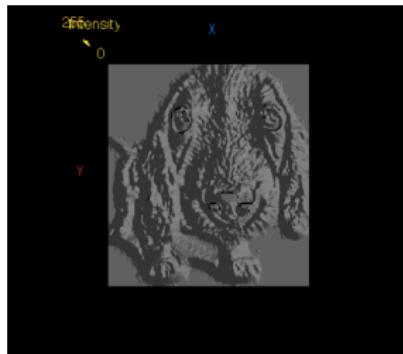
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# Finding Waldo



image /

- How can we use what we just learned to find Waldo?

# Finding Waldo



image  $I$

filter  $F$

- Is correlation a good choice?

## A Slight Detour: Correlation in Matrix Form

- Remember correlation:

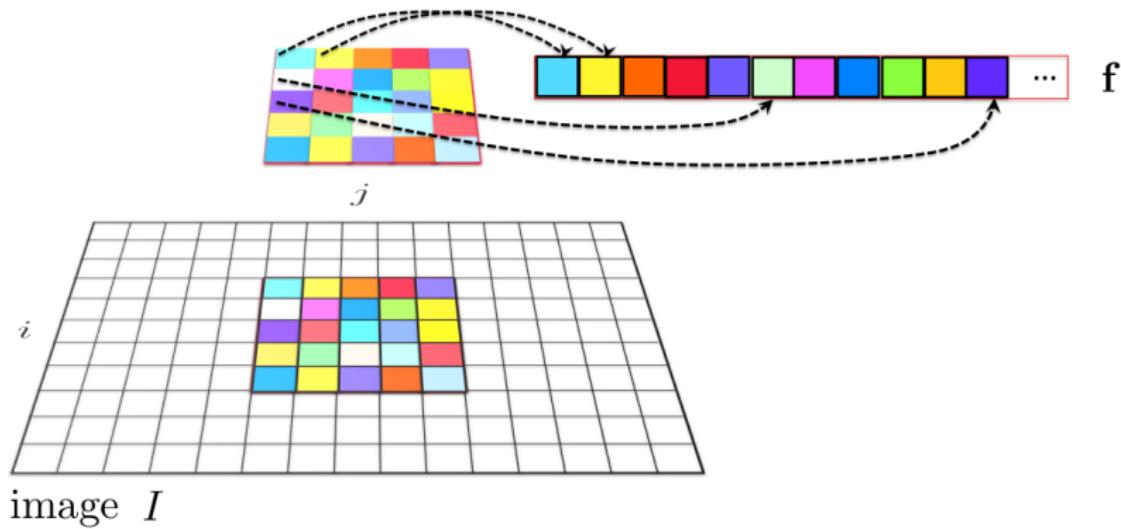
$$G(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k F(u, v) \cdot I(i + u, j + v)$$

- Can we write that in a more compact form (with vectors)?

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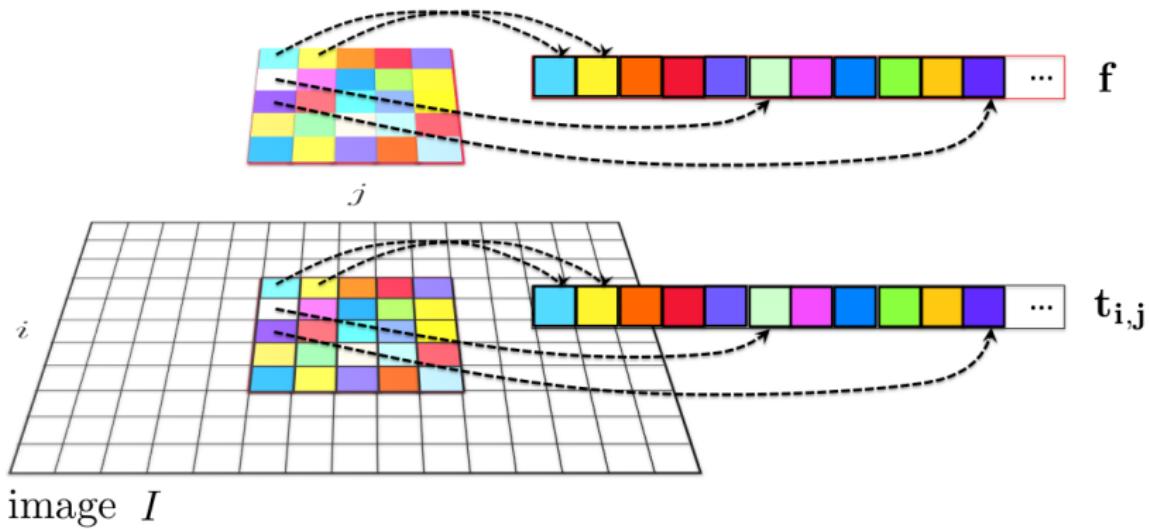
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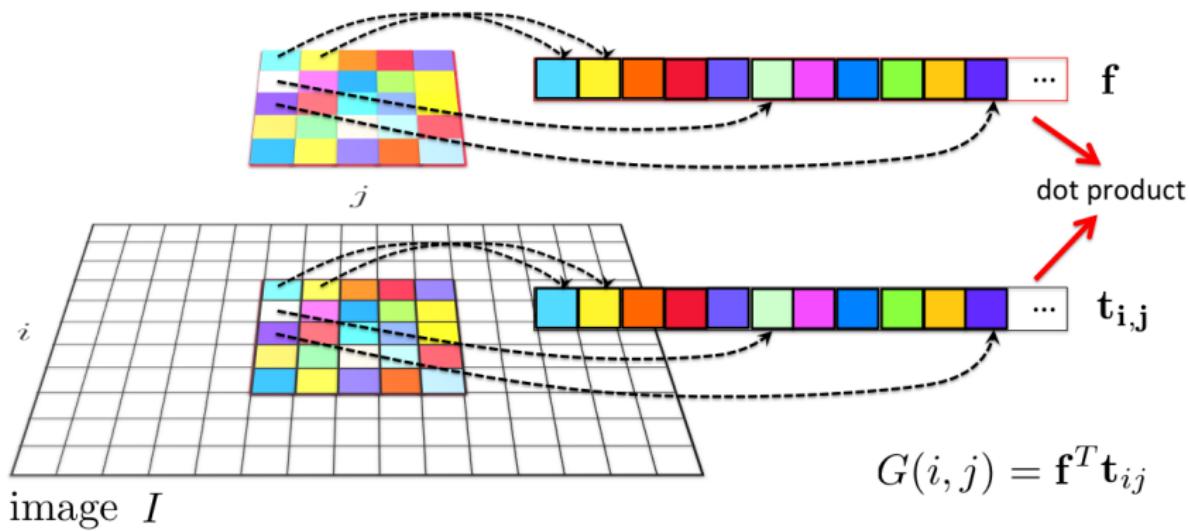
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- Can we write that in a more compact form (with vectors)?
- Define  $\mathbf{f} = F(:)$ ,  $T_{ij} = I(i - k : i + k, j - k : j + k)$ , and  $\mathbf{t}_{ij} = T_{ij}(::)$

$$G(i, j) = \mathbf{f} \cdot \mathbf{t}_{ij}$$

where  $\cdot$  is a dot product

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- Homework:** Can we write full correlation  $G = F \otimes I$  in matrix form?

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- Finding Waldo: How could we ensure to get the best “score” (e.g. 1) for an image crop that looks exactly like our filter?
- Normalized cross-correlation:**

$$G(i, j) = \frac{\mathbf{f}^T \mathbf{t}_{ij}}{\|\mathbf{f}\| \|\mathbf{t}_{ij}\|}$$

# Back to Waldo

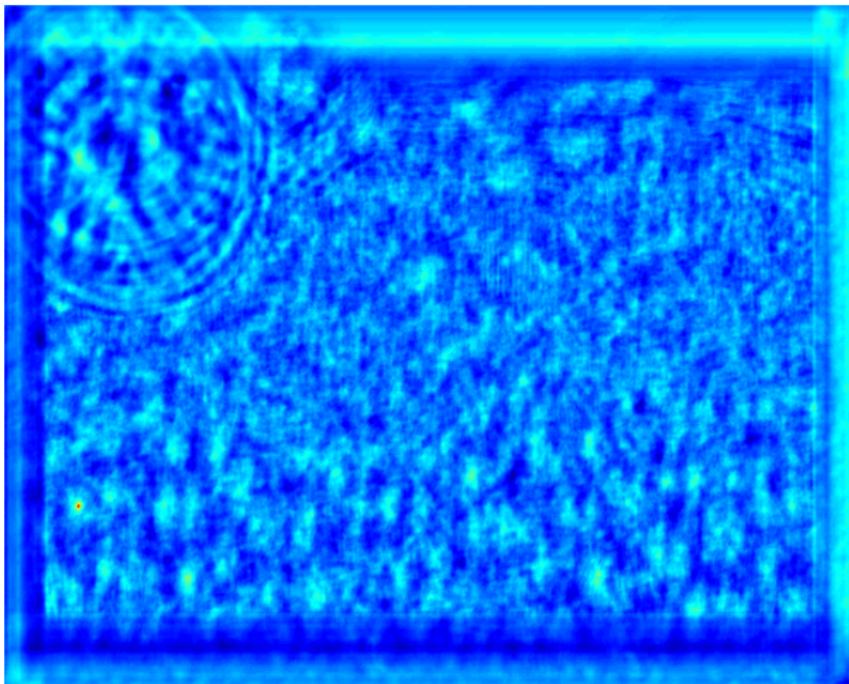


image  $I$



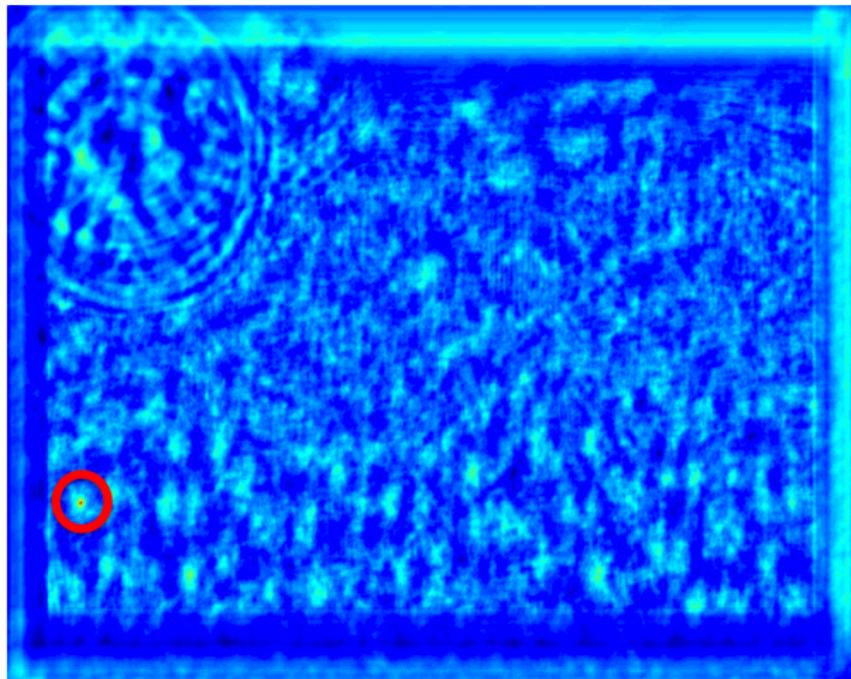
filter  $F$

## Back to Waldo



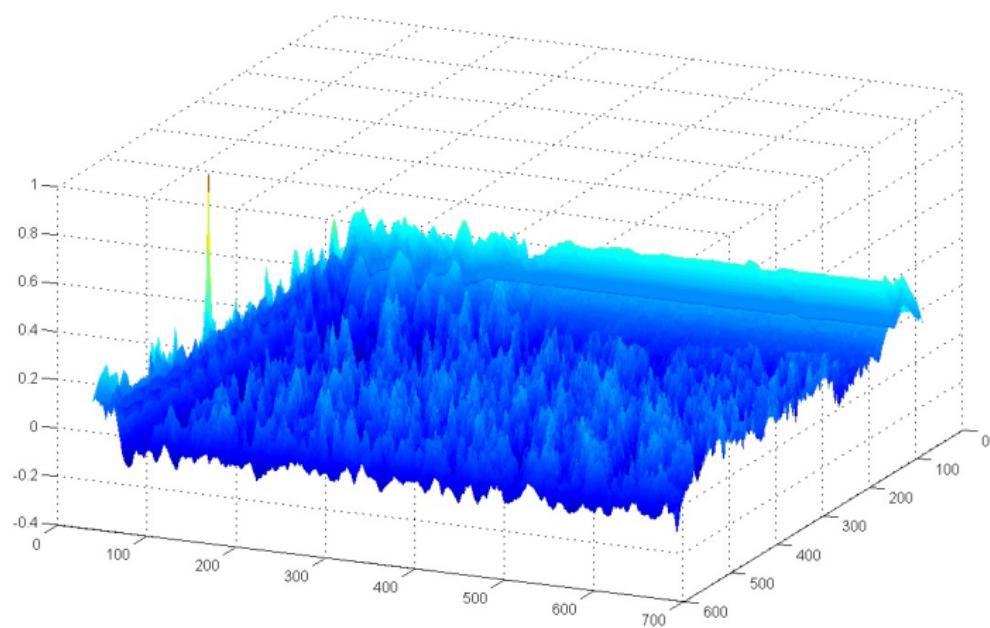
- Result of normalized cross-correlation

## Back to Waldo



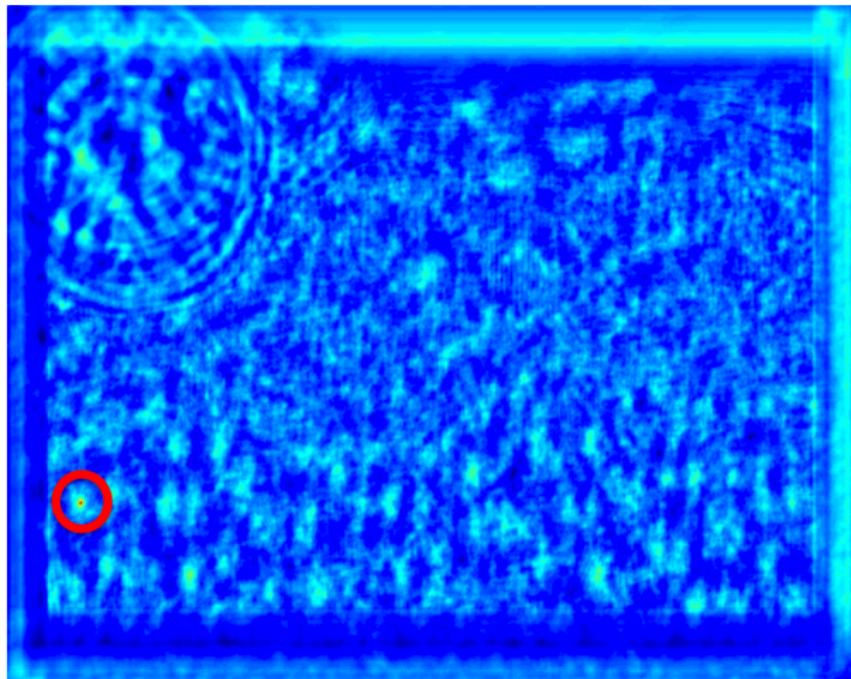
- Find the highest peak

# Back to Waldo



- Find the highest peak

## Back to Waldo



- Find the highest peak

# Back to Waldo



And put a bounding box (rectangle the size of the template) at the point!

# Back to Waldo



- **Homework:** Do it yourself! Code on class webpage. Don't cheat ;)
- We covered till here on Tuesday, Sep 12th 2017

# Example of Correlation

- What is the result of filtering the impulse signal (image)  $I$  with the arbitrary filter  $F$ ?

0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0
0	0	0	0	0	0	0	0



a	b	c
d	e	f
g	h	i

$$F(i, j)$$

$$I(i, j)$$


$$G(i, j)$$

[Source: K. Grauman]

# Convolution

- **Convolution** operator

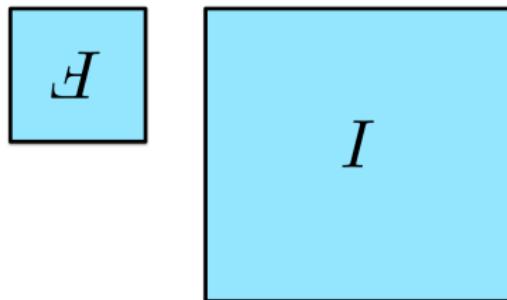
$$G(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k F(u, v) \cdot I(i - u, j - v)$$

# Convolution

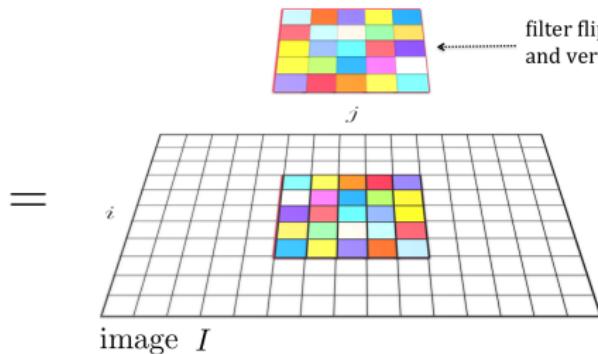
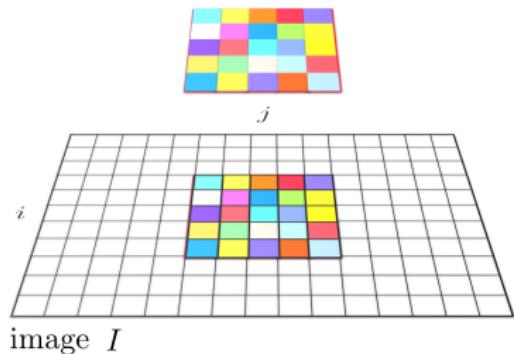
- **Convolution** operator

$$G(i, j) = \sum_{u=-k}^k \sum_{v=-k}^k F(u, v) \cdot I(i - u, j - v)$$

- **Equivalent** to flipping the filter in both dimensions (bottom to top, right to left) and apply correlation.



# Correlation vs Convolution



# Correlation vs Convolution

- For a Gaussian or box filter, how will the outputs  $F * I$  and  $F \otimes I$  differ?

# Correlation vs Convolution

- For a Gaussian or box filter, how will the outputs  $F * I$  and  $F \otimes I$  differ?
- How will the outputs differ for:

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

# “Optical” Convolution

- Camera Shake



Figure: Fergus, et al., SIGGRAPH 2006

- Blur in out-of-focus regions of an image.



Figure: Bokeh: <http://lullaby.homepage.dk/diy-camera/bokeh.html>

Click for more info

[Source: N. Snavely]

# Properties of Convolution

Commutative :  $f * g = g * f$

Associative :  $f * (g * h) = (f * g) * h$

Distributive :  $f * (g + h) = f * g + f * h$

Assoc. with scalar multiplier :  $\lambda \cdot (f * g) = (\lambda \cdot f) * g$

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- **Homework:** Why is this good news?
- **Hint:** Think of complexity of convolution and Fourier Transform
- What if we wanted to undo the result of convolution?

## Separable Filters: Speed-up Trick!

- The process of performing a convolution requires  $K^2$  operations per pixel, where  $K$  is the size (width or height) of the convolution filter.

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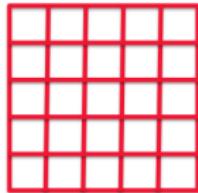
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- And it is the outer product of two filters:

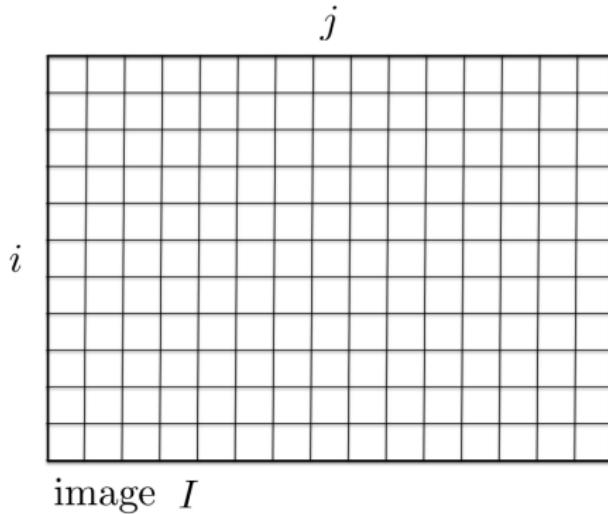
$$\mathbf{F} = \mathbf{v} \mathbf{h}^T$$

[Source: R. Urtasun]

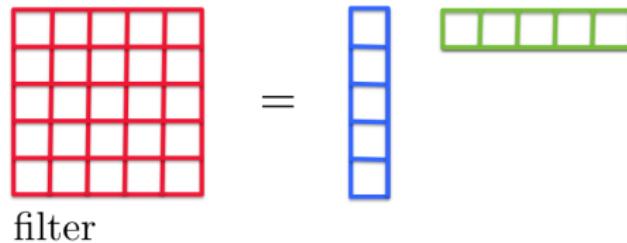
# How it Works



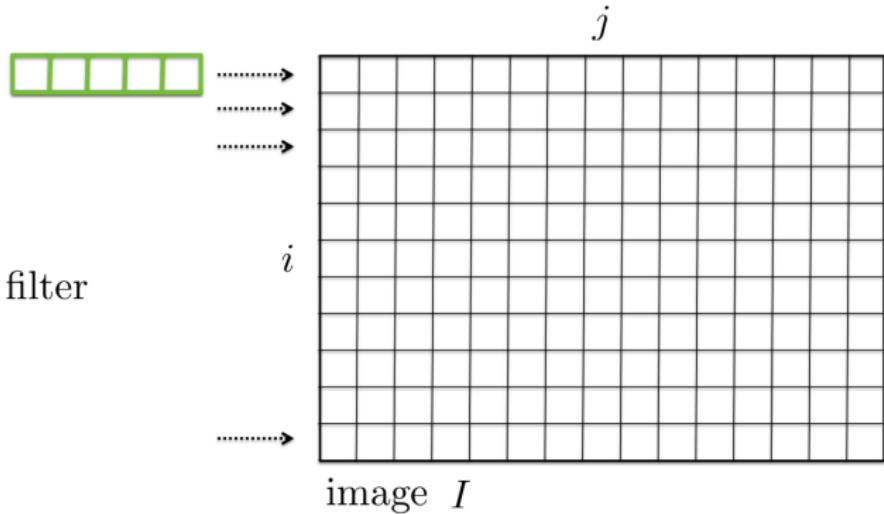
filter



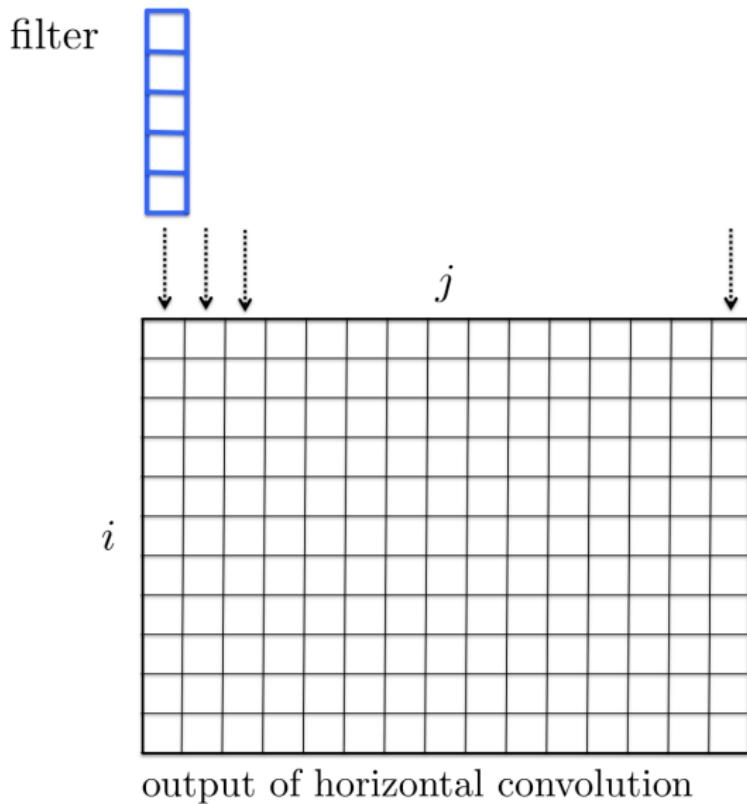
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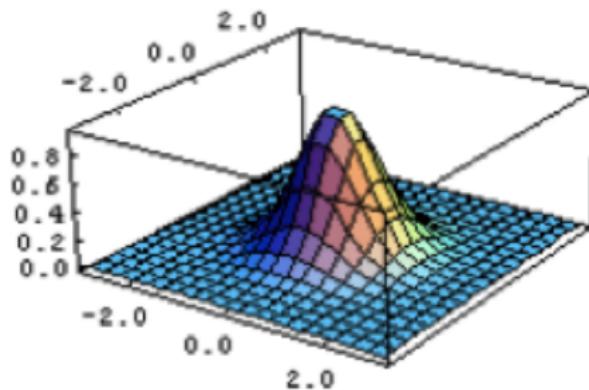
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## Separable Filters: Gaussian filters

- One famous separable filter we already know:

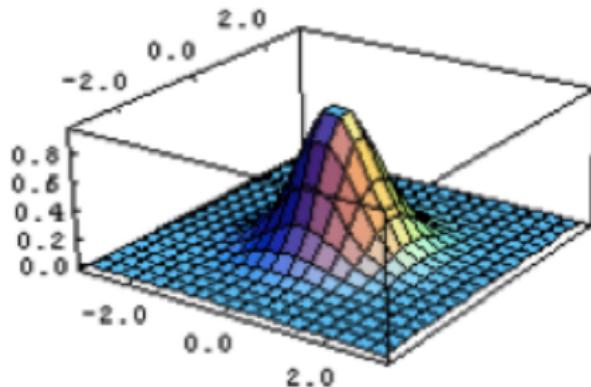
$$\text{Gaussian} : f(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{\sigma^2}}$$



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$$\text{Gaussian} : f(x, y) = \frac{1}{2\pi\sigma^2} e^{-\frac{x^2+y^2}{\sigma^2}}$$
$$= \left( \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{x^2}{\sigma^2}} \right) \cdot \left( \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{\sigma^2}} \right)$$



# Let's play a game...

Is this separable? If yes, what's the separable version?

$$\frac{1}{K^2} \begin{array}{|c|c|c|c|} \hline 1 & 1 & \cdots & 1 \\ \hline 1 & 1 & \cdots & 1 \\ \hline \vdots & \vdots & 1 & \vdots \\ \hline 1 & 1 & \cdots & 1 \\ \hline \end{array}$$

[Source: R. Urtasun]

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What does this filter do?

[Source: R. Urtasun]

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$\frac{1}{16}$	1	2	1
	2	4	2
	1	2	1

[Source: R. Urtasun]

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[Source: R. Urtasun]

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$\frac{1}{8}$	-1	0	1
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	-1	0	1

[Source: R. Urtasun]

# Let's play a game...

Is this separable? If yes, what's the separable version?

$$\frac{1}{8} \begin{bmatrix} -1 & 0 & 1 \\ -2 & 0 & 2 \\ -1 & 0 & 1 \end{bmatrix}$$
$$\frac{1}{2} \begin{bmatrix} -1 & 0 & 1 \end{bmatrix}$$

What does this filter do?

[Source: R. Urtasun]

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- Matlab:  $[\mathbf{U}, \mathbf{S}, \mathbf{V}] = \text{SVD}(\mathbf{F})$ ;
- $\sqrt{\sigma_1} \mathbf{u}_1$  and  $\sqrt{\sigma_1} \mathbf{v}_1^T$  are the vertical and horizontal filter.

[Source: R. Urtasun]

# Summary – Stuff You Should Know

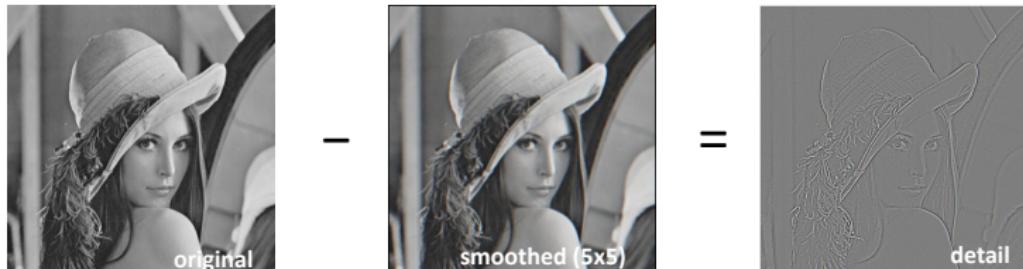
- **Correlation:** Slide a filter across image and compare (via dot product)
- **Convolution:** Flip the filter to the right and down and do correlation
- **Smooth** image with a Gaussian kernel: bigger  $\sigma$  means more blurring
- **Some** filters (like Gaussian) are **separable**: you can filter faster. First apply 1D convolution to each row, followed by another 1D conv. to each column

## Matlab functions:

- IMFILTER: can do both correlation and convolution
- CORR2, FILTER2: correlation, NORMXCORR2 normalized correlation
- CONV2: does convolution
- FSPECIAL: creates special filters including a Gaussian

# Edges

- What does blurring take away?



[Source: S. Lazebnik]

Next time:  
Edge Detection